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A REVIEW ON DESIGN METHODS FOR COMPLIANT MECHANISMS

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Abstract. This paper gives a brief overview and comparison of the methods applied in the design of compliant mechanisms. Since the first research works on the subject appeared in the 1980s, several methods have being conceived to analyze and design these mechanisms that gain part of their motion from the deflection of flexible members rather than from movable joints only. The scope and limitations of the most widely used design tools in the field; pseudo-rigid model based methods, optimization based methods), and a novel inverse design method are investigated and discussed.

1 INTRODUCTION

A traditional *rigid-body mechanism* consists of rigid links connected at movable joints, and its motion is composed of rigid-body translations and/or rotations. Nowadays, many mechanisms are designed to derive some mobility by elastic deformation in one or more elements, that is, they gain at least some of their mobility from the deflection of flexible members rather than from movable joints only. This latter group is widely known as *compliant mechanisms*. According to how the flexibility is distributed in the system, a compliant mechanism can be classified in two main categories: mechanisms with *distributed compliance*, and mechanisms with *concentrated compliance*.

Systems with *concentrated compliance* behave like classic *rigid link mechanisms*, where kinematic joints are replaced with flexible hinges, and in consequence methods conceived to design rigid body mechanisms can be modified and applied successfully in this case. Design methods for mechanisms with concentrated compliance design had its genesis in the works of Ashok Midha in the 1980s. With his co-workers he developed a tool to classify and design mechanisms with concentrated compliance (Midha et al. (1994)). Later, Howell and Midha (1994) introduced the pseudo-rigid model concept, where flexible links are modeled as rigid links connected by kinematic joints and torsional springs, and this tool allows to design compliant mechanisms with methods conceived to design rigid mechanisms. Howell (2001). Murphy et al. (1996) developed a method based in graph theory to design compliant mechanisms, used since the 1960 decade in design rigid mechanisms. This methods allowed to generate several different topologies starting from an initial mechanism by using a systematic enumeration process, creating atlases of mechanisms (an atlas is a topological design space constituted only by connected topologies in non-isomorphic ways (Pucheta and Cardona, 2010)). Pucheta (2008) introduced a synthesis tool to conceive plane rigid and/or compliant mechanisms, using graph theory to generate different topologies and the pseudo-rigid method to analyze each one of them.

Methodologies to design mechanisms with *distributed compliance* appeared in the middle of the 1990s. In this case the mechanism is treated as a continuum flexible structure, and Continuum Mechanics design methods are used instead of rigid body kinematics. Ananthasuresh (1994) pioneered the use of structural optimization applied to the design of compliant mechanisms with distributed compliance, by adapting the homogenization method and using the displacement of one point in the mechanism as the objective function. Alternative structural optimization procedures seek different objective functions, like the minimization of the mechanism's deformation energy Frecker et al. (1997), or the maximization of the energy efficiency Hetrick and Viota (1999). A more recent technique applied to mechanisms with distributed compliance was introduced by Lu and Kota (2003) and Lu and Kota (2005), where a load-path methodology and genetic algorithms are used to design compliant mechanisms with shape morphing starting from a domain discretized by an exhaustive set of truss or beam elements.

In parallel and in the same period of time, research and development of a new method to design structures, mechanisms and machine parts began. These new methods allows the designer to determine the initial shape of a piece such that it attains the given design shape under the effect of service loads. They are as formally known as an *inverse design problem* Beck and Woodbury (1998), or simply as an *inverse problem*. Despite the importance of direct methods, inverse methods constitute a very useful tool that allows engineers to conceive designs in less time and at much lower costs than the ones involved in traditional experimental and direct computational design, and avoid the trial and error approach used in the design process. Finite element models for the inverse design of two- and three-dimensional isotropic elastic continuum bodies subjected to large deformations have been proposed by Govindjee and Mihalic (1996), Govindjee and Mihalic (1998) and Yamada (1997) for isotropic behavior. More recently, Lu *et al* Lu et al. (2007) and Fachinotti *et al* Fachinotti et al. (2008) developed three-dimensional models for the inverse design of orthotropic hyperelastic solids. Also, inverse finite element was applied to the design of shells Zhou and Lu (2008).

We present here a *new method to design compliant mechanisms based in an inverse finite element beam model*, since most flexible links and flexible hinges are prismatic shaped and can be modeled as beam-type elements (simplificative hypothesis). It consist in computing the undeformed (reference) configuration knowing the deformed shape of the body and the loads applied. This method was presented in Albanesi et al. (2010) as an extension of our previous work in inverse design methods (Fachinotti et al., 2008). It is a novel and original method in the field of compliant mechanisms, as there is no background of inverse finite elements methods among the procedures used to design compliant systems.

1.1 Mechanism Synthesis

The most widely known task in mechanism design is *analysis*, and its used to determine the characteristic motion of the mechanism. It's important to recall that to carry out this task a fully defined design is needed. On the other hand, the essence of mechanisms *synthesis* is to find the mechanisms for a given motion (kinematic synthesis), and deals with the systematic design of mechanisms for a specified performance. A given design problem typically has many different solutions, and therefore it involves iterations between kinematic synthesis and analysis.

There are three customary tasks for kinematic synthesis: *path generation*, *rigid-body guidance* and *function generation*. In path generation, a point of the mechanism is required to travel along a specified path or preset points. In the case of rigid-body guidance (also called motion generation) the position of one or more elements are prescribed, such that a rigid body is moved through a specified motion. When the guidance comprises the movement a flexible link rather than a rigid body, it implies the guidance through a sequence of discrete prescribed *precision shapes* in addition to the precision points in rigid-body motion, and this is known as *compliant-segment motion generation* (Saggere and Kota, 2001), (Albanesi et al., 2007). Function generation is the correlation of the input and output links of the mechanism.

The major categories of synthesis include *type*, *number* and *dimensional* synthesis. In type synthesis, the type of mechanism best suited to solve the problem is sought (e.g. linkages, gears, cams, etc). Number synthesis, which may be considered a subset of the latter (type synthesis), involves the determination of the number of links and degrees of freedom a mechanisms should have to perform the task. Dimensional synthesis is carried on after a suitable topology has been developed through a previous synthesis procedure (e.g. type synthesis), and consists in the determination of the mechanism significant geometry to accomplish a specified task and performance (e.g. link length, area, angles, ratios, etc).

2 THE PSEUDO-RIGID BODY MODEL

The pseudo-rigid body model (PRBM) (Howell and Midha, 1994), (Howell and Midha, 1996), (Howell, 2001) is used to model the deflection of flexible members using rigid body components that have equivalent force-deflection characteristics. Rigid link mechanisms theory may then be used to analyze the compliant mechanism. Different types of compliant segments require different pseudo-rigid models that predicts the deflection path and force-deflection relationship of a flexible segment. Figure 1 depicts the pseudo-rigid body model of a large-deflection beam, in which it has been assumed that the almost circular path can be accurately modeled by two rigid links joined at a pivot along the beam (Howell, 2001). This model opened the wealth of information available in rigid body mechanisms synthesis to be used in compliant mechanisms design. The approach is useful for designing mechanisms to perform a traditional task of kinematic synthesis path following, function generation and rigid-body guidance without concern for the energy storage in the flexible members. This flexure based compliant mechanisms can be divided in two main categories: planar (two dimensional) and spatial (three dimensional), depending on the design and overall motion of the mechanism.

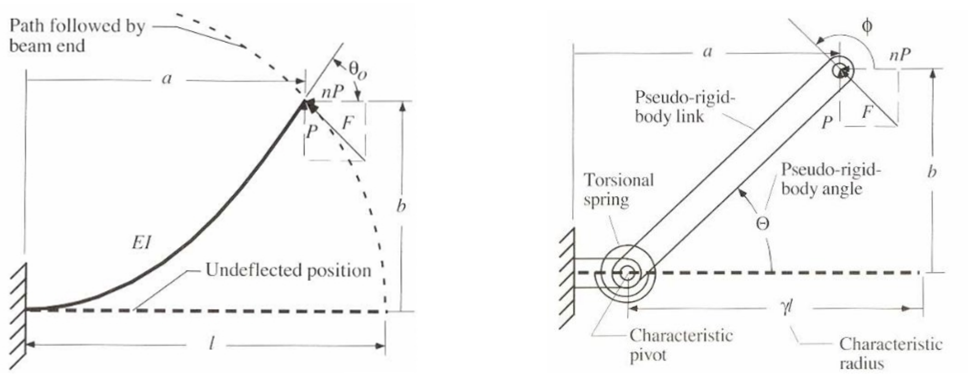

Figure 1: Large-deflection beam (left) and its pseudo-rigid body model (right) (Howell, 2001)

The inverse design situation, called Rigid-Body Replacement Synthesis, is also very easy to achieve by identifying rigid-body components as the PRBM of flexible members to synthesize (Pucheta and Cardona, 2010). In the early design stages, PSBM may serve as a fast and efficient method of evaluating many different trial designs to meet the specific design objectives. If a designer relies solely on prototyping or full numerical analysis, an initial design must be obtained before it can be modeled or built. The pseudo-rigid-body model, on the other hand, may be used to obtain a preliminary design which may then be optimized. Once a design is obtained such that it meets the specified design objectives, it may be further refined using methods such as nonlinear finite element analysis, and it may then be prototyped and tested. In addition, approximate dynamical information can be obtained with this model.

PRBM is particularly useful in combination with other methods, such as mechanism enumeration in the form of atlases (Murphy et al., 1996), and the automated exploration of atlases of compliant mechanisms proposed addressed by Pucheta and Cardona (2007), Pucheta (2008) for rigid and compliant planar linkage mechanisms.

3 OPTIMIZATION METHODS

Fully flexible mechanisms can be viewed as flexible continua and treated as such in their analysis and synthesis. In such case, Continuum Mechanics-based methods are used and structural optimization techniques (continuum optimization) are used to design mechanisms. This is the case of mechanisms with *distributed compliance*, where a large portion of a structure deforms when it's loaded. Most of classical optimization algorithms solve the same mathematical problem: minimize an objective function $f(x)$ (if minimizing $f(x)$ improves the design), under a set of restrictions c, by varying the value of one or more design variables x_j between specified bounds (Vanderplaats, 1984)

minimize

$$f(x)$$

subject to

$$g_j(x) \leq 0 \; ; j = 1, 2, ..., m$$

$$h_k(x) = 0 \; ; k = 1, 2, ..., m$$

$$\underline{c_l} \leq c_l(x) \leq \overline{c_l} \; ; l = 1, 2, ..., m$$

by variation of the values on their bounds

$$\underline{x_i} \leq x_i \leq \overline{x_i} \; ; i = 1, 2, ..., n$$

The structural optimization of a continuum mechanism can be separated in thee main levels: *topology*, *shape* and *size* optimization (Howell, 2001). The more general level is topology optimization, where material connectivity among various portions of the compliant mechanism (inputs, outputs and connections between them) are determined. Given a design domain where the mechanisms has to fit, the algorithms considers all possible ways of distributing the material in the domain, which brings a multitude of possible design in the hand of a designer without the need of any commitments or initial topological proposals. This stage of structural optimization in mechanisms with distributed compliance is the equivalent to the type synthesis stage in mechanisms with concentrated compliance. Shape optimization deals with the shape that individual segments of the mechanisms must acquire, once a topology has been established. Appropriate design variables are needed to identify the shape of compliant portions of the mechanism and many types of shapes can be included in the search space (e.g Bezier and spline interpolation curves, and the coordinate of the nodes of a FEM mesh are ways to vary the shape in the procedure). This stage may need remeshing of the continuum if the mesh is highly distorted by the shape changes along the optimization. Sensitivity analysis may also be needed to evaluate the influence of shape changes on the objective and constraint functions. At the lowest level, once the topology and shape of the mechanism are defined, the last step is size optimization where the design variables are the cross-section, thickness dimension of beams or truss-like segments, thickness of plates and so on.

In most of the topology optimization methods the material is iteratively removed, either by reducing the density of an element or by eliminating the element completely (Pucheta and Cardona, 2010). These methods are called *continuous material density parametrization* and *ground structure parametrization* respectively, Howell (2001).

Ground structure parametrization is a discrete synthesis approach in which the mechanism is represented by a network of truss or beam elements (known as the ground structure (Bendsoe, 1995), depicted in Figure 2, and in which the topological synthesis is first solved using discrete algorithms (Lu and Kota, 2003) and (Lu and Kota, 2005). The method varies the cross section of each individual element, and when the area of the cross-section of any element become smaller than a defined tolerance then that element is removed. As the procedure advances towards convergence, some elements will be removed for the ground structure, and the remaining elements will define the topology and shape of the mechanism. Then, several continuum sizing methods (continuum optimization) are applied for each topology. This procedure gives not only the optimal topology but also the optimal size and shape. However, when the mesh is refined the optimization problem becomes large, and the manner in which the exhaustive set of elements is defined determines the type of solution we obtain.

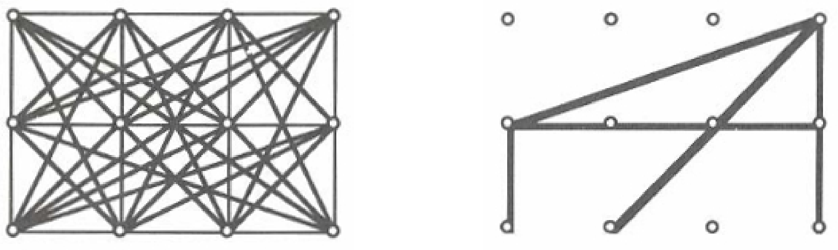

Figure 2: Ground structure parametrization: ground structure (left) and possible topology after removal of some elements (right) (Howell, 2001)

On the other hand, continuous material density parametrization is a more general approach in which the algorithm varies the material density at each point in the design domain. This method uses a rectangular grid mesh and defines an artificial material density function that is limited in two bounds, and it derives from a more rigorous method known as the homogenization method (Bendsoe, 1995). If at a certain iteration the density function reaches a very small value, i.e the lower bound, it implies that the element is made of very soft (artificial) material, and makes it absent from the structure. When the function takes a value of the upper bound it becomes the solid portion of the optimal mechanism. If the value of function is in between the bounds, then it becomes the transition region. In a grey-scale area (Figure 3), *black* represents the solid elements, *white* the void elements and the transition region is the *gray areas*. Gray areas are the main drawbacks of continuum methods since manufacturing methods with intermediate artificial material density are expensive (Pucheta and Cardona, 2010). To overcome this inconvenient, algorithms have been developed to push the design variables to either one of the limits.

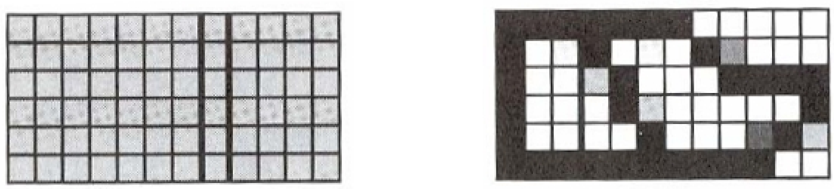

Figure 3: Continuous material density parametrization: (Howell, 2001)

3.1 Optimization algorithms

In the context of optimization of structures and compliant mechanisms local approximations techniques are widely used: *sequential quadratic programming* (SQP), *generalized method of moving asymptotes* (GMMA) and *globally convergent method for moving asymptotes*, (GCM) among others (Albanesi et al., 2006).

- SQP is a *feasible direction method* in which the first step is to generate a search direction by solving a sub-problem with quadratic objective functions and linear restrictions, and the algorithm tries to improve the design in such direction (references). The search direction and the objective function are both expanded using Lagrange multipliers, and an exterior penalty is used to free the one-dimensional search from restrictions (Vanderplaats, 1984), (Patnaik et al., 1996), (Schittkowski, 2005), (Schittkowski and Zillober, 2005).

- GMMA is an asymptotic optimization algorithm in which the objective functions and the restrictions can be treated separately because each function has it own moving asymptote. By changing the asymptotic parameters a new family of convex approximation is generated, and this property adds robustness to the method. As the sign of the first derivative remains unaltered trough the optimization (even with the variation of design variables), this algorithm has a monotonic behavior, (Remouchamps and Radovcic, 2002).
- GCM derives from GMMA. Its a second order method and as such it needs certain information from a previous iteration (the fist iteration is always of order one). Unlike GMMA, the approximation of GCM is a not monotonic function, and it's suitable for problems where the objective function has a non-linear response to the variation of the design variables,(Remouchamps and Radovcic, 2002).

4 INVERSE METHODS

Finite element models for the inverse design of two- and three-dimensional isotropic elastic continuum bodies subjected to large deformations have been proposed by Govindjee and Mihalic (1996), Govindjee and Mihalic (1998) and Yamada (1997) for isotropic behavior, recently extended to orthotropic materials by Fachinotti et al. (2008). We introduce a novel design method based on an *inverse finite element beam model* Albanesi et al. (2010) as an extension of our previous work in inverse design methods (Fachinotti et al., 2008). The problem consists in computing the initial shape of the beam such that it attains the design shape under the effect of service loads. This formulation has immediate applications in fields such as compliant mechanism analysis and it is a novel and original method in this field mechanisms as there is no background of inverse finite elements methods among the procedures used to design compliant systems. To this end we have formulated the inverse finite element model of non-linear beam proposed by Cardona and Géradin (1988) and Géradin and Cardona (2000). Compliant mechanisms necessarily have low mass and very high flexibility, so large displacements behavior ought to be considered. We further assumed that beam cross-sections remain straight but the beam can undergo shear strains. For the purpose of flexible mechanism analysis and synthesis, a simplified beam theory with a linear-elastic constitutive relation is adopted. Although flexibility effects were introduced using a large displacements hypothesis with finite rotations, we assumed that the strains which resulted were small.

The non-linear beam model relies on three kinematic hypotheses: *the beam is straight when unloaded*, *beam cross sections remain plane during deformation* and *shear deformation of the neutral axis is allowed*, and it relates strain and stress measures in the deformed configuration $\mathcal{B}$ with the same measures in the undeformed configuration $\mathcal{B}_0$ of the beam, Figure 4.

The current "inverse" beam element that represents the deformed configuration $\mathcal{B}$ is a straight, mixed linear-linear finite element, and the unknowns are the trace of the neutral axis on the cross section $\boldsymbol{X}_0$ and the Cartesian rotational vector $\boldsymbol{\psi}$ (used to parametrize rotations), both in $\mathcal{B}$. After elastic deformation, the basis in $\mathcal{B}$ $\{\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{e}_3\}$ transforms to $\{\boldsymbol{E}_1,\boldsymbol{E}_2,\boldsymbol{E}_3\}$ in $\mathcal{B}_0$ according to the orthogonal transformation

$$\boldsymbol{e}_i = \boldsymbol{R}\boldsymbol{E}_i \qquad i = 1, 2, 3 \tag{1}$$

where the operator $\boldsymbol{R}$ is formally a linear operator on the abstract three dimensional space and represents the physical rotation between the two basis

$$\boldsymbol{R}(\boldsymbol{\psi}) = \boldsymbol{I} + \frac{\sin\psi}{\psi}\widetilde{\boldsymbol{\psi}} + \frac{1-\cos\psi}{\psi^2}\widetilde{\boldsymbol{\psi}}\widetilde{\boldsymbol{\psi}} \tag{2}$$

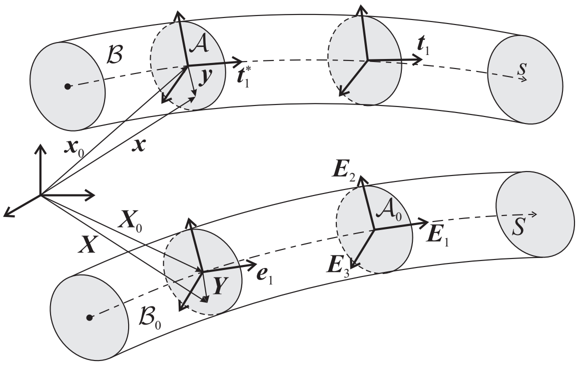

Figure 4: Description of beam kinematics

and $\widetilde{*}$ is the spin operator applied to the vector $*$. The unknowns $\boldsymbol{X}_0$ and $\boldsymbol{\psi}$ of the inverse problem in $\mathcal{B}$ are are approximated by

$$\boldsymbol{X}_0(s) = \varphi_1(s)\boldsymbol{X}_0^1 + \varphi_2(s)\boldsymbol{X}_0^2 \tag{3}$$

$$\boldsymbol{\psi}(s) = \varphi_1(s)\boldsymbol{\psi}^1 + \varphi_2(s)\boldsymbol{\psi}^2 \tag{4}$$

where φ_i is the linear shape function associated to node i, being $i = 1, 2$. After discretising the equilibrium equations following the standard Galerkin finite element method (Zienkiewicz and Taylor, 2000) the non-linear system of algebraic equations results in

$$\boldsymbol{F}_{int} - \boldsymbol{F}_{ext} = \boldsymbol{0} \tag{5}$$

where $\boldsymbol{F}_{int}$ and $\boldsymbol{F}_{ext}$ are the vectors of internal and external forces respectively, and it's solved using the Newton-Raphson method (Zienkiewicz and Taylor, 2000). The rate of convergence of the inverse beam model is quadratic (order 2), as it should be in any method using the Newton-Raphson algorithm. The error of the inverse beam model compared to an exact reference solution was computed for the nodes position $\|\boldsymbol{X}_0 - \boldsymbol{X}_0{}^{ref}\|$ and for the rotation angle $\|\boldsymbol{\psi} - \boldsymbol{\psi}^{ref}\|$, measured at different element sizes corresponding to 1,2,4,8,12,16,20 and 24 elements. The exact reference solution is computed through an elliptic integral solution (which may be used under the assumption that the beam is linearly elastic, inextensible, rigid in shear and of constant cross section). First, the elliptic integral algorithm is used to determine the beam tip position $\boldsymbol{x}_{tip}$ and $\boldsymbol{y}_{tip}$ (Howell, 2001). Then, $\boldsymbol{x}_{tip}$ and $\boldsymbol{y}_{tip}$ are used as boundary conditions to numerically solve the elastica differential equation. This two steps allows us to determine the node coordinates of the entire beam used as a reference solution, and not only at the tip (which is the only available solution found in large-deflection beam bibliography). The beam analyzed has length $L = 2 \times 10^3$ mm, cross-section height h = 60 mm, cross-section width h = 30 mm, Young's module $E = 2.1 \times 10^5$ N/mm^2, Poisson ratio $\nu = 0$ (this value is chosen because the elliptic solution assumes the beam is rigid in shear), and the load applied is P = 1 $\times$ 10^5 N. This results is depicted in Figure 5.

4.1 Stability check and feasibility of a design: critical points

Certain points of an equilibrium path have special significance. A structure that is initially stable may lose stability as it moves to another equilibrium position. The general form of the Newton-Raphson method used to find the solution of the system (the roots of the equation

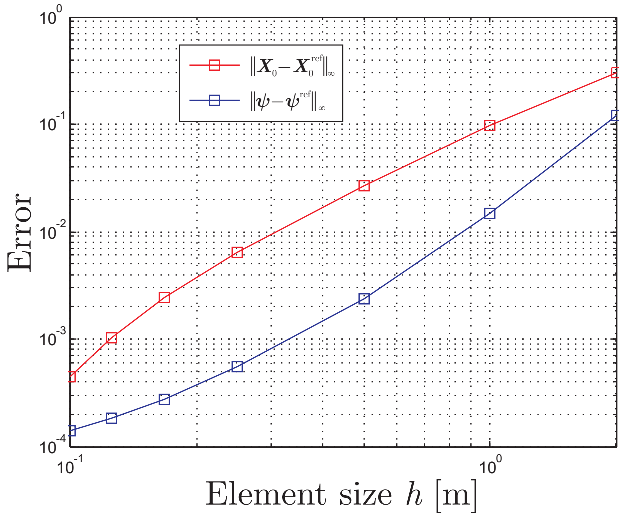

Figure 5: Convergence of the inverse beam model

system) is

$$\boldsymbol{x}_j = \boldsymbol{x}_{j-1} - \boldsymbol{K}_t(\boldsymbol{x}_{j-1})^{-1}\boldsymbol{F}(\boldsymbol{x}_{j-1}) \tag{6}$$

where $\boldsymbol{K}_t(\boldsymbol{x}_{j-1})$ is tangent stiffness matrix of the system (derivatives of the function $\boldsymbol{F}(\boldsymbol{x}_{j-1})$ with respect to the vector $\boldsymbol{x}_{j-1}$ at iteration $j-1$). Under certain conditions, that transition is associated with the occurrence of *critical points* at which $\boldsymbol{K}_t$ becomes singular. Critical points have been classified into *limit points* (at which the tangent to the equilibrium path is horizontal) and *bifurcation points* (at which two or more equilibrium paths cross). Equilibrium points that are not critical are called regular.

Stability is assessed by comparing the potential energy of actual configurations with that of the equilibrium position. If all previous states have a higher potential energy, the equilibrium is stable. If at least one state has a lower (equal) potential energy the equilibrium is unstable (neutrally stable).

Direct methods to detect critical points in the beam deflection analysis consist in searching for critical points without being concerned with tracing equilibrium paths up to those points. Two of the simplest direct evaluations procedures are the *determinant* and *spectrum* test of $\boldsymbol{K}_t$ (Allgower and Georg, 1987), (Crisfield, 2000), (Felippa, 2010). Since the tangent stiffness matrix is singular at critical points, the most intuitive procedure consists in computing the determinant of $\boldsymbol{K}_t$. This approach is generally impractical for several reasons: 1) analytical expressions for the determinant are complicated, 2) the estimation would be very expensive, and 3) the determinant is an ill-behaved function. The spectrum test consists in looking at the eigenvalues λ of the tangent matrix Kt. The set of λ_i's are the solution of the eigenproblem

$$\boldsymbol{K}_t = \lambda_i \boldsymbol{z}_i \tag{7}$$

where $\boldsymbol{z}_i$ are the eigenvectors corresponding to the eigenvalues λ_i. If $\boldsymbol{K}_t$ is real and symmetric (with real eigenvalues) we can apply the following procedure to detect critical points:

- If all $\lambda_i < 0$ the equilibrium position is strongly stable
- If all $\lambda_i \geq 0$ the equilibrium position is neutrally stable

- If some $\lambda_i > 0$ the equilibrium position is unstable

In the case of inverse analysis, the mechanisms evolves from the deformed configuration (design shape) to the undeformed configuration (manufacturing shape), such that this manufacturing shape attains the design shape under the effect of service loads. In the presence of critical points however, one cannot assure that the computed manufacturing shape will attain the design shape once loaded. For this reason, we have adopted the *lowest eigenvalue test* criterion to evaluate stability at each loadstep. If at a given loadstep the lowest eigenvalue is null or negative, then that design is classified as non-feasible.

4.2 Compliant mechanism design with the inverse finite element method

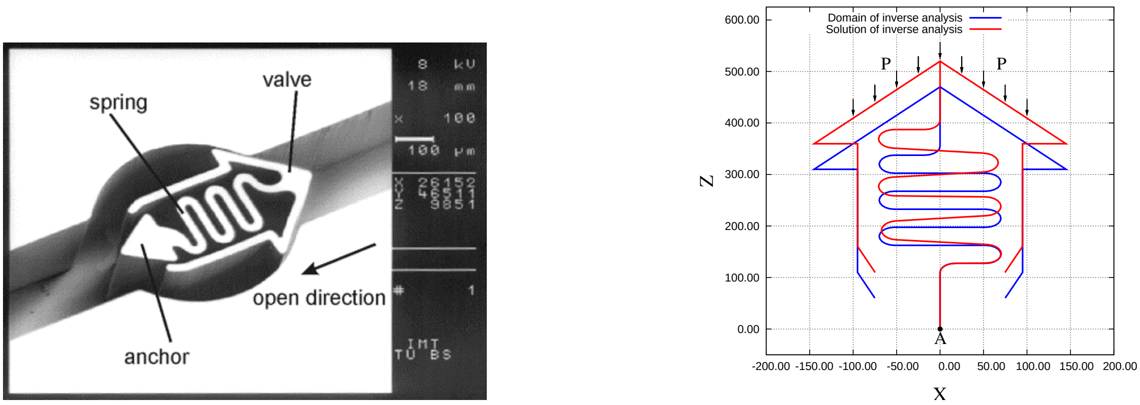

Figure 6: Compliant check valve proposed by Seidemann et al. (2001) (left) and its inverse analysis (right)

A few examples of the inverse finite element method applied to the design of compliant biomedical devices will be presented. The first is a *passive microvalves* proposed by Seidemann et al. (2001) to seal a 200 μm microfluidic channel, Figure 6. Passive microvalves (also known as a check valve) only open to forward pressure showing diode-like characteristics (Kwang and Chong, 2006). This geometry was discretized with 85 flexible beam segments with constant rectangular cross section ($h = 15$ μm high and $b = 20$ μm wide). The point labeled $\boldsymbol{A}$ in the figure is the anchor that holds the valve in position, and $\boldsymbol{P} = 1.286$KPa is the service pressure (we remark that the pressure acts only in the portion of the arrow shape valve that seals the microchannel) applied in four loadsteps. The solution was obtained in 6 iterations. At each loadstep the lowest eigenvalues were positive, so the solution of the inverse analysis is a feasible design.

The next example is the inverse analysis of a microgripper proposed by Kohl et al. (2000), depicted in Figure 7. Its a 2x3.9x0.1 mm^3 microgripper with a stress-optimized geometry, built in a shape memory alloy (SMA) conceiving a high flexibility design. Due to the geometry (two circular beams on top and a folded beam structure at the bottom) this microgripper has a distributed compliance behavior. When the microgripper is acted by a force in the beam structure at the bottom, the circular beams are deformed and the gripping jaws are closed. It was modeled with 160 beam segments with cross-section height and width $h = 0.5$ mm and $b = 0.5$ mm respectively. $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ correspond to the ground fixation points, and it's acted by a single input force $P = 160$ μN. A solution was obtained in 11 iterations and as all lowest eigenvalues were positive it consists in a feasible design.

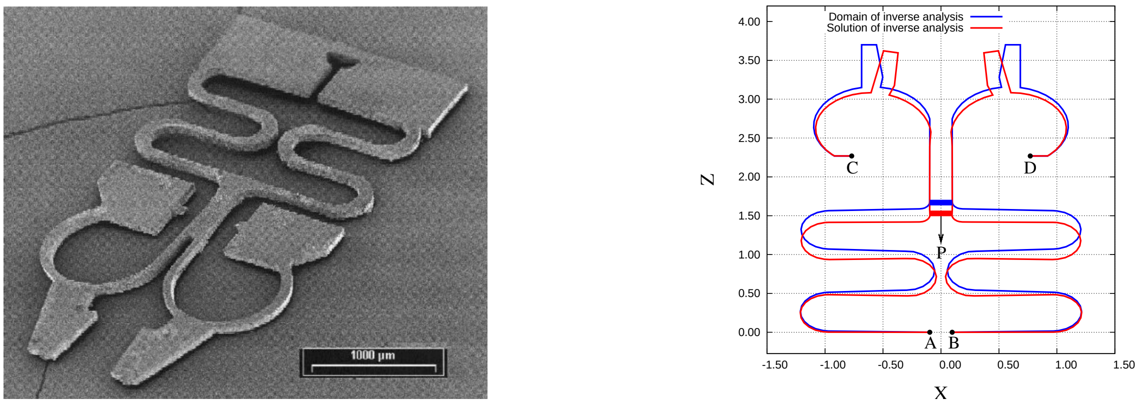

Figure 7: SMA microgipper proposed by Kohl et al. (2000) (left) and its inverse analysis (right)

4.3 Advantages and/or Disadvantages of the Inverse Model as a Design Tool

It is convenient to make some final remarks of the advantages and inconveniences found in the design process while using inverse analysis. In all the applications analyzed, the problems were reproduced with much lower computational costs using the inverse beam model. The inverse model allows to perfectly match the desired shapes of the objects, e.g. in the microgripper example leads to maximize the contact points between the gripper and the object to grab, and this is a clear advantage. The model also allows to reduce stress concentration in distributed compliance mechanisms.

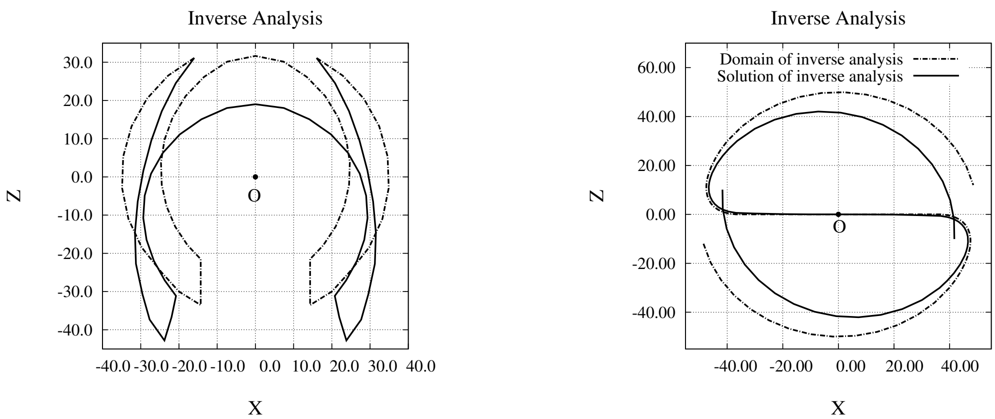

Figure 8: Non-valid results for the compliant gripper (left) and compliant clutch (right), (Albanesi et al., 2009)

However, some draw-backs were found while analyzing other applications, i.e. a compliant gripper and a compliant clutch (Albanesi et al., 2009). In some cases, intersections of beam elements appeared in undeformed configuration, even thought the inverse analysis started from a valid deformed geometry. This is depicted in the left part of Figure 8. This type of problems also lead to a trial and error process (changing the beam cross-section, or the material), until the intersection disappears. A possible workaround to this problem would be to implement a simple contact problem of the type node-to-segment with penalty (Puso and Laursen, 2004) and soft contact algorithms with friction for 3D beams (Litewka, 2007), in order to impose restrictions of the solution and eliminate unfeasible designs.

Another draw-back is that the evolution of the model, as it evolves between the deformed to the undeformed configuration is unknown. Even if the convergence rate is quadratic, this could lead to unfeasible designs, because at a certain iteration a beam element may fall outside of the desired design domain (violating design constraints). In the compliant clutch example for instance, we may encounter cases where the tip of the clutch shoes engage contact with the outer drum at an undesired engine speed, as it evolves from the closed-undeformed configuration to the deformed configuration (Figure 8 at right). The obtention of an undeformed solution inside a prescribed design space without self-crossings is the most challenging difficulty encountered when using inverse design methods.

5 CONCLUSIONS

The many advantages of compliant mechanisms compared to their rigid-body counterparts have produced a growing interest in compliant mechanism synthesis methods. A brief review of the most utilized design methods for compliant mechanisms; pseudo-rigid-body-model, optimization and a novel inverse design method was presented. The scope and properties of each is described in the context of mechanisms synthesis. Optimization algorithms and stability analysis are also discussed.

The pseudo-rigid-body-model is as approximate method to predict the deformation of flexible segments that can be easily combined with other powerful methods such as graph theory and mechanisms atlases, in order to create an automatic mechanism search and enumeration tool. The inverse analysis model permits to find the initial shape of a beam such that it attains the given design shape under the effect of service loads. The inverse analysis begins at the deformed configuration (mechanisms design requirement) and the undeformed configuration is sought (manufacturing shape of the mechanism). Structural optimization techniques are more general design methods, in which a multitude of possible designs are obtained without the need of any commitments or initial topological proposals. However, they are expensive method compared to pseudo-rigid-body-model and inverse analysis model.

At present work is being carried out to create an automatic design tool based in the inverse method in order to overcome the main drawbacks or this model, and to extend the model to other type of elements such as membrane and 3D solids.

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