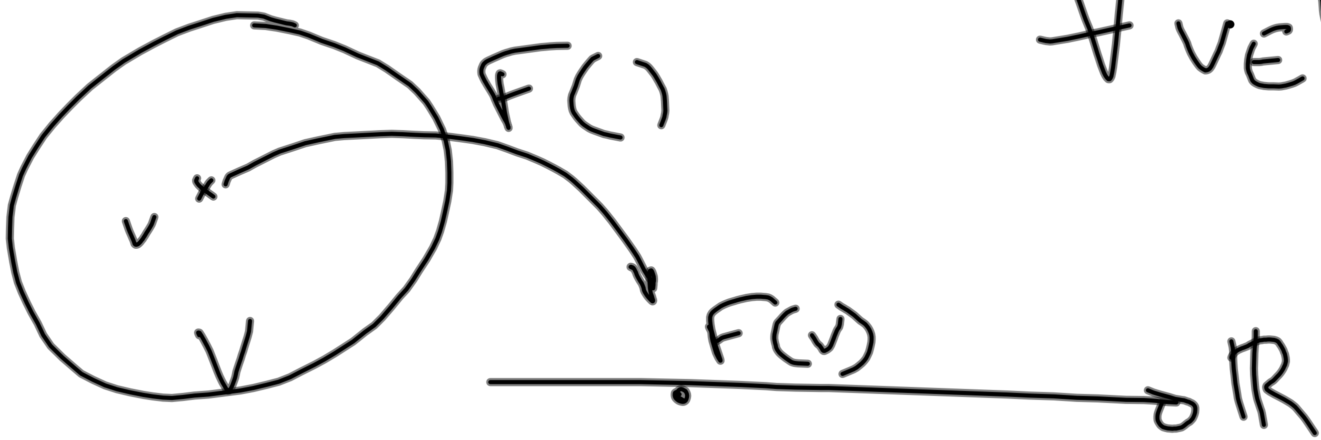
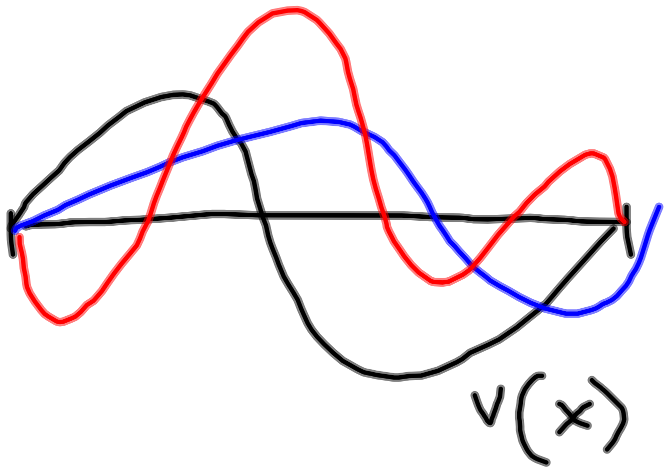
ED: $\Big\{ \Longrightarrow$ Prob Variaci

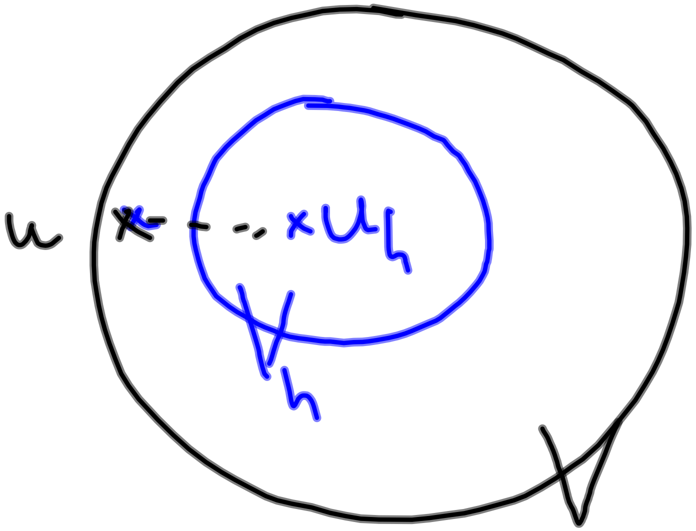
(M) $\underset{\in V}{u}$? / $F(u) \leq F(v)$ $\forall v \in V$





$u = ?$

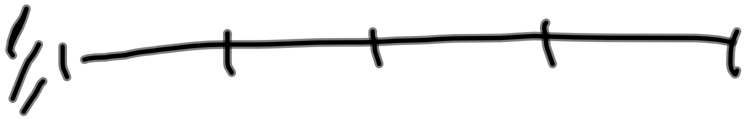
$u /$ $\underline{F(u) \text{ míni}}$

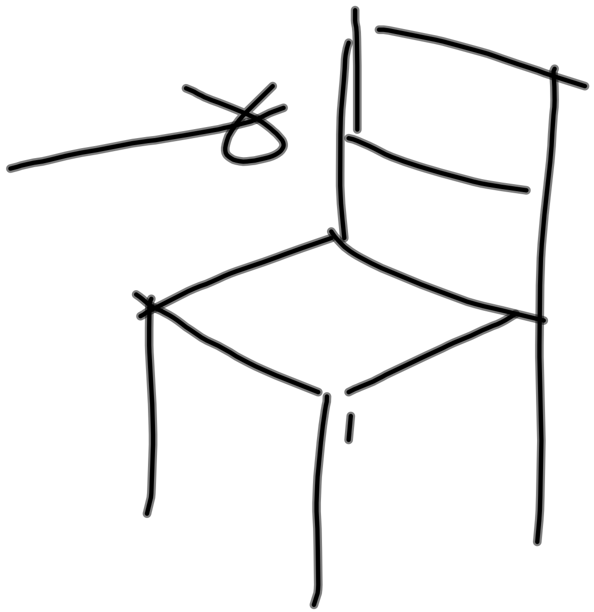


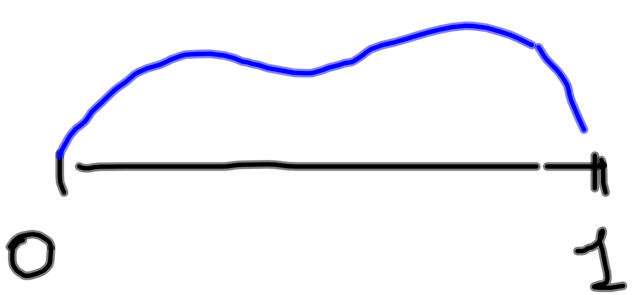
V_h dimensión finita

$F(u_h) \leq F(v)$

$\forall v \in V_h$



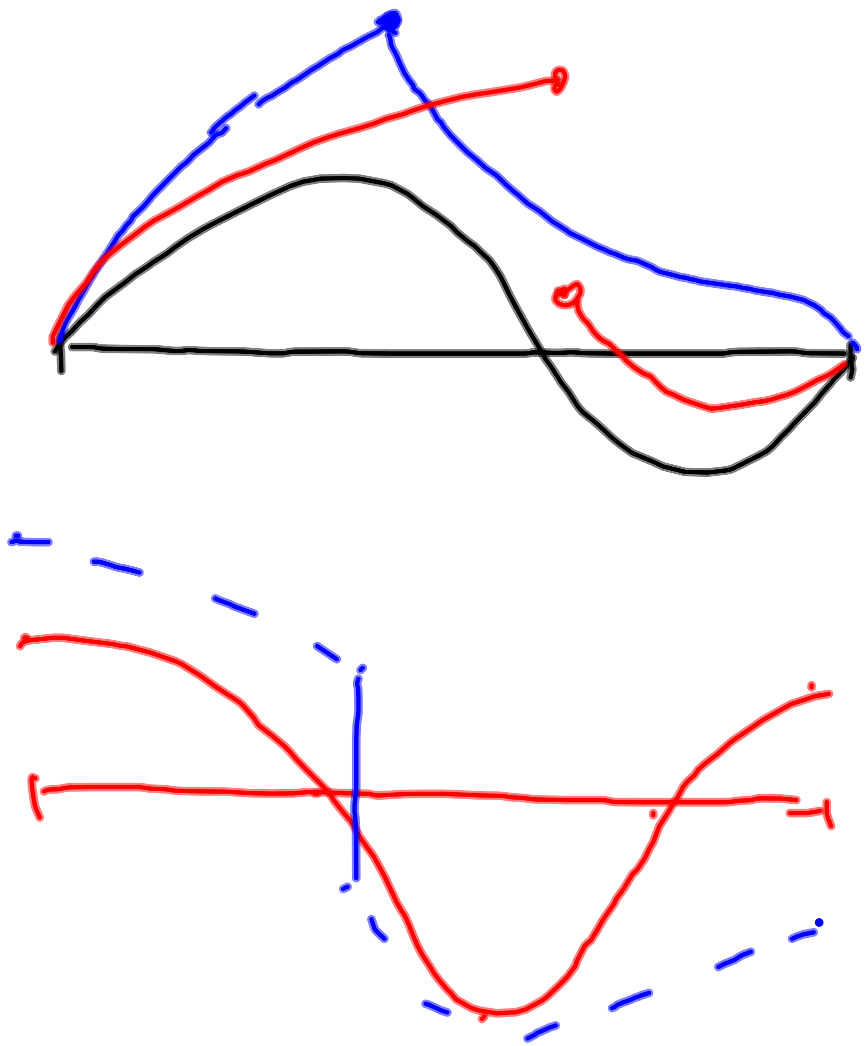




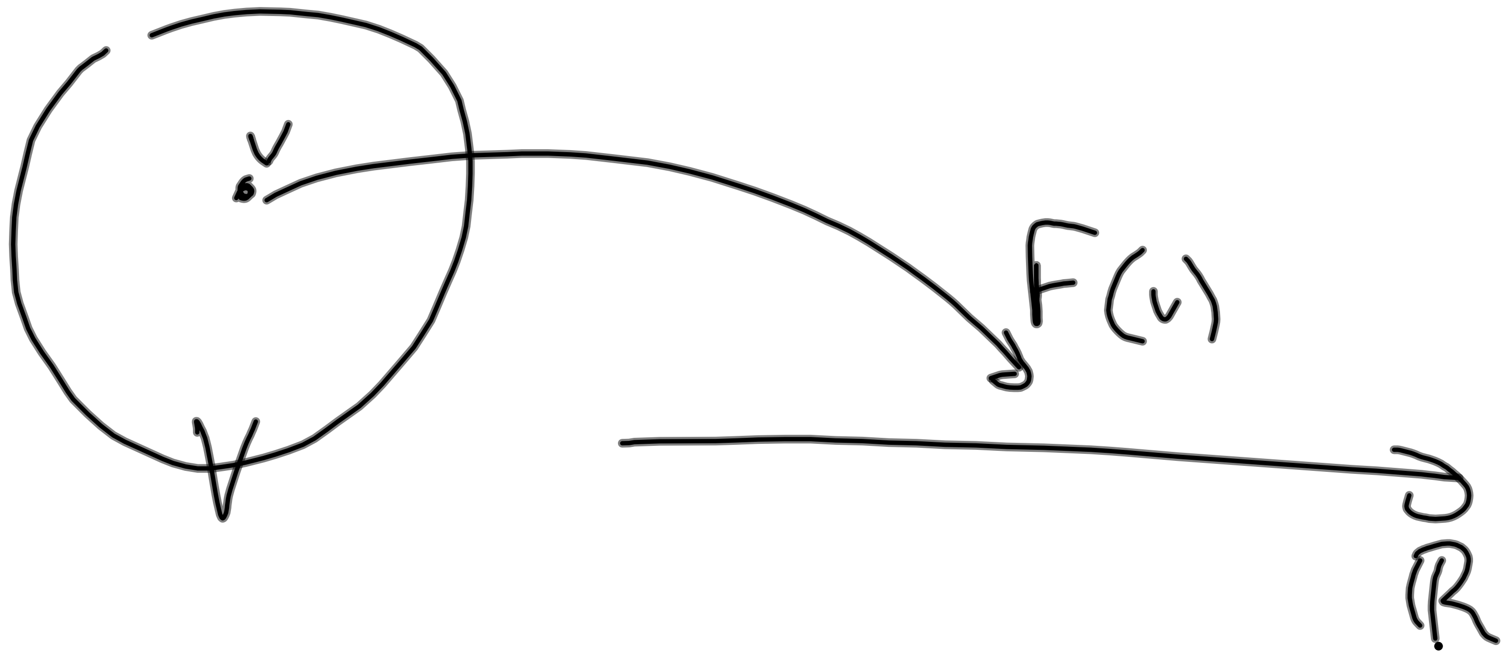
$$-\frac{d^2u}{dx^2} = f$$

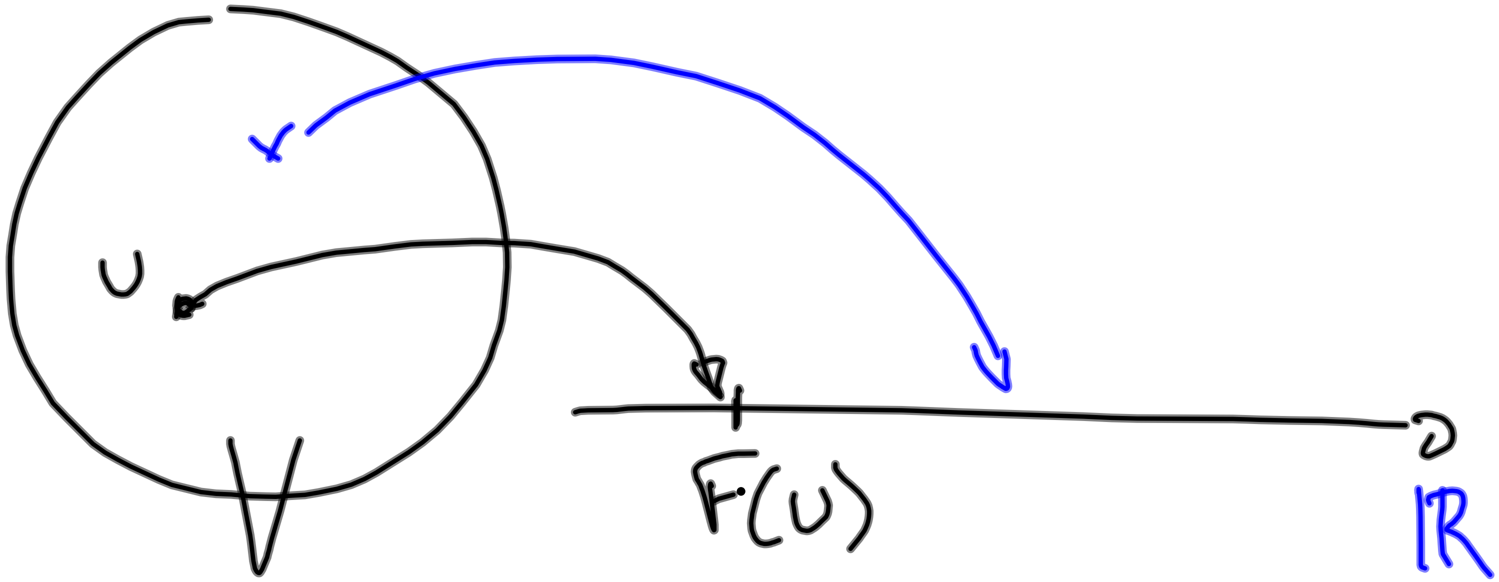
$$u(0) = u(1) = 0$$

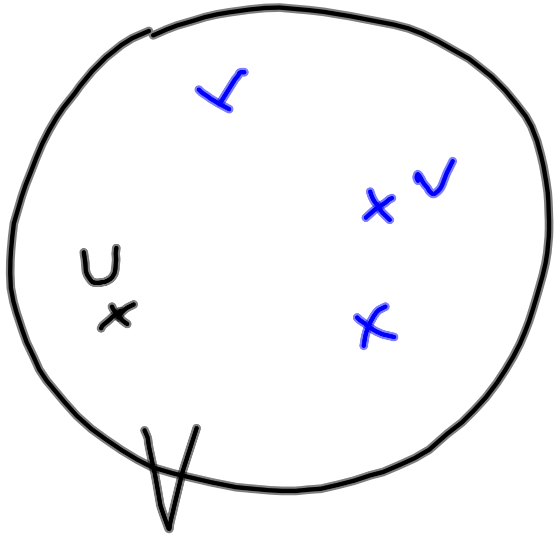

$T_x \sin\theta_x$
θ
$T_{x+dx} \times \sin\theta$



$$F(v) = \frac{1}{2}\int_0^1 \left(\frac{dv}{dx}\right)^2 dx - \int_0^1 f v \, dx$$




v
u
V
F(u)
IR



f dato

$$(u', v') = (f, v) \quad \forall v \in V$$

$$\int_0^1 \frac{du}{dx} \frac{dv}{dx} \, dx = \int f v \, dx$$

$$-(u'',v) = -\int_0^1 u''v\,dx \overset{\text{IPP}}{=} \underbrace{\cancel{-u'v\Big|_0^1}}_{-u'(1)\underbrace{v(1)}_{0}+u'(0)\underbrace{v(0)}_{0}} + \int_0^1 u'v'\,dx = (u',v')$$

$$(u', v') = (f, v) \quad \forall v \in V$$

$$w = v - u \quad \in V$$

$$(u' + w', u' + w') = \int (u' + w')(u' + w')\, dx =$$

$$= \int u' u' + \ldots$$

$$F(u+\varepsilon v) = \frac{1}{2}\left(u'+\varepsilon v',\, u'+\varepsilon v'\right) - \left(f,\, u+\varepsilon v\right)$$

$$= \frac{1}{2}(u',u') + \underbrace{\frac{1}{2}(\varepsilon v',u')}_{\underbrace{\frac{1}{2}\varepsilon(v',u') + \frac{1}{2}\varepsilon(u',v')}_{\varepsilon(u',v')}} + \frac{1}{2}(u',\varepsilon v') + .$$

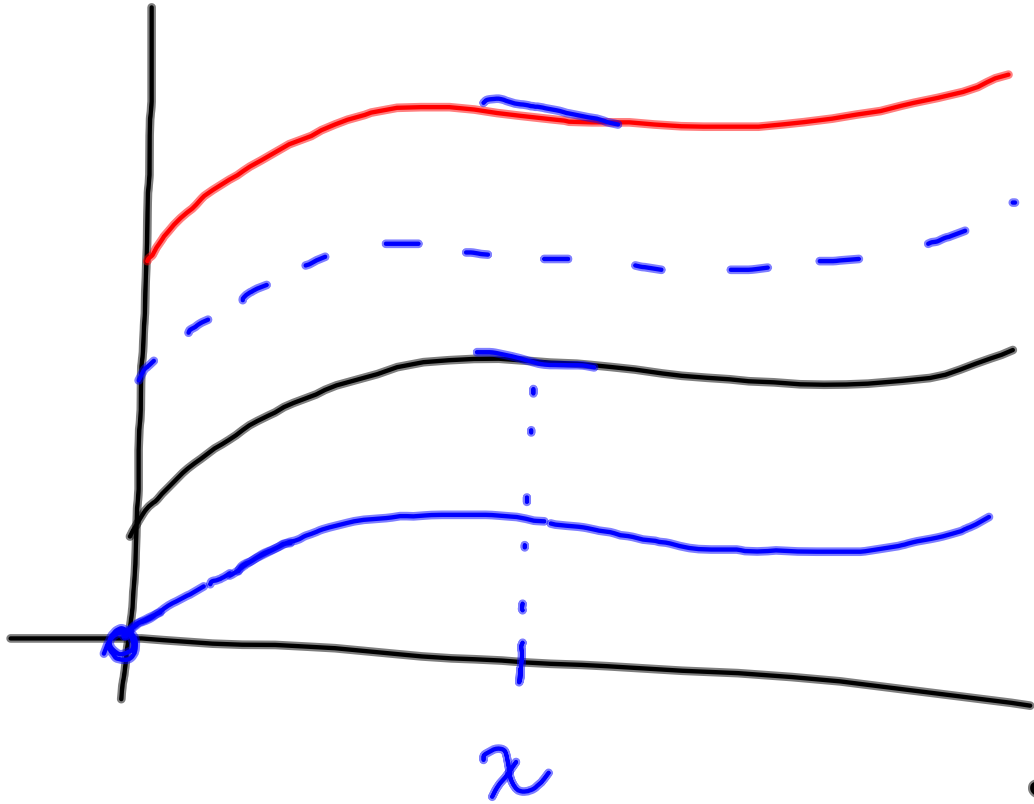
$$(D) \Longrightarrow (V) \rightleftarrows (M)$$

u_1

u_2

$$(u_1' - u_2', v') = 0 \qquad \forall v \in V$$

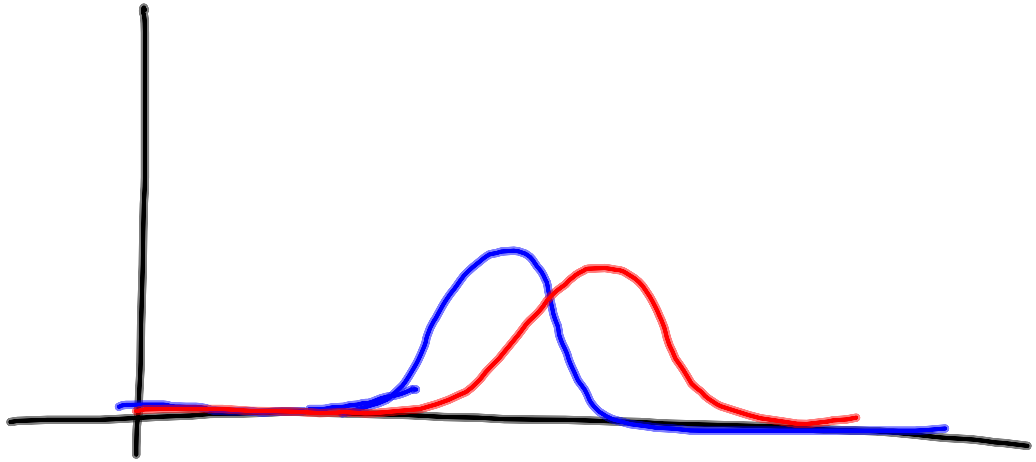
Sea $v = u_1 - u_2$



SOL UNICA

$$(D) \Longrightarrow (V) \Longleftrightarrow (M)$$

$$\underset{?}{\not\Longleftarrow}$$



$$\int \underbrace{(u'' + f)}_{0} \, v \, dx = 0$$

$$\forall v$$

$$(u_h', v') = (f, v) \quad \forall v \in V_h$$

$$(u_h', \varphi_i') = (f, \varphi_i) \quad i = 1, M$$

$$u_h = \sum \xi_j \varphi_j \Rightarrow u_h' = \sum \xi_j \varphi_j'$$

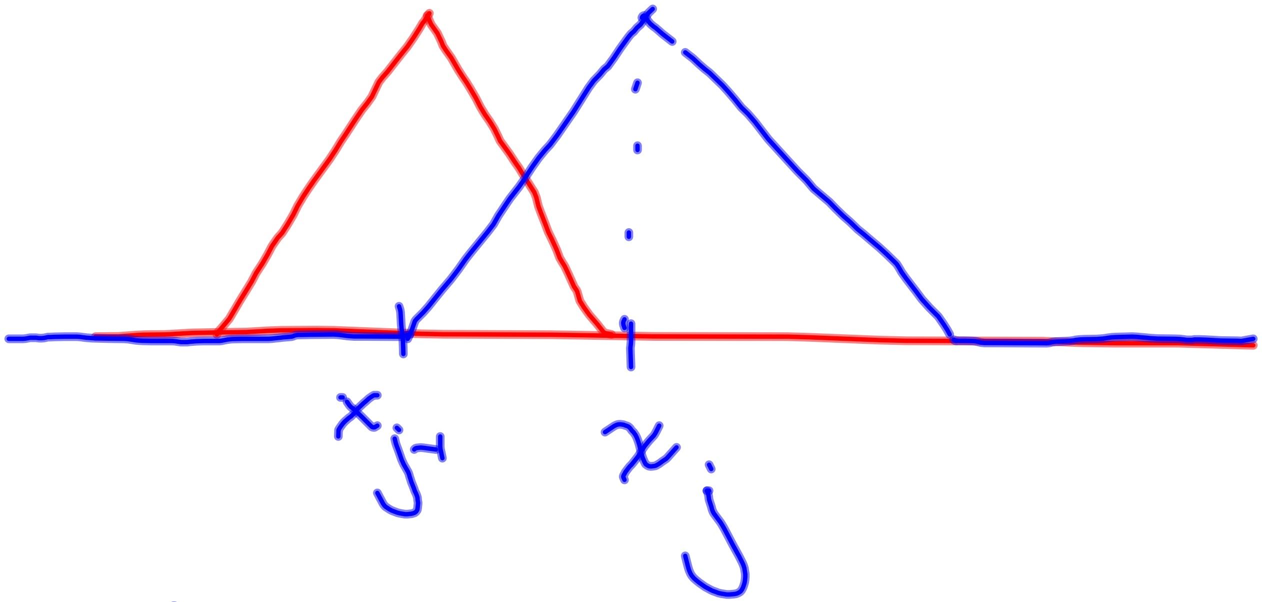
$$\left(\sum_{j=1}^{M} \xi_j \varphi_j', \varphi_i'\right) = (f, \varphi_i)$$

$$i = 1, \dots M$$

$$\sum_{i=1}^{m} \underline{\xi_i} \underbrace{(\varphi_i', \varphi_j')} = \underbrace{(f, \varphi_j)}_{j=1,m}$$

$$a_{ij} = \int_0^1 \varphi_i' \varphi_j' \, dx$$

$$\sum_{i=1}^{m} \xi_i a_{ij} = b_j$$



$$A > 0 \iff x^T A x > 0 \quad \forall x \neq 0$$