$$h = \frac{1}{M+1}$$

$$- \quad (V_h) \quad (u_h', v') = (f, v) \quad \forall v \in V_h$$

$$1 \quad (V) \quad (u', v') = (f, v) \quad \forall v \in V$$

$$\forall v \in V_h$$

$$\| \quad (\underbrace{u' - u_h'}_{\text{error}}, v') = 0 \quad \forall v \in V_h$$

Ecuación del error

$$((u - u_h)', v') = 0 \quad \forall v \in V_h$$

$u, v, w, f, \dots$

$ED(\Omega)$

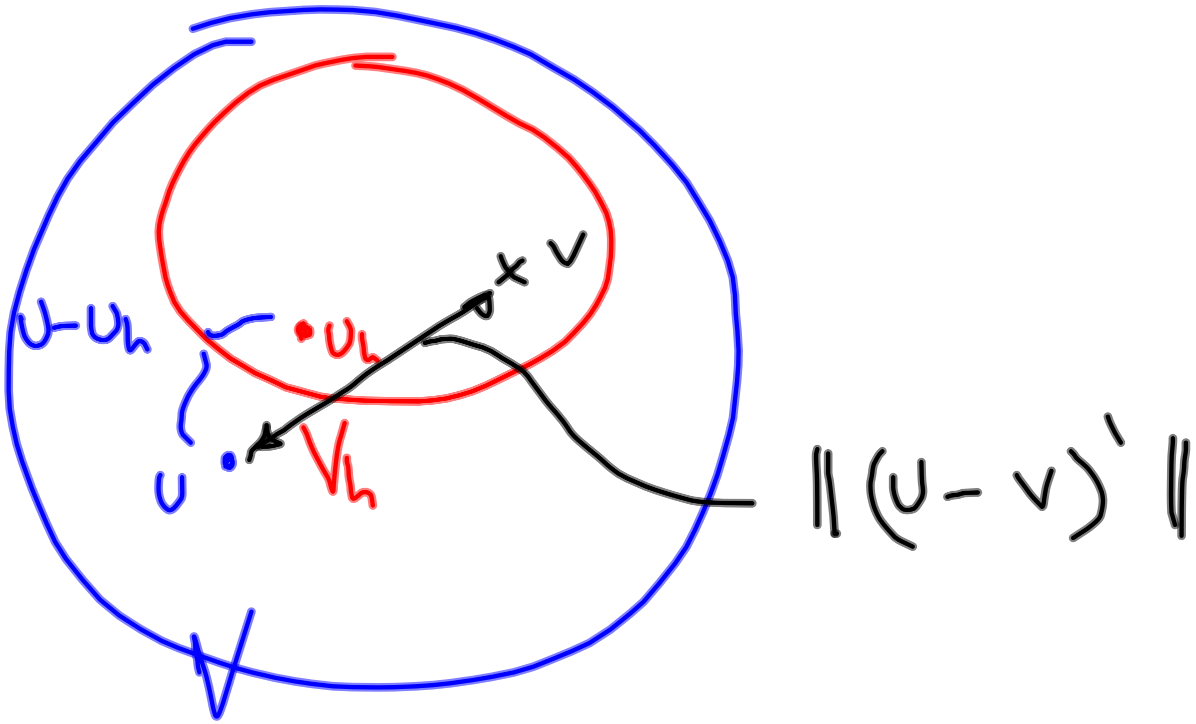
$u : \Omega \longrightarrow \mathbb{R}$

(Ec dif escalar)

$-u'' = f.$

u función : $[0,1] \longrightarrow \mathbb{R}$

$$\|(u-u_h)'\| \le \|(u-v)'\| \quad \forall v \in V_h$$



$$((u-u_h)', v') = 0 \qquad \forall v \in V_h$$

$$w = u_h - v \in V_h \Longrightarrow ((u-u_h)', w') = 0$$

$$\|(u-u_h)'\|^2 = ((u-u_h)', (u-u_h)') +$$

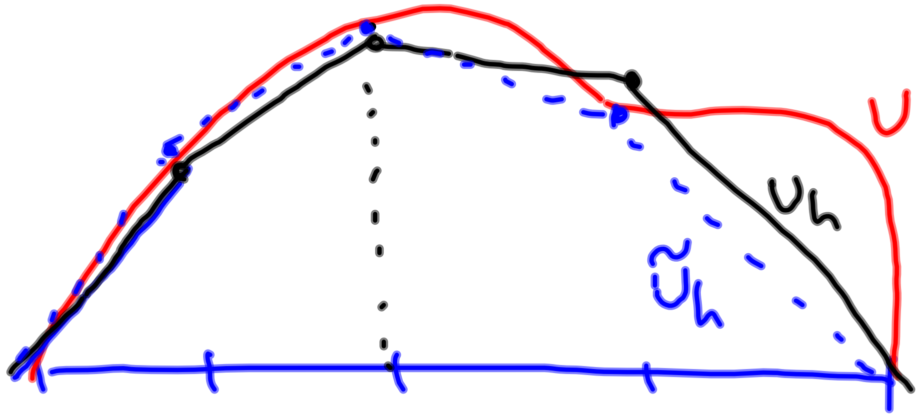
$$0 = + ((u-u_h)', w') =$$

$$= ((u-u_h)', (\underbrace{u-u_h+w}_{u-v})')$$

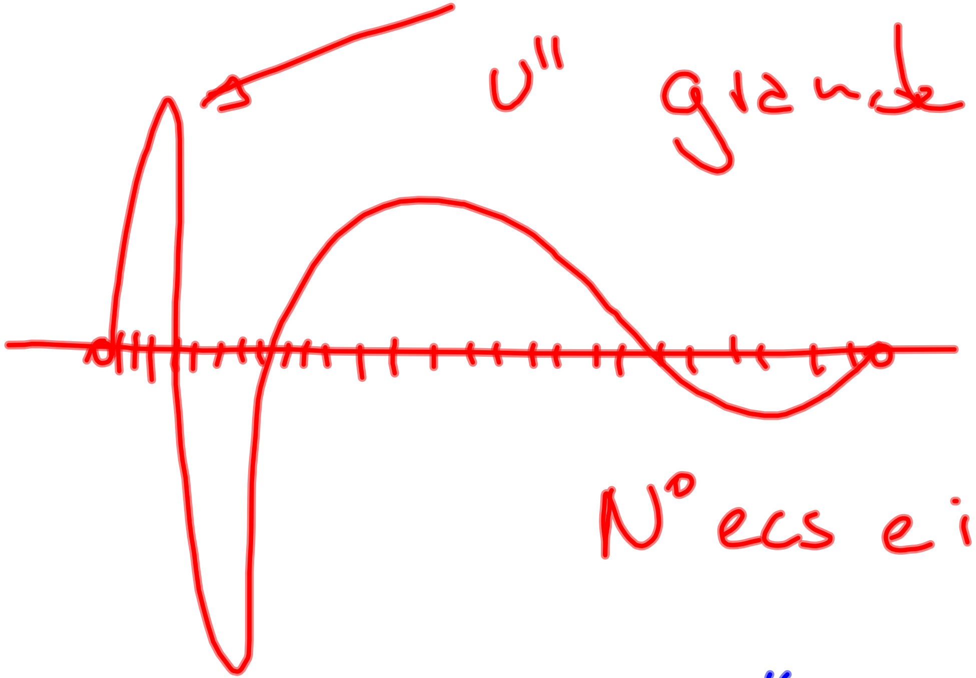
C.S.

$$= ((u-u_h)', (u-v)') \le \cancel{\|(u-u_h)'\|}\, \|(u-v)'\|$$

$$\|(u-u_h)'\| \le \|(u-v)'\| \quad \forall v \in V_h$$


u
u_h
$\tilde u_h$

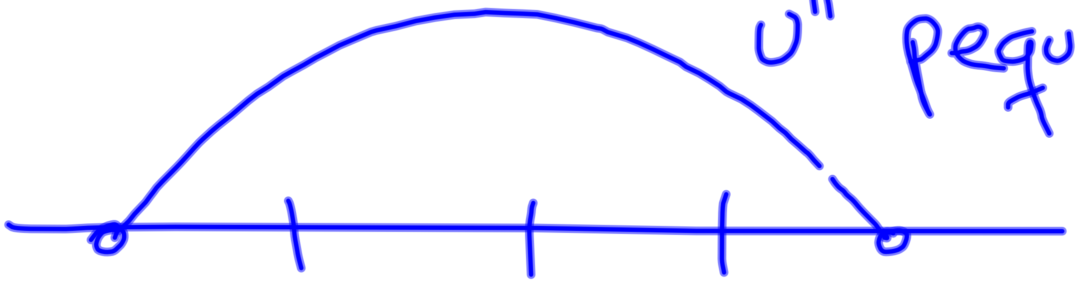
u'' grande!



ERROR ↑

Nº ecs e incógnitas ↑

u'' pequeño

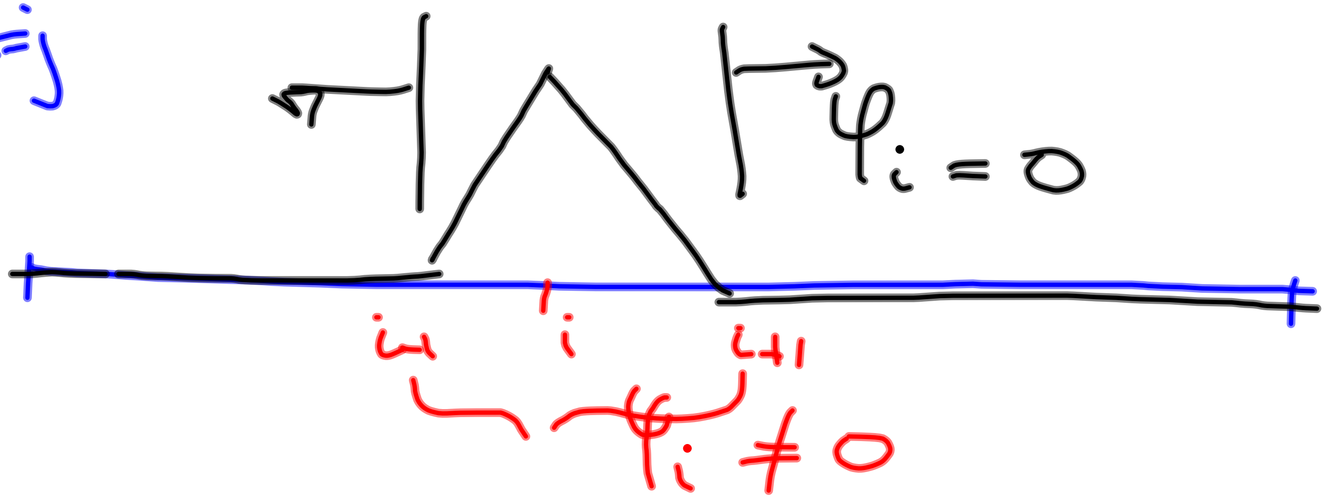


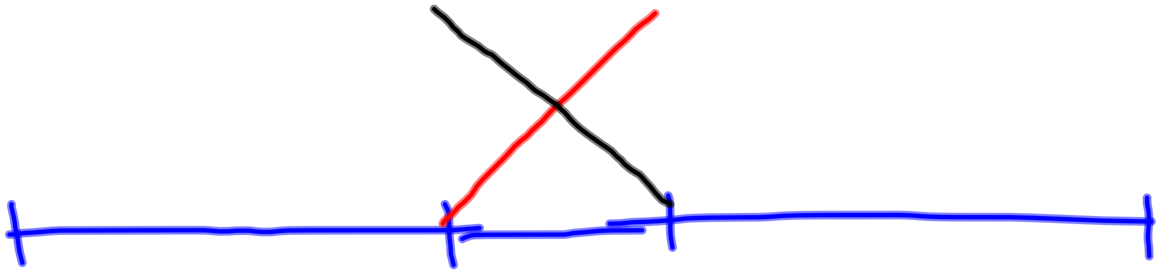
ERROR ↓

Nº ecs ↓ costo ↓

$$(\varphi_i', \varphi_j') = \int_0^1 \varphi_i' \varphi_j' \, dx \neq 0 \quad |i-j| \le 1$$

$i=j$

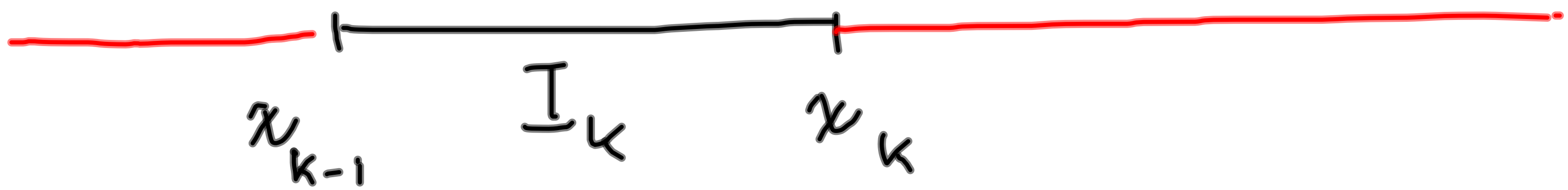




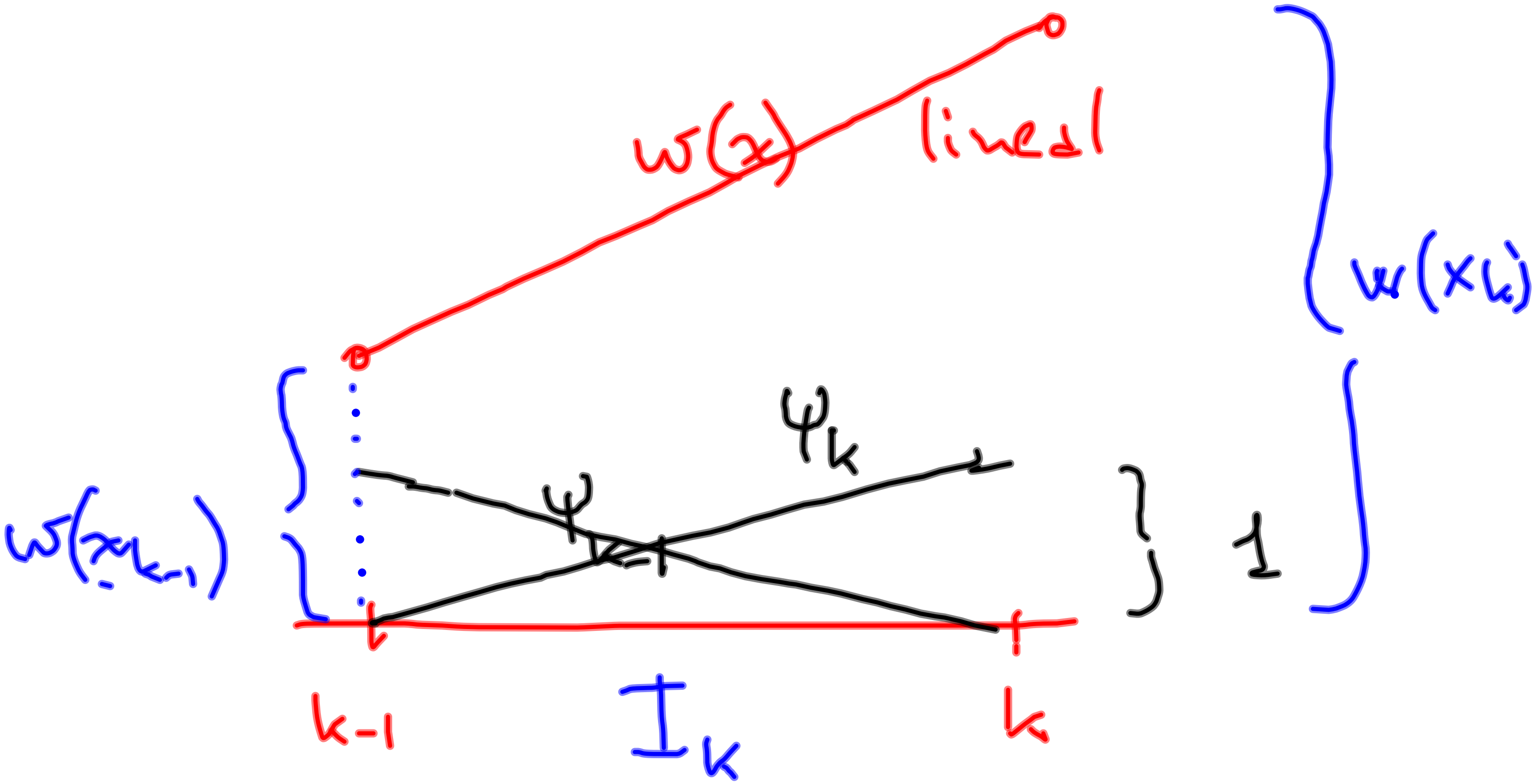
I_K

$$\Omega = \bigcup_{k=1,m+1} I_K = [0,1]$$

$$\int_0^1 = \sum_{k=1,..m+1} \int_{I_K}$$



$$\varphi_i \neq 0 \text{ en } I_k \iff \begin{matrix} i = k-1 \\ \text{ó} \\ i = k \end{matrix}$$



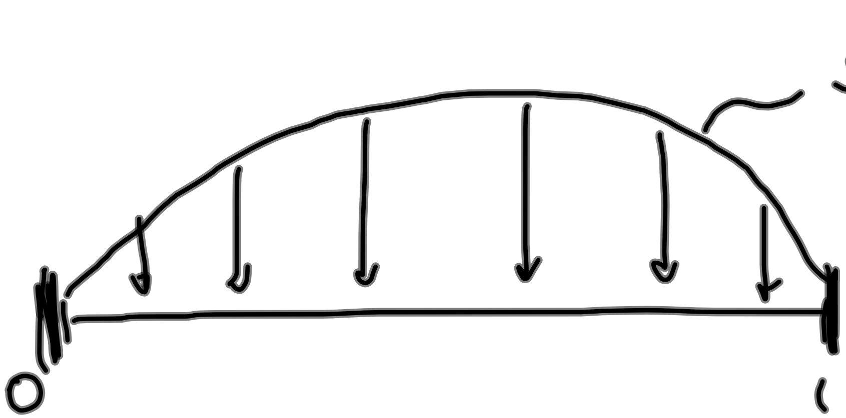
$$w(x) = w(x_{k-1}) \cdot \psi_{k-1}(x) + w(x_k)\, \psi_k(x)$$

$$\varphi_i(x_j) = \delta_{ij} \qquad k-1, \quad k$$

$$\alpha_k + \beta_k \, x_k = 1$$

$$\alpha_k + \beta_k \, x_{k-1} = 0$$

$$\begin{bmatrix} 1 & x_k \\ 1 & x_{k-1} \end{bmatrix} \begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$-u'' = \sin(\pi x)$$

Sol exacta: $u(x) = \frac{1}{\pi^2} \sin(\pi x)$

$$\frac{x_k - x}{h_k} \Longrightarrow \begin{array}{l} x_{k-1} \longrightarrow 1 \\ x_k \longrightarrow 0 \end{array}$$

$$\frac{x - x_{k-1}}{h_k} \Longrightarrow \begin{array}{l} x_k \longrightarrow 1 \\ x_{k-1} \longrightarrow 0 \end{array}$$

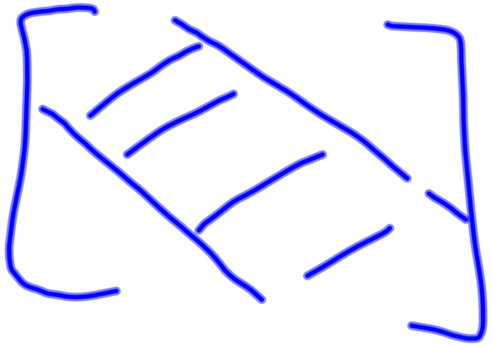
$$\underline{\underline{A}}\,\underline{\xi} = \underline{b}$$

M nodos $\Longrightarrow$ M ecs y
M incógnitas

Resolver sistema "~~lleno~~" $\Longrightarrow$
costo $O(M^3)$

Sistema "banda"
M ecs
n ancho de banda

n



Custo p/ matriz banda:

$$O(M n^2) \ll O(M^3)$$