$$\int v \times \left(-\Delta u + u\right) = \int f \times v$$

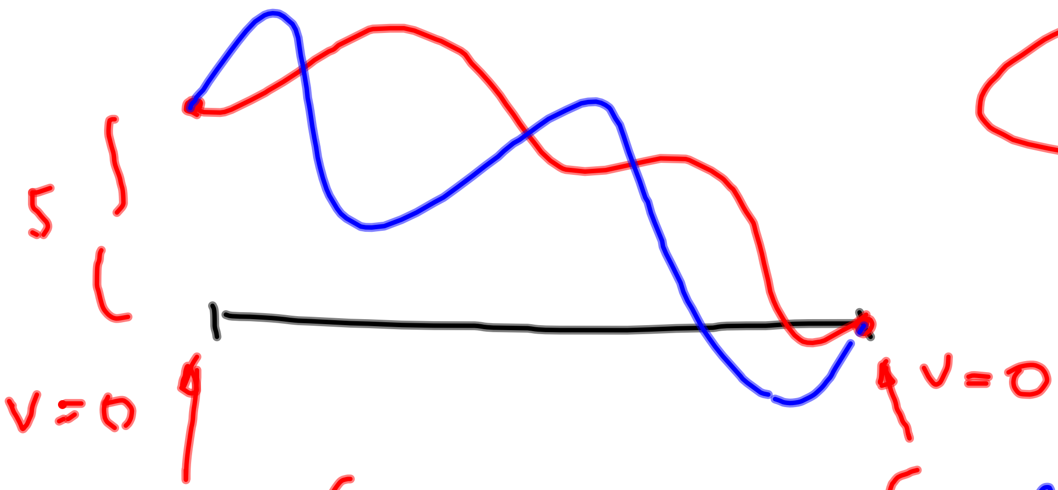
$$\underbrace{-\int v \Delta u \, d\Omega}_{\displaystyle \int \nabla v \cdot \nabla u \, d\Omega - \int_{\Gamma} v \, n \cdot \nabla u \, d\Omega} + \int v u \, d\Omega - \int f v \, d\Omega$$

$$\int_{\Omega} v u + \nabla v \cdot \nabla u - \int_{\Gamma} v \, n \cdot \nabla u = \int f v \, d\Omega$$

$$\forall v \in H^1$$

$$\underbrace{\int_\Omega uv + \nabla v \cdot \nabla u \, d\Omega}_{a(u,v)} - \cancel{\int_\Gamma v \, \underbrace{n \cdot \nabla u}_{=0} \, d\Gamma} = \underbrace{\int_\Omega f v \, d\Omega}_{(f,v)} \quad \forall v \in H^1(\Omega)$$

$$a(u,v) = (f,v) \quad \forall v \in H^1(\Omega)$$



$$-\Delta u = f$$

$$u(0) = 5$$

$$u(1) = 0$$

$$\int v(-\Delta u) - \int v f = \text{res "ponder"}$$

$$|a(v,w)| = \Big| \underbrace{\int v w + \nabla v \cdot \nabla w \, d\Omega}_{\text{prod interno en } H^1(\Omega)} \Big| \leq C$$

$$\|v\|_{H^1(\Omega)} \, \|w\|_{H^1(\Omega)} =$$

$$= \left[a(v,v)\right]^{1/2} \left[a(w,w)\right]^{1/2}$$

$$|L(v)| = \left|\int f v \, d\Omega\right| = \left|(f,v)_{L^2(\Omega)}\right| \leq$$

$$|L(v)| \leq \underbrace{\|f\|_{L^2(\Omega)}}_{\lambda} \|v\|_{L^2(\Omega)}$$

1) Sim ok

2) Cont $\Big\{$ ø, → bilineal ok

3) Cont $\Big\{$ ø, → lineal ok

4) Coercividad ?

$\exists \alpha$? / $\underbrace{a(v,v)}$ $\geq \alpha \|v\|^2_{H^1(\Omega)}$

$$\int v^2 + \nabla v \cdot \nabla v \, d\Omega = \|v\|^2_{H^1(\Omega)}$$

SI, $\alpha = 1$

$$\left| a(v,w) \right| = \left| \int v' w' \, dx \right| = \left| (v', w')_{L_2(I)} \right| \leq$$

$v, w \in H_0^1(I) \Longrightarrow$ v, w cuad int

v', w' cuad int

$\in L_2(I)$

$$\leq \|v'\|_{L_2(I)} \|w'\|_{L_2(I)} \leq \|v\|_{H_0^1(I)} \|w\|_{H_0^1(I)}$$

$\int v'^2 \, dx$

$\boxed{\gamma = 1}$

$\int_I v^2 + v'^2 \, dx$

$$\left| \underline{a(v,v)} \right| \geq \alpha \, \|v\|^2_{H^1(I)} \quad ? \qquad \exists \alpha \, ?$$

$$\int v' v' \, dx$$

$$\|v\|^2_{L_2(\Omega)} \leq C\left(\underbrace{\|v\|^2_{L_2(\Gamma)}}_{0} + \|\nabla v\|^2_{L_2(\Omega)}\right)$$

$\forall v \in H^1(\Omega)$

$C \quad a(v,v) = \int_\Omega \nabla v \cdot \nabla v \, d\Omega \geq \cancel{\frac{1}{C}} \|v\|^2_{L_2(\Omega)}$

$+$

$v \in H^1_0(\Omega) \Longrightarrow \|v\|_{L_2(\Gamma)} = 0$

$$\underline{a(v,v) =} \qquad \underline{\int \nabla v \nabla v \, d\Omega}$$

$$(C+1)\, a(v,v) \geq \int v^2 \, d\Omega + \int \nabla v \cdot \nabla v \, d\Omega$$

$$\left(\frac{d^4u}{dx^4} = f\right) v$$

$$\underbrace{\int_I v \frac{d^4u}{dx^4}\, dx}_{-\int_I \frac{dv}{dx}\frac{d^3u}{dx^3}\, dx + \cancel{v\frac{d^3u}{dx^3}\Big|_0^1}} = \int_I v f\, dx$$

$$v(0) = v(1) = 0$$

$$\int_I \frac{d^2v}{dx^2}\frac{d^2u}{dx^2} - \cancel{\frac{dv}{dx}\frac{d^2u}{dx^2}\Big|_0^1} \qquad \frac{dv}{dx}(0) = \frac{dv}{dx}(1) = 0$$

$$\underbrace{\int_I \frac{d^2v}{dx^2}\frac{d^2u}{dx^2}\,dx}_{} = \int_I v f\,dx \qquad \forall v \in H_0^2(I)$$

necesitamos der seg cuad int $\Rightarrow H^2$

$$v(0) = v(1) = v'(0) = v'(1) = 0$$

$u ? \in H_0^2(I)$

$$(v,w)_{H^2} = \int (vw + v'w' + v''w'')\,dx$$

SIMETRIA OK

$$a(v, w) = \int v'' w''$$

CONTINUIDAD:

$$|a(v, w)| = \left| \int v'' w'' \, dx \right| = (v'', w'')_{L_2(I)} \leq$$

$$\leq \|v''\|_{L_2(I)} \|w''\|_{L_2(I)} \leq$$

$\gamma = 1$!

$$\leq \|v\|_{H^2(I)} \|w\|_{H^2(I)}$$

$$\frac{d^4 u}{dx^4} = f \Longrightarrow \frac{d^2}{dx^2}\frac{d^2}{dx^2} u = f$$

$$\Delta^2 u = f \Longrightarrow \Delta(\Delta u) = f$$

$$D^{\alpha} v = \frac{\partial^{|\alpha|} v}{\partial x_1^{\alpha_1} \, \partial x_2^{\alpha_2}} \qquad \underline{\alpha} = (\alpha_1, \alpha_2)$$

$$|\alpha| = \alpha_1 + \alpha_2$$

<u>Ejemplo:</u> $\alpha = (1, 2)$

$$D^{\alpha} v = \frac{\partial^3 v}{\partial x_1 \, \partial x_2^2}$$

$$\int_\Omega f v \, d\Omega = \int v \, \Delta^2 u \, d\Omega = \quad \cdots$$

$$\underbrace{\int_\Omega \Delta v \, \Delta u \, d\Omega}_{a(u,v)} = \underbrace{\int f v \, d\Omega}_{L(v)}$$

SIMETRIA OK

CONTINUIDAD

$$|a(u,v)| = \left| \int \Delta v \, \Delta u \, d\Omega \right| = \left| (\Delta u, \Delta v)_{L_2(\Omega)} \right| \cdots$$

$a(\cdot,\cdot)$ es V-elip c/ $\alpha = \dfrac{1}{1+C+C^2}$

$$\|\Delta v\|^2_{L_2(\Omega)} = \int_\Omega (\Delta v)^2 \, d\Omega = \sum_{i,j=1}^{2} \int_\Omega \frac{\partial^2 v}{\quad}$$

$$\Delta v = \frac{\partial^2 v}{\partial x_1^2} + \frac{\partial^2 v}{\partial x_2^2}$$

$$(\Delta v)^2 = \left(\frac{\partial^2 v}{\partial x_1^2}\right)^2 + 2\,\frac{\partial^2 v}{\partial x_1^2}\,\frac{\partial^2 v}{\partial x_2^2} + \left(\frac{\partial^2 v}{\partial x_2^2}\right)^2$$

VER !