$$u_{i,j} = \frac{\partial u_i}{\partial x_j}$$

$$u_{i,jj} = \frac{\partial^2 u_i}{\partial x_j \partial x_j} = \frac{\partial^2 u_i}{\partial x_1 \partial x_1} + \frac{\partial^2 u_i}{\partial x_2 \partial x_2} +$$

$$+ \frac{\partial^2 u_i}{\partial x_3 \partial x_3} =$$

$$= \frac{\partial^2 u_i}{\partial x_1^2} + \frac{\partial^2 u_i}{\partial x_2^2} + \frac{\partial^2 u_i}{\partial x_3^2}$$

$$\operatorname{div} \underline{\underline{\sigma}} \equiv \sigma_{ij,j} =$$

$$= \sigma_{i1,1} + \sigma_{i2,2} + \sigma_{i3,3}$$

$$\frac{\partial \sigma_{i1}}{\partial x_1} + \frac{\partial \sigma_{i2}}{\partial x_2} + \frac{\partial \sigma_{i3}}{\partial x_3}$$

$$\underline{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \begin{matrix} \longleftarrow \in P_1(k) \\ \longleftarrow \in P_1(k) \end{matrix}$$

$$\underline{v}\Big|_k \in \left[P_1(k)\right]^2$$

$$\int_\Omega v_i f_i = \int_\Omega \left(-\Delta u_i + p_{,i}\right) v_i$$

$$\int_\Omega v_i f_i \, d\Omega = \int_\Omega \nabla u_i \nabla v_i \, d\Omega - \cancel{\int p \overbrace{v_{i,i}}^{0} \, d\Omega} - \cancel{\int_\Gamma \frac{\partial u_i}{\partial n} \underbrace{v_i}_{0} \, ds} + \cancel{\int_\Gamma p \, n_i v_i \, ds}$$

$$\underbrace{\mu \int_\Omega \nabla u_i \nabla v_i \, d\Omega}_{a(\underline{u}, \underline{v})} = \underbrace{\int_\Omega v_i f_i \, d\Omega}_{L(\underline{v})}$$

$$\underline{v} = \begin{pmatrix} \partial\varphi/\partial x_2 \\ -\partial\varphi/\partial x_1 \end{pmatrix} = \mathrm{rot}(\varphi)$$

función línea de corriente

$$v \in H_0^1 \Longrightarrow \varphi \in H_0^2$$

$$\mathrm{div}\,\underline{v} = \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = \frac{\partial^2 \varphi}{\partial x_1 \partial x_2} - \frac{\partial^2 \varphi}{\partial x_1 \partial x_2} = 0$$

$$\mu \int_\Omega \underbrace{\nabla u_i}\, : \nabla v_i \, d\Omega$$

$$\begin{cases} \underline{\nabla} u_1 = \underline{\nabla}\left(\dfrac{\partial \varphi}{\partial x_2}\right) = \begin{pmatrix} \dfrac{\partial^2 \varphi}{\partial x_1 \partial x_2} \\[2ex] \dfrac{\partial^2 \varphi}{\partial x_2^2} \end{pmatrix} \\[6ex] \underline{\nabla} u_2 = \underline{\nabla}\left(-\dfrac{\partial \varphi}{\partial x_1}\right) = \begin{pmatrix} -\dfrac{\partial^2 \varphi}{\partial x_1^2} \\[2ex] -\dfrac{\partial^2 \varphi}{\partial x_1 \partial x_2} \end{pmatrix} \end{cases}$$

$$\underline{\nabla} u_i : \underline{\nabla} v_i = \begin{Bmatrix} \varphi_{,12} \\ \varphi_{,22} \end{Bmatrix} \begin{Bmatrix} \psi_{,12} \\ \psi_{,22} \end{Bmatrix} + \begin{Bmatrix} +\varphi_{,11} \\ +\varphi_{,12} \end{Bmatrix} \begin{Bmatrix} +\psi_{,11} \\ +\psi_{,12} \end{Bmatrix} =$$

$$\underline{u} = \operatorname{rot} \varphi$$

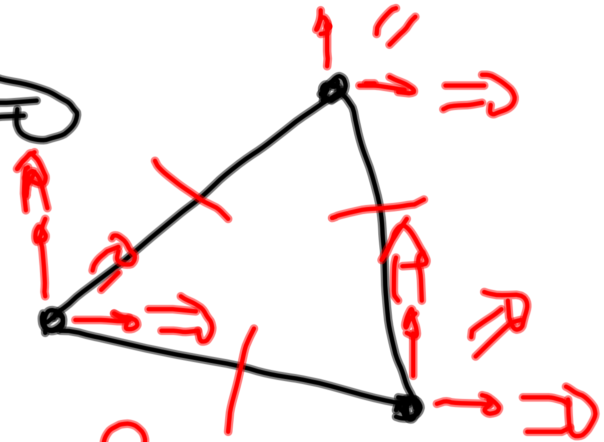
$$\underline{v} = \operatorname{rot} \psi$$

$$= \underbrace{\psi_{,11}\varphi_{,11} + 2\psi_{,12}\varphi_{,12} + \psi_{,22}\varphi_{,22}}$$

$$a(\psi, \varphi) = \int_\Omega \quad \cdot \quad d\Omega$$

$$\underline{\underline{\varphi(x,y) \in H_0^2}}$$

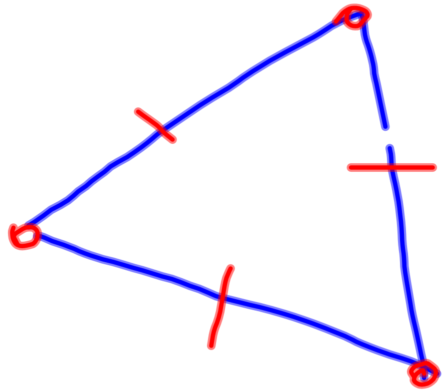
$H^2_0(\Omega) \Longrightarrow$



21 gdl

$\varphi(x,y) \in P_5(K)$

$$\varphi(x,y) = p^T(x,y)\,\underset{\sim}{c}$$



1) $\varphi(x_k, y_k) = \varphi^k$

2) $\varphi_{,x}(x_k, y_k) = \varphi^k_{,x}$

$\varphi_{,y}(x_k, y_k) = \varphi^k_{,y}$

$$\underline{\underline{A}}\,\underline{c} = \underline{\Phi} \leftarrow \text{gdl nodals}$$

$$\begin{cases} \varphi_e \text{ nodos} \\ \varphi_{,x} \text{ " " } \\ \vdots \end{cases}$$

f (coord nodales)

$$\underline{c} = \underline{\underline{A}}^{-1}\,\underline{\Phi}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{\partial^2}{\partial x^2}\left(\underline{p}^T(x,y)\,\underline{c}\right) = \frac{\partial^2 \underline{p}^T}{\partial x^2}\,\underline{c}$$

$$\frac{\partial^2 \varphi}{\partial x \partial y}$$

$$\frac{\partial^2 \varphi}{\partial y^2}$$

$$\int \nabla v \, \nabla u \, d\Omega$$

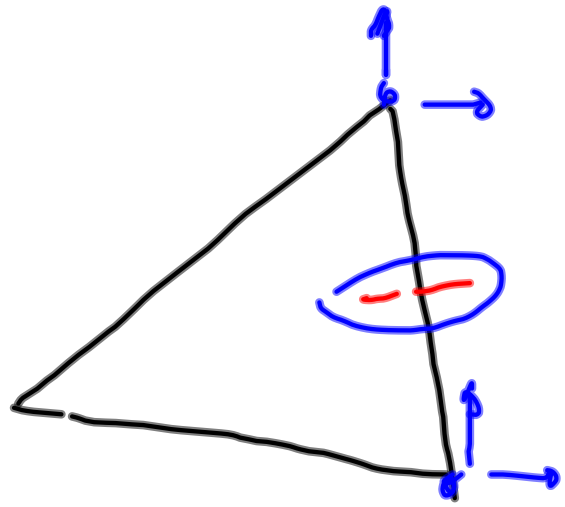
$$u = N_I U_I$$

$$v = N_I V_I$$

$$a(u,v) = L(v) \quad \underbrace{\forall v \in V}$$

$$\equiv$$

$$v = N_I V_I \Longrightarrow \forall V_I \in \mathbb{R}^N$$



$$\varphi^{12}_{,n} = n_x \varphi^{12}_{,x} + n_y \varphi^{12}_{,y}$$

$$\simeq n_x \left(\frac{\varphi^1_{,x} + \varphi^2_{,x}}{2} \right) +$$

$$+ n_y \left(\frac{\varphi^1_{,y} + \varphi^2_{,y}}{2} \right)$$

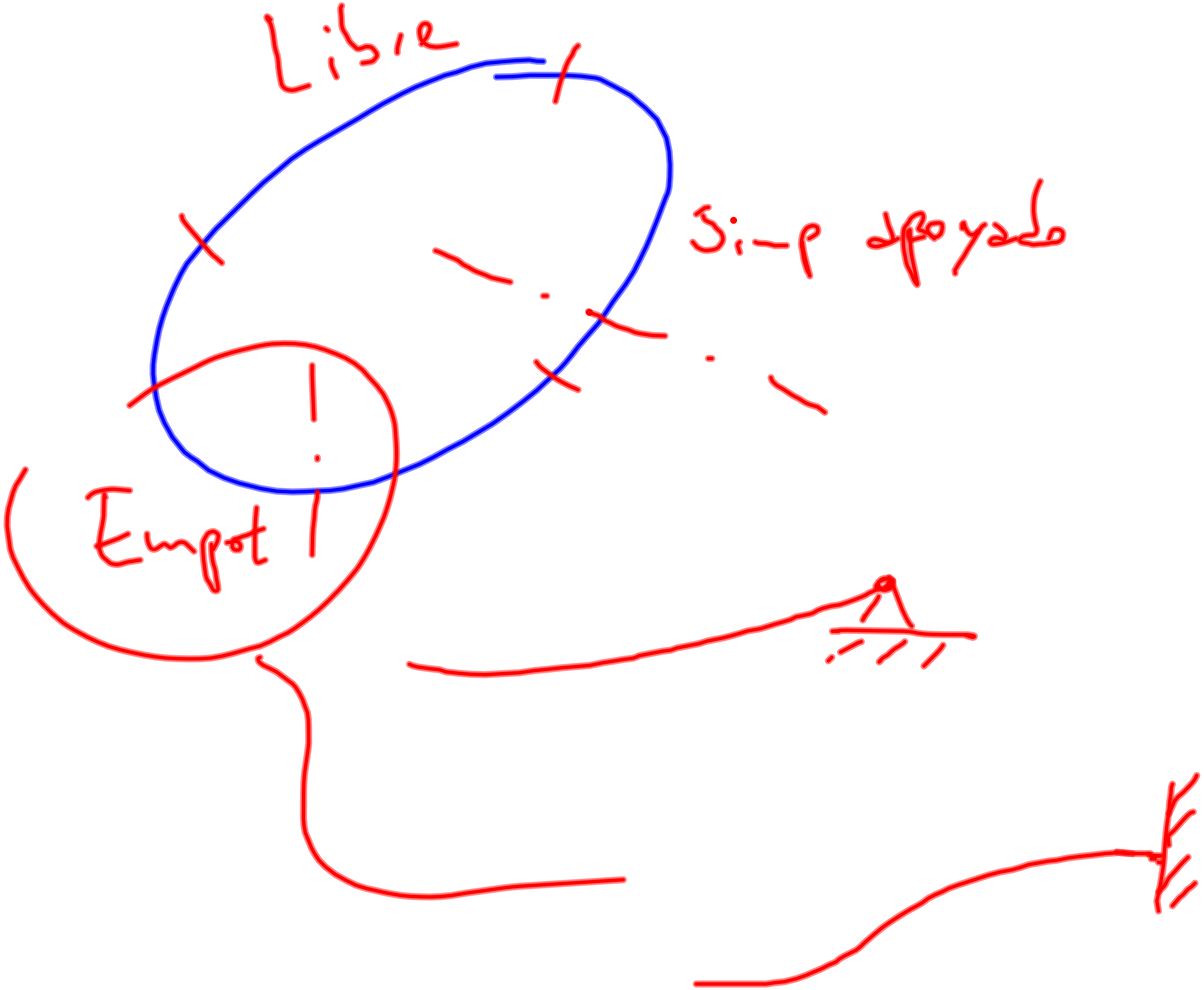
$$K = \mu A^{-T} \begin{bmatrix} 0 & 0 \\ 0 & \rho \end{bmatrix} A^{-1}$$

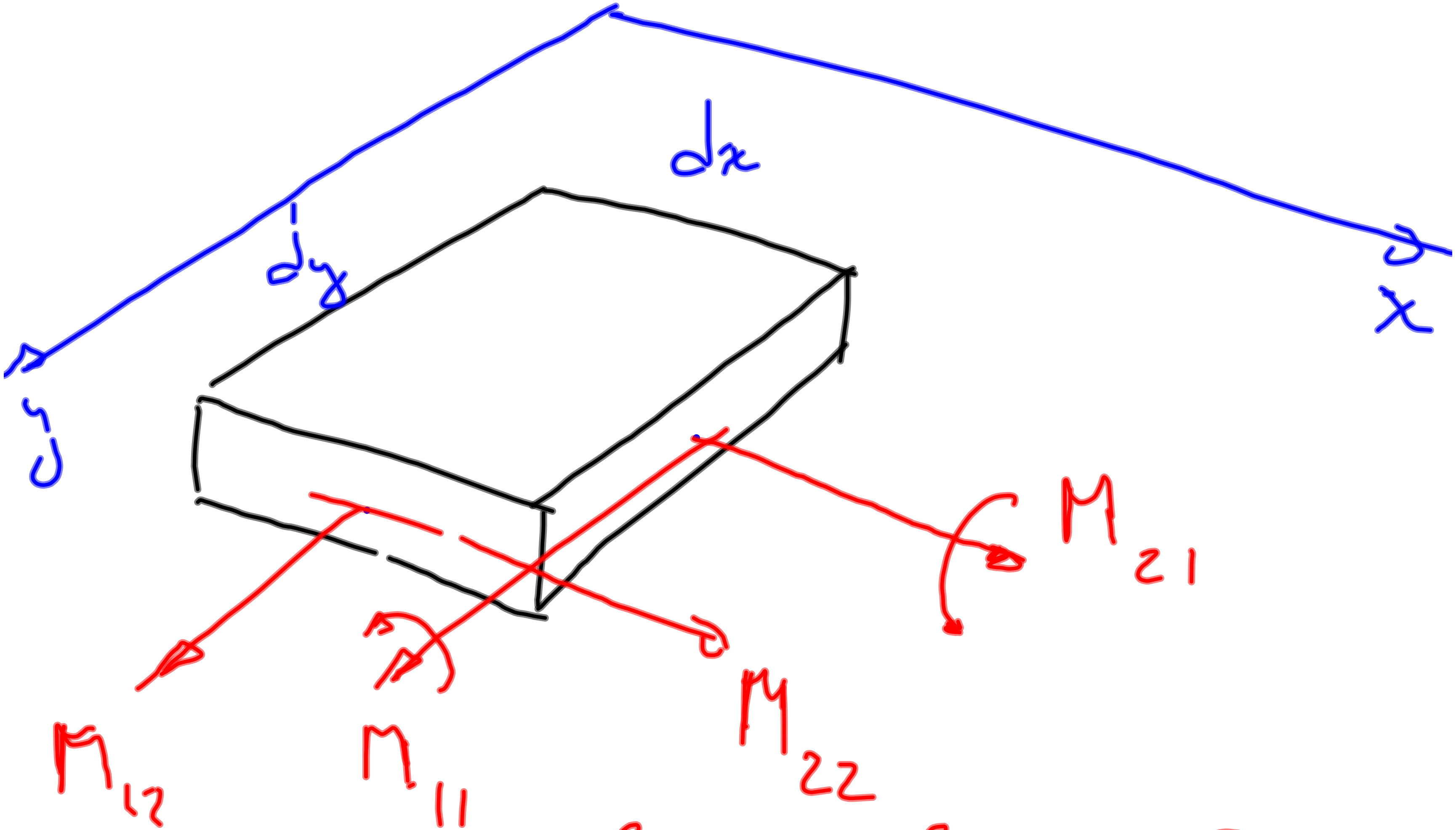
$$a(\psi, \varphi) = \psi^T \mu A^{-T} \begin{bmatrix} 0 & 0 \\ 0 & \rho \end{bmatrix} A^{-1} \underset{21}{\underset{\sim}{\varphi}}$$

$$\underline{\varphi} = \begin{pmatrix} \varphi_{1:18} \\ \varphi_{19:21} \end{pmatrix} = \begin{pmatrix} \varphi_{1:18} \\ D\varphi_{1:18} \end{pmatrix} = \begin{bmatrix} I \\ D \end{bmatrix} \varphi_{1:18}$$

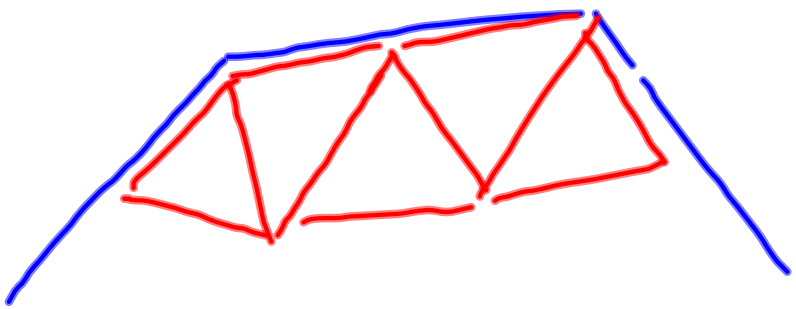
$$a(\psi_{1:180}, \psi_{1:18}) = \Psi_{\mu} B^T P B \psi$$

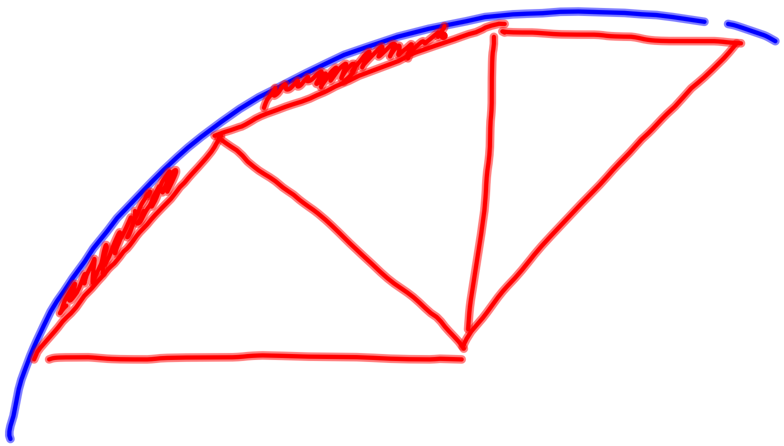
$$B = A^{-1}\begin{bmatrix} I \\ D \end{bmatrix}$$

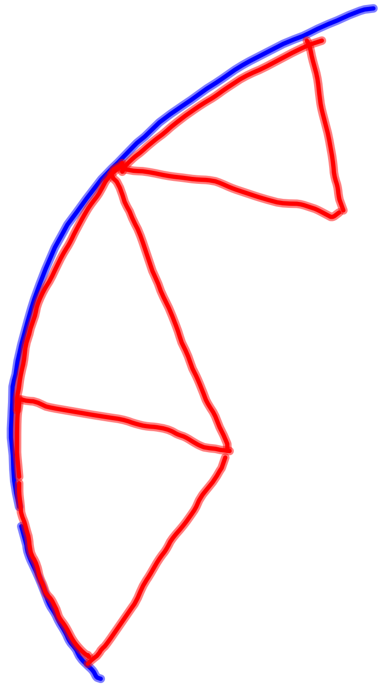

Libre
Simp apoyado
Empot !

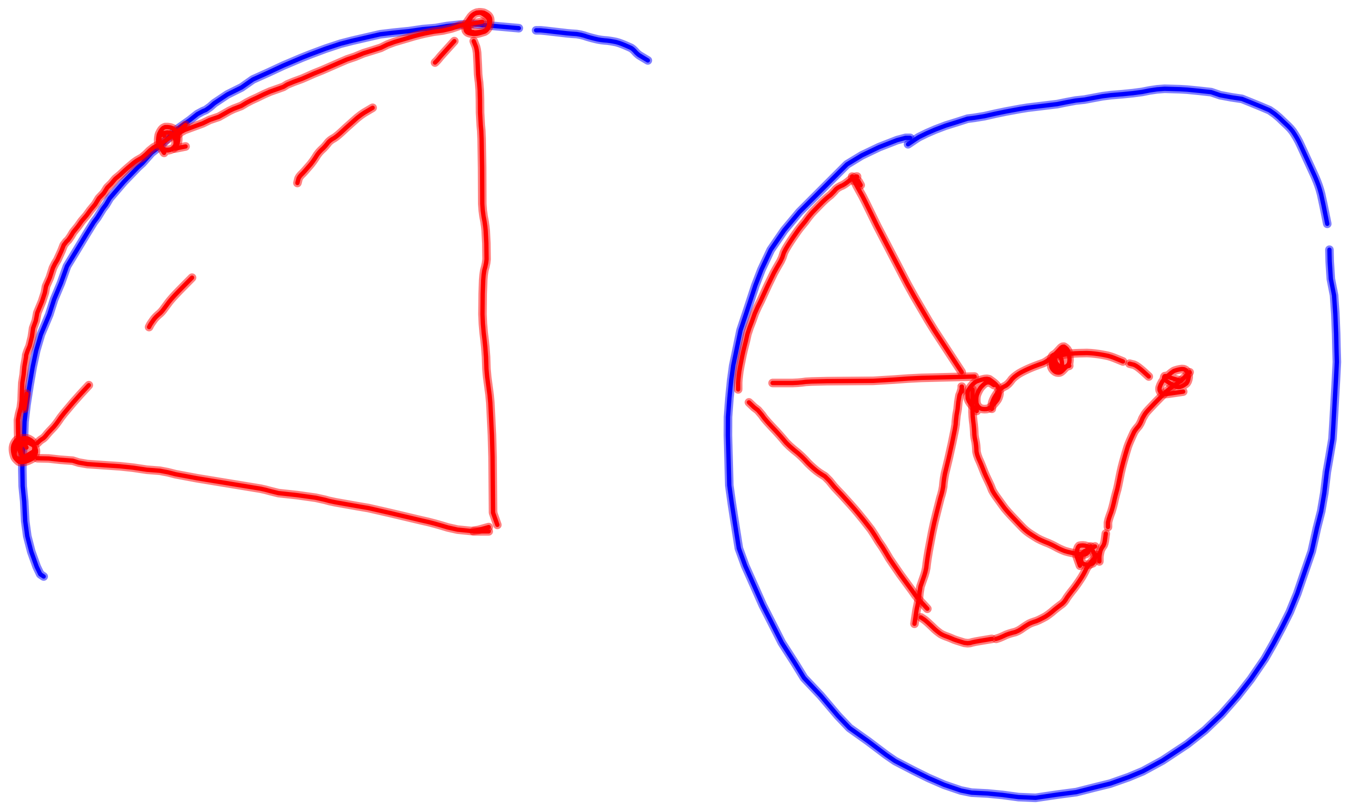


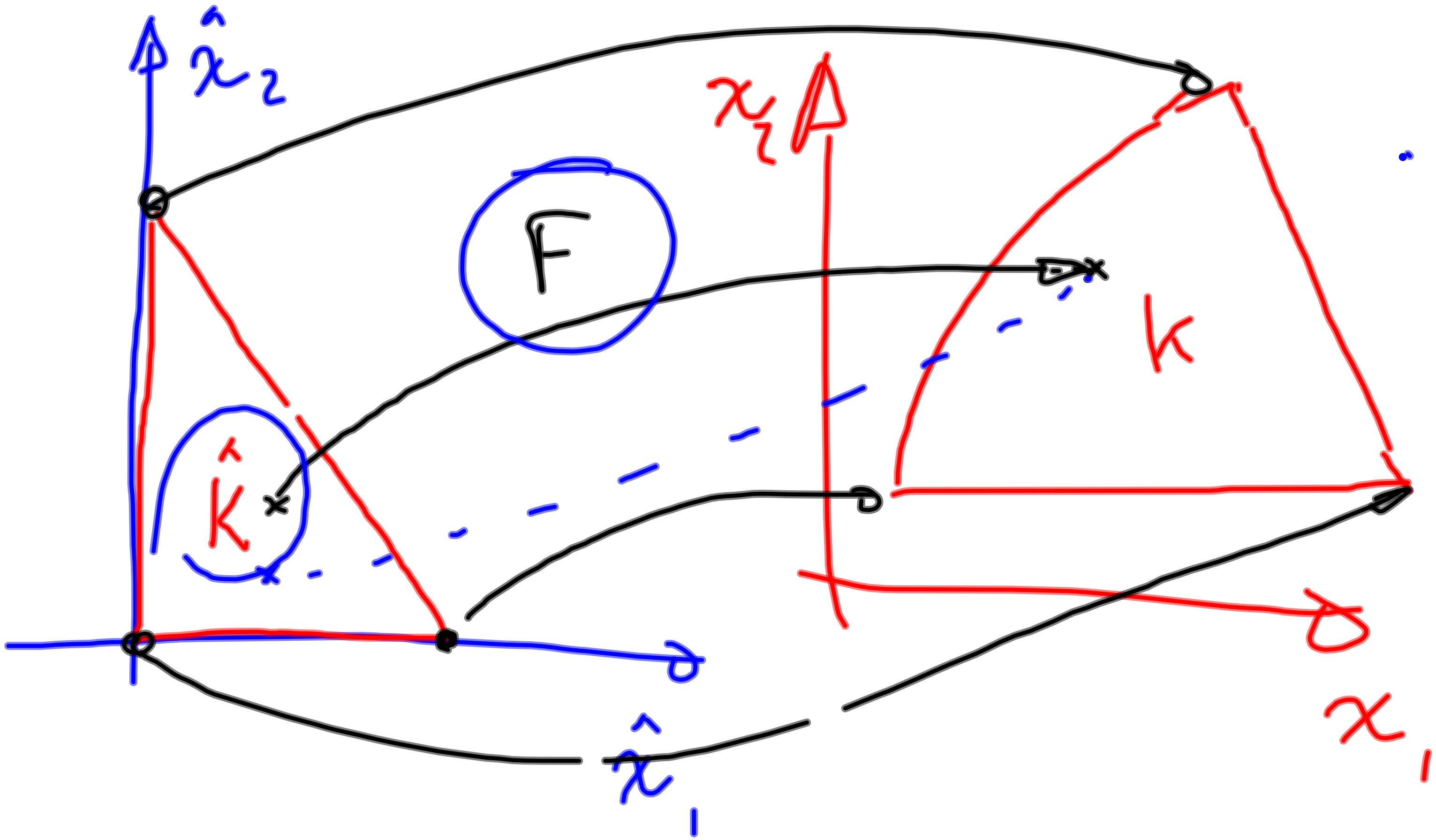
$$M_{ij,ij} = f \Rightarrow \frac{\partial^2 M_{11}}{\partial x_1^2} + 2\frac{\partial^2 M_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M_{22}}{\partial x_2^2} = f$$





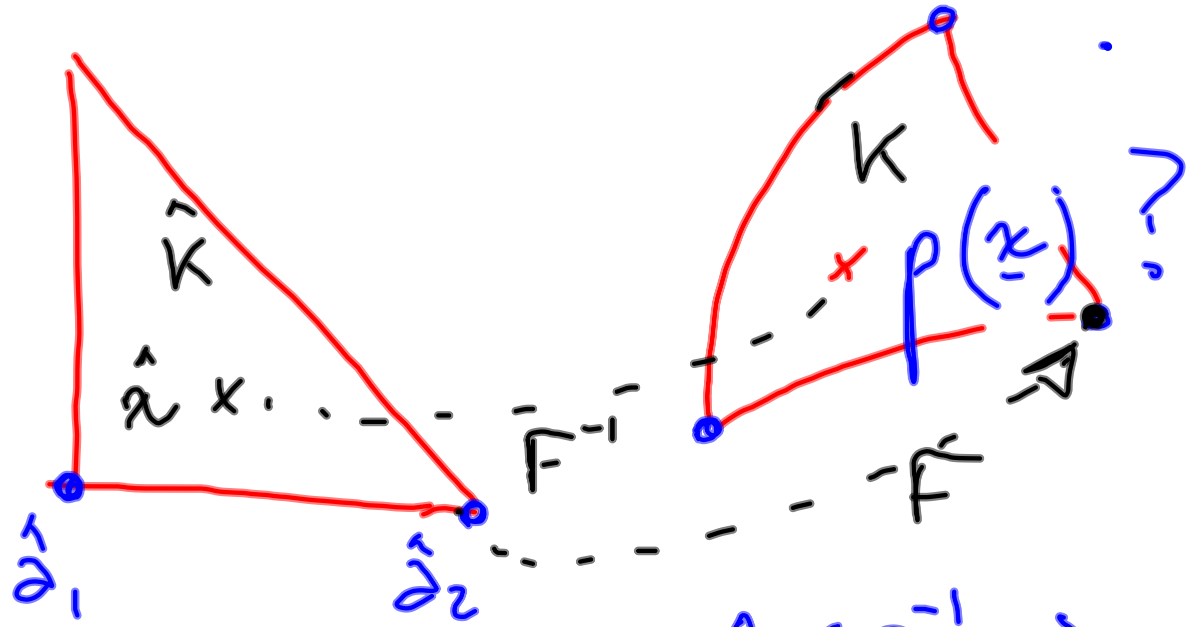






$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = F \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix}$$

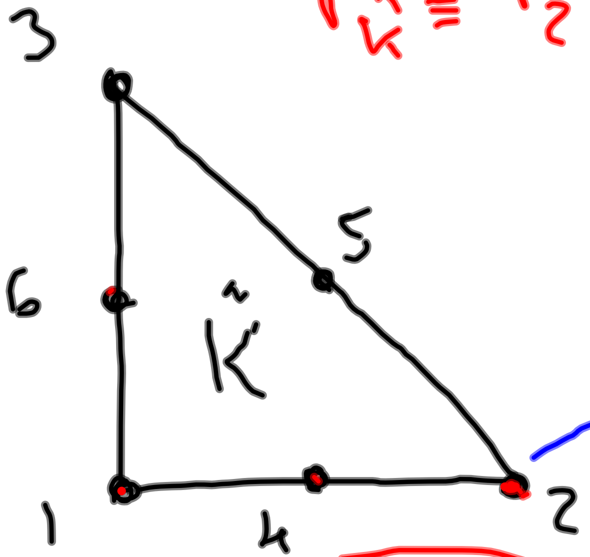
$\Sigma_{\hat K}$

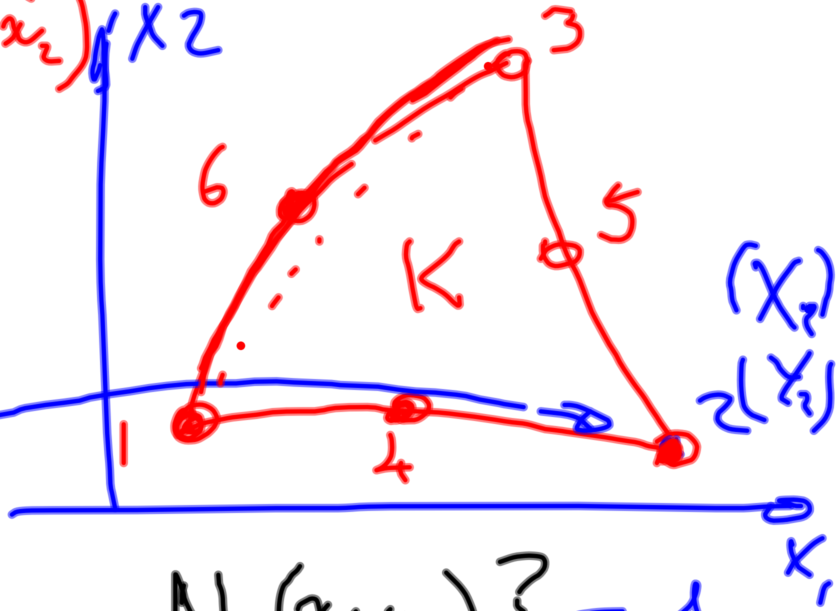


$$p(\underline{x}) = \hat p\left(\underline{F}^{-1}(x)\right)$$

Σ_K = valors de la funció en $a_i = F(\hat a_i)$

$P_{\hat K} \equiv P_2(\hat x_1, \hat x_2)$





$N_2(x_2, y_2) \overset{?}{=} 1$

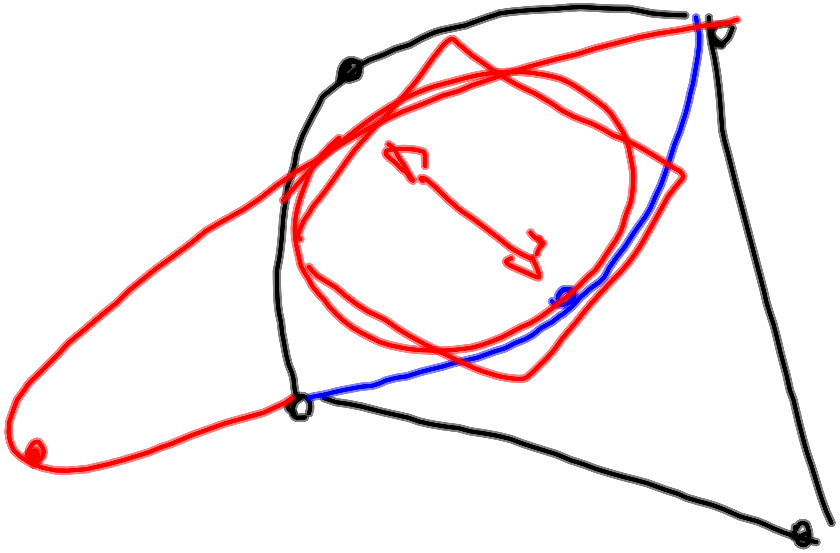
$$u = \sum N_I(\hat x_1, \hat x_2)\, U_I$$

fcs cuad

X_I, Y_I :

$\boxed{\text{coord de nodos del elem-to } K}$

$$F \left\| \begin{aligned} x_1 &= \sum N_I(\hat x_1, \hat x_2)\, X_I \\ x_2 &= \sum N_I(\hat x_1, \hat x_2)\, Y_I \end{aligned} \right.$$



F ? $|d|$