Guía 1, Ej. 5

$S \in CT(3) \rightarrow$ comps. S_{ijk}

$T \in CT(2) \rightarrow$ comps. T_{ij}

W con comps. $W_{ijkem} = S_{ijk} T_{em}$

Demostrar que $W \in CT(5)$

Transforma como $CT(5)$ cuando cambio de $\{\underline{e}_i\}$ a $\{\underline{e}'_i\}$?

$$\underline{e}'_i = Q_{ij}\, \underline{e}_j$$

Comps. de W en $\{\underline{e}'_i\}$

$$W'_{ijkem} = S'_{ijk} T'_{em} = Q_{ip} Q_{jq} Q_{kr} S_{pqr} \times Q_{es} Q_{mt} T_{st}$$

$$= Q_{ip} Q_{jq} Q_{kr} Q_{es} Q_{mt} \underbrace{S_{pqr} T_{st}}_{W_{pqrst}}$$

$$\Rightarrow W \in CT(5)$$

b) $U_{ijk} = S_{ijp} T_{kp}$

$U \in CT(3)$?

$$U'_{ijk} = S'_{ijp} T'_{kp} = Q_{ir} Q_{js} Q_{pt} S_{rst}$$

$$\times\, Q_{ku} Q_{pv} T_{uv}$$

$$= Q_{ir} Q_{js} Q_{ku} \underbrace{Q_{pt} Q_{pv}}_{\underbrace{[Q^T Q]_{tr}}_{\delta_{tr}}} S_{rst} T_{uv}$$

$$T_{ut}$$

$$= Q_{ir} Q_{js} Q_{ku} \underbrace{S_{rst} T_{ut}}_{U_{rsu}}$$

$\Rightarrow U \in CT(3)$

$$
\begin{array}{ccc|c}
 & & & v_1 \\
 & & & v_2 \\
 & & & v_3 \\
\hline
0 & 1 & 0 & v_2 \\
-1 & 0 & 0 & -v_1 \\
0 & 0 & 1 & v_3
\end{array}
$$

$$\Rightarrow Qv = \begin{bmatrix} v_2 \\ -v_1 \\ v_3 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$v_1 = v_2 = -v_1 = 0$$

$$v_1 = -v_2 = 0$$

$$\underbrace{(ST)}_{\text{Tensor}}\underbrace{(u)}_{\text{vector}} = S\underbrace{(Tu)}_{\text{Vector}}$$

$$[ST]_{ij} = S_{ik}T_{kj}$$

$$[(ST)u]_i = (ST)_{ij}u_j = S_{ik}\underbrace{T_{kj}u_j}_{(Tu)_k} = [S(Tu)]_i$$

$$\Rightarrow (ST)u = S(Tu)$$

$$\left[(\underline{u} \otimes \underline{v})\,\underline{w}\right]_i = (\underline{u} \otimes \underline{v})_{ij}\, w_j$$

$$= u_i \underbrace{v_j w_j}_{\underline{v} \cdot \underline{w}} = (\underline{v} \cdot \underline{w})\, u_i$$

$$= \left[(\underline{v} \cdot \underline{w})\,\underline{u}\right]_i$$

$$\Rightarrow (\underline{u} \otimes \underline{v})\,\underline{w} = (\underline{v} \cdot \underline{w})\,\underline{u}$$

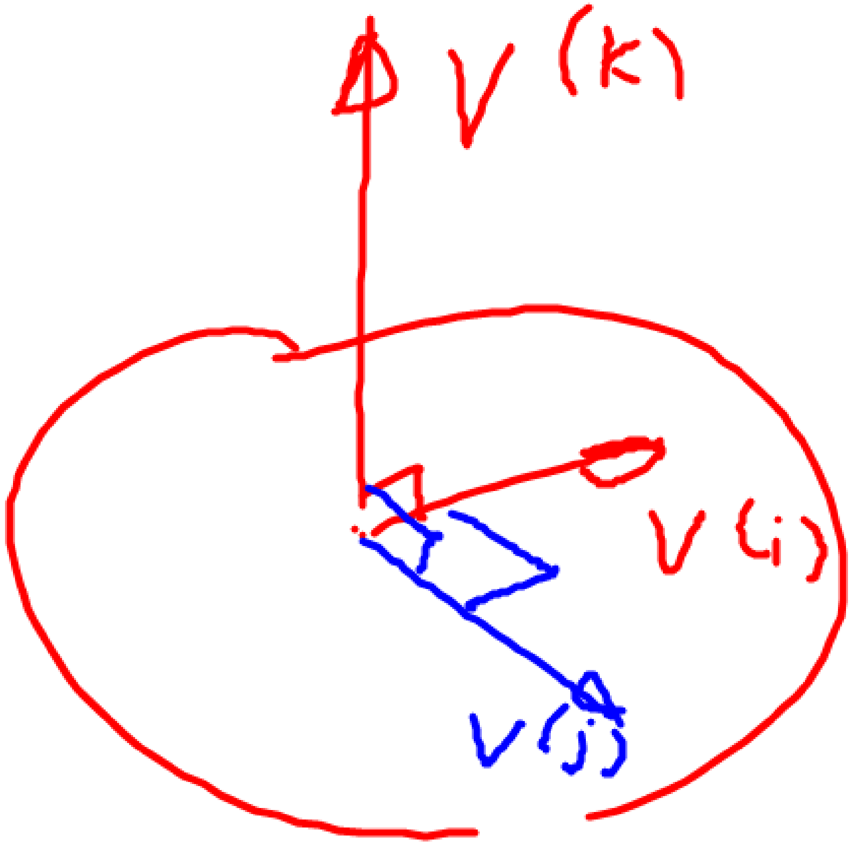
S simétrico $\Rightarrow S_{ij} = S_{ji}$

$$\underline{\underline{S}} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

A antisimétrico $\Rightarrow A_{ij} = -A_{ji}$

$$\underline{\underline{A}} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ -A_{12} & A_{22} & A_{23} \\ -A_{13} & -A_{23} & A_{33} \end{bmatrix}$$

$A_{11} = -A_{11} = 0$



$\lambda_i = \lambda_j \neq \lambda_k$

$$T = \lambda_1 V^{(1)} \otimes V^{(1)} + \lambda_2 V^{(2)} \otimes V^{(2)}$$

$$+ \lambda_3 V^{(3)} \otimes V^{(3)}$$

Si $e_i = V^{(i)}$ $\Rightarrow T_{11} = \lambda_1$

$T_{22} = \lambda_2$

$$\Rightarrow \underline{\underline{T}} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$T_{33} = \lambda_3$

$T_{ij} = 0$ para $i \neq j$

$$\{e_i\},\ \{v^{(i)}\}$$

Si $e'_i \equiv v^{(i)}$

comps. en $\{e_i\}$

$$T' = Q\,T\,Q^T \Rightarrow T = Q^T\,T'\,Q$$

comps. en $\{e'_i\}$

$$Q_{ij} = e'_i \cdot e_j = v^{(i)} \cdot e_j = [v^{(i)}]_j$$

$\underline{v}^{(i)}$: fila i de $\underline{\underline{Q}}$

$$|W| = \begin{vmatrix} 0 & W_{12} & W_{13} \\ -W_{12} & 0 & W_{23} \\ -W_{13} & -W_{23} & 0 \end{vmatrix} =$$

$$= -W_{12} W_{13} W_{23} + W_{13} W_{12} W_{23}$$

$$= 0$$

$$[Qu]_i \cdot [Qv]_i = u_i \cdot (Q^T Q v)_i \ ?$$

$$Q_{ij} u_j \, Q_{ik} v_k = u_j \underbrace{\underbrace{Q^T_{ji} Q_{ik}}_{(Q^T Q)_{jk}} v_k}_{(Q^T Q v)_j}$$

$$= u \cdot (Q^T Q v)$$