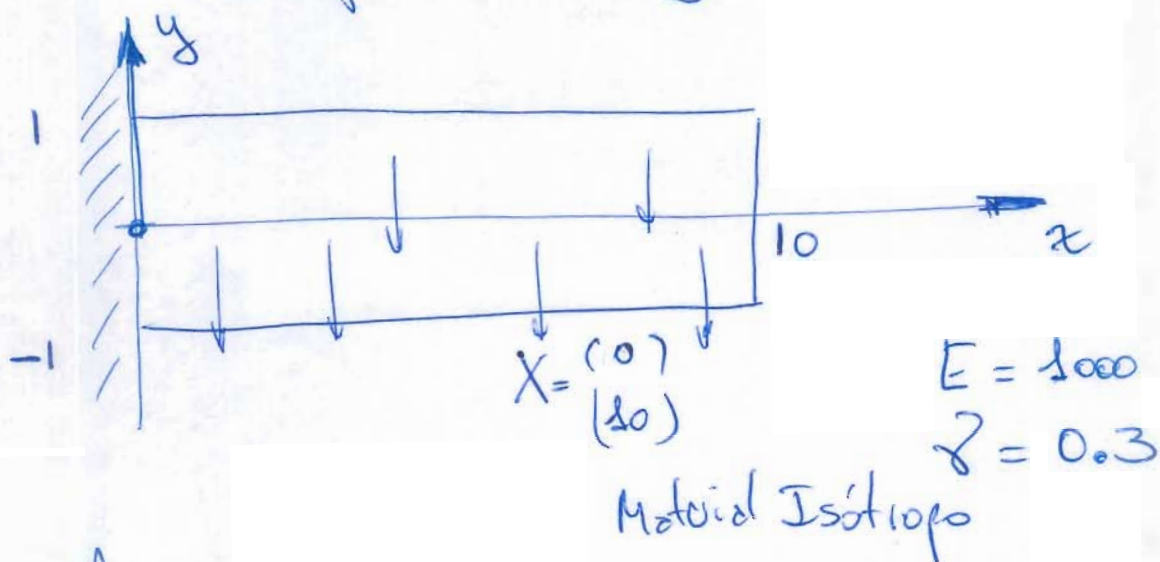


P.T.V.

$$\int_V X_i \delta u_i dV + \int_{S_T} T_i \delta u_i dS = \int_V \sigma_{ij} \delta e_{ij} dV$$

Sea el problema siguiente:



Aplicaciones

$$\underline{u} = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{bmatrix} 1 & x & y & x^2 & xy & y^2 & 0 \\ 0 & x & y & x^2 & xy & y^2 \end{bmatrix}$$

$$\begin{pmatrix} U_{00} \\ U_{01} \\ U_{11} \\ U_{02} \\ U_{12} \\ U_{22} \\ V_{00} \\ V_{01} \\ V_{11} \\ \vdots \\ V_{22} \end{pmatrix}$$

Notas que debe ser

$$u(x=0, y) = 0$$

Luego

$$u = U_{00} + x U_{01} + y U_{11} + x^2 U_{02} + xy U_{12} + y^2 U_{22}$$

$$= U_{00} + y U_{11} + y^2 U_{22} = 0$$

$$\Rightarrow \begin{aligned} U_{00} &= 0 \\ U_{11} &= 0 \\ U_{22} &= 0 \end{aligned}$$

$$v(x=0, y) = 0$$

$$v = V_{00} + x V_{01} + \dots = 0$$

$$\Rightarrow \begin{aligned} V_{00} &= 0 \\ V_{11} &= 0 \\ V_{22} &= 0 \end{aligned}$$

Luego, diremos:

$$\underline{u} = \begin{pmatrix} u \\ v \end{pmatrix} = \underbrace{\begin{bmatrix} x & x^2 & xy & 0 & 0 & 0 \\ 0 & 0 & 0 & x & x^2 & xy \end{bmatrix}}_{\underline{N}} \begin{pmatrix} U_{01} \\ U_{02} \\ U_{12} \\ V_{01} \\ V_{02} \\ V_{12} \end{pmatrix}$$

$$\therefore \underline{\delta u} = \underline{N} \underline{\delta U}$$

$$e_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

Lo escribiremos vectorialmente:

$$\begin{pmatrix} e_{11} \\ e_{22} \\ 2e_{12} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} [x \quad x^2 \quad xy] \\ \frac{\partial}{\partial y} [x \quad x^2 \quad xy] \\ \frac{\partial}{\partial y} [x \quad x^2 \quad xy] \\ + \frac{\partial}{\partial x} [x \quad x^2 \quad xy] \end{pmatrix} \begin{pmatrix} U_{01} \\ U_{02} \\ U_{12} \\ + \\ V_{01} \\ V_{02} \\ V_{12} \\ + \\ V_{01} \\ V_{02} \\ V_{12} \end{pmatrix}$$

Trabaja con:

$$= \underbrace{\begin{bmatrix} 1 & 2x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & x \\ 0 & 0 & x & 1 & 2x & y \end{bmatrix}}_{\underline{\underline{B}}} \begin{pmatrix} U_{01} \\ U_{02} \\ U_{12} \\ V_{01} \\ V_{02} \\ V_{12} \end{pmatrix}$$

Para un material isotrópico

$$\sigma_{ij} = \lambda e_{xx} \delta_{ij} + 2G e_{ij}$$

En estado plano de tensiones:

$$\sigma_{xx} = \frac{E}{1-\nu^2} (e_{xx} + \nu e_{yy})$$

$$\sigma_{yy} = \frac{E}{1-\nu^2} (e_{yy} + \nu e_{xx})$$

$$\sigma_{xy} = \frac{E}{1+\nu} \epsilon_{xy}$$

(4)

Luego, ~~relacion~~ constitutiva:

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = E \underbrace{\begin{bmatrix} \frac{1}{1-\nu^2} & \frac{\nu}{1-\nu^2} & 0 \\ \frac{\nu}{1-\nu^2} & \frac{1}{1-\nu^2} & 0 \\ 0 & 0 & \frac{1}{2(1+\nu)} \end{bmatrix}}_{\underline{C}} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \end{pmatrix}$$

~~En~~

En forma:

$$\underline{\epsilon} = \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \end{pmatrix} = \underline{B} \underline{U} \Rightarrow \delta \underline{\epsilon} = \underline{B} \delta \underline{U}$$

$$\underline{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} = \underline{C} \underline{\epsilon}$$

$$\begin{aligned} \sigma_{ij} \delta \epsilon_{ij} &= \sigma_{11} \delta \epsilon_{11} + \sigma_{12} \delta \epsilon_{12} + \sigma_{22} \delta \epsilon_{22} + \sigma_{21} \delta \epsilon_{21} \\ &= \underline{\sigma} \cdot \delta \underline{\epsilon} \end{aligned}$$

↑ Nota el 2!



Hilsej el riv

U

$$\int_V \delta \underline{u}^T \underline{X} dV = \int_V \delta \underline{e}^T \underline{Q} dV$$

$$\int_V \delta \underline{u}^T \underline{N}^T \underline{X} dV = \int_V \delta \underline{u}^T \underline{B}^T \underline{C} \underline{e} dV =$$

$$= \int_V \delta \underline{u}^T \underline{B}^T \underline{C} \underline{B} \underline{u} dV$$

$\delta \underline{u}, \underline{u}$  sale de l signo integral:

$$\cancel{\delta \underline{u}^T} \int_V \underline{N}^T \underline{X} dV = \int_V \underline{B}^T \underline{C} \underline{B} dV \underline{u}$$

o o

$$\int_V \underline{B}^T \underline{C} \underline{B} dV \underline{u} = \int_V \underline{N}^T \underline{X} dV$$

$$\underline{K} \underline{u} = \underline{F}$$