(2) $Q = \int_{-h}^{h} \rho u(y)\,dy = \frac{\rho \alpha}{2\mu}\int_{-h}^{h}(h^2-y^2)\,dy = \frac{\rho\alpha}{2\mu}\left(h^2 y - \frac{y^3}{3}\right)\Big|_{-h}^{h} =$

$$= \frac{\rho\alpha}{2\mu}\left(h^3 - \frac{h^3}{3} + h^3 - \frac{h^3}{3}\right) = \frac{\rho\alpha h^3}{3\mu}$$

$$V_m = \frac{1}{2h}\int_{-h}^{h} u(y)\,dy = \frac{1}{2h}\,\frac{\alpha h^3}{3\mu} = \frac{\alpha h^2}{6\mu}$$

$$\tau = \mu\frac{\partial u}{\partial y} = -\cancel{\mu}\,\frac{2\alpha y}{2\cancel{\mu}} = -\frac{\alpha y}{2}$$

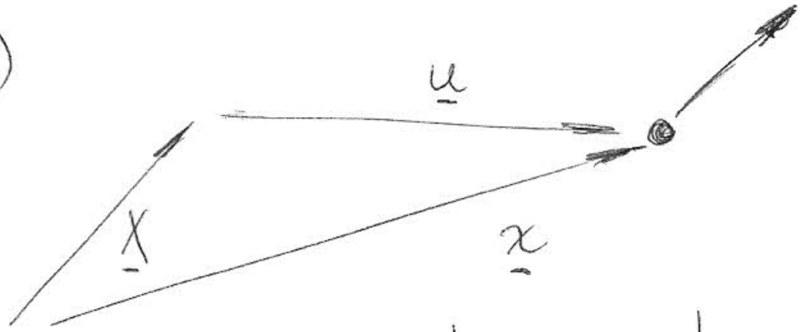
$$\tau\big|_{h} = -\frac{\alpha h}{2} \qquad \tau\big|_{-h} = \frac{\alpha h}{2}$$

(3)

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{bmatrix} e^t & 0 & e^t - 1 \\ 0 & 1 & e^t - e^{-t} \\ 0 & 0 & 1 \end{bmatrix}}_{J} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$$

a) $\det J = e^t \neq 0$

b)



$$\underline{v} = \frac{d\underline{x}}{dt}$$

$$v_1 = \frac{dx_1}{dt} = e^t X_1 + e^t X_3$$

$$v_2 = e^t X_3 + e^{-t} X_3$$

$$v_3 = 0$$

$$X_1 = e^{-t} x_1 + \left(e^{-t} - 1\right) x_3$$

$$X_2 = x_2 + \left(e^{-t} - e^t\right) x_3$$

$$X_3 = x_3$$

$$v_1 = e^t\left(e^{-t} x_1 + \left(e^{-t} - 1\right) x_3\right) + e^t x_3 = x_1 + x_3$$

$$v_2 = \left(e^t + e^{-t}\right) x_3$$

$$v_3 = 0$$

(d) $$\frac{D}{Dt}\int \rho P_{ij}\, dV = \int_V \rho \frac{DP_{ij}}{Dt}\, dV$$

$$\frac{D}{Dt}\int_V \rho P_{ij}\, dV = \int_V \left[\frac{D}{Dt}(\rho P_{ij}) + \rho P_{ij}\frac{\partial v_k}{\partial x_k}\right] dV =$$

$$= \int_V \left[\rho \frac{DP_{ij}}{Dt} + P_{ij}\frac{D\rho}{Dt} + \rho P_{ij}\frac{\partial v_k}{\partial x_k}\right] dV =$$

$$= \int_V \left[\rho \frac{DP_{ij}}{Dt} + P_{ij}\underbrace{\left(\frac{D\rho}{Dt} + \rho\frac{\partial v_k}{\partial x_k}\right)}_{0 \text{ por continuidad}}\right] dV =$$

$$= \int_V \rho \frac{DP_{ij}}{Dt}\, dV \qquad \left(\frac{D}{Dt}\int_V \rho\, dV = 0\right)$$

$$\frac{D}{Dt}\int_V A\, dV = \int_V \frac{DA}{Dt}\, dV + \int A \frac{\partial v_i}{\partial x_i}\, dV =$$

$$= \int_V \frac{\partial A}{\partial t}\, dV + \int_V \left(v_i\frac{\partial A}{\partial x_i} + A\frac{\partial v_i}{\partial x_i}\right) dV =$$

$$= \int_V \frac{\partial A}{\partial t}\, dV + \int_V \frac{\partial}{\partial x_i}(v_i A)\, dV$$

$$= \int_V \frac{\partial A}{\partial t}\, dV + \int_S A v_i n_i\, dS$$