GENERATION OF TURBULENT INLET VELOCITY CONDITIONS FOR LARGE EDDY SIMULATIONS

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Intro and General Objectives

▷ Large eddy simulation (LES) has become an attractive approach to solve fluid flow problems due to the improvement of computational power.

▷ In order to obtain a fully developed turbulence flow without a significant increase in computational costs one must assure that the turbulence inlet conditions are adequately prescribed.

▷ Several methods are available (Tabor and Baba-Ahmadi, *Computer and Fluids*, 2010):

- ▷ Precursor simulation methods: *cyclic domains, preprepared library, concurrent library generation*.
- ▷ Synthesis method: *Fourier techniques, digital filter based method, proper orthogonal decomposition analysis*.

Intro and General Objectives

Objective of this research:

- ▷ Develope a synthesis turbulence method to be used in the generation of inflow boundary conditions for LES with application to turbulent fluid flows.

- ▷ Key points of the proposed technique:
  - ▷ Statistical properties of turbulence.
  - ▷ Isotropic and anisotropic turbulence.
  - ▷ Spectral target function.

Modified discretizing and synthesizing random flow generation (MDSRFG)

▷ The proposed methodology is based on previous approaches:

▷ Random flow generation technique (RFG, Smirnov et al, *Journal of Fluids Engineering*, 2001):

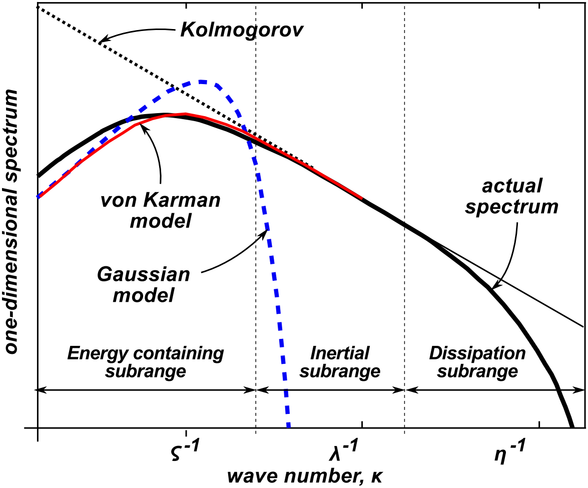
-Inhomogeneous and anisotropic turbulence flow.
-Time and spatial scales incorporated in the formulation.
-Included in the software FLUENT (spectral synthesizer).
-Gaussian's spectral model only.

▷ Discretizing and synthesizing random flow generation method (DSRFG, Huang et al, *Journal of Wind Engineering and Industrial Aerodynamics*, 2010):

-Any model spectrum can be used.
-Inhomogeneous and anisotropic turbulence flow.
-Highly parallelizable algorithmic implementation.
-Time correlation?.
-What about frequency interval size (Δf)?.

MDSRFG (cont.)

Huang et al. (2010) analysis:



MDSRFG (cont.)

According to Huang et al. (2010) (based on the work of Smirnov et al. (2001) and Kraichnan (1970)) an inhomogeneous and anisotropic turbulent flow field $\mathbf{u}(\mathbf{x}, t)$ can be synthesized as follows:

$$u_i(\mathbf{x}, t) = \sum_{m=1}^{M} \sum_{n=1}^{N} \left[p_i^{m,n} \cos\left(\tilde{k}_j^{m,n} \tilde{x}_j + \omega_{m,n} t \right) + q_i^{m,n} \sin\left(\tilde{k}_j^{m,n} \tilde{x}_j + \omega_{m,n} t \right) \right],$$

where M and N are the number of wavenumbers (k_m) considered in the discretization of the target spectrum and the sample size of each k_m, respectively, and $\omega_{m,n} \in N(0, k_m U_{\mathrm{avg}})$.

MDSRFG (cont.)

and

$$p_i^{m,n} = \mathrm{sign}(r_i^{m,n})\sqrt{\frac{4}{N}E_i(k_m)\frac{(r_i^{m,n})^2}{1+(r_i^{m,n})^2}},$$

$$q_i^{m,n} = \mathrm{sign}(r_i^{m,n})\sqrt{\frac{4}{N}E_i(k_m)\frac{1}{1+(r_i^{m,n})^2}}.$$

$$\tilde{x} = \frac{x}{L_s},$$

$$\tilde{k}^{m,n} = \frac{k^{m,n}}{k_0},$$

$$L_s = \theta_1\ \sqrt{L_u^2 + L_v^2 + L_w^2}.$$

where $r_i^{m,n} \in N(0,1)$, θ_1 is a scalar value between 1 and 2 used to adjust the spatial correlation.

MDSRFG (cont.)

$$\boxed{p_i^{m,n} = \operatorname{sign}(r_i^{m,n})\sqrt{\frac{4}{N}E_i(k_m)\frac{(r_i^{m,n})^2}{1+(r_i^{m,n})^2}}}$$

$$\boxed{q_i^{m,n} = \operatorname{sign}(r_i^{m,n})\sqrt{\frac{4}{N}E_i(k_m)\frac{1}{1+(r_i^{m,n})^2}}}$$

these factors align the energy spectrum according to the anisotropy conditions of the turbulence, providing a sinthesized velocity series that must satisfy the mean square values in each of the spatial coordinate axes.

MDSRFG (cont.)

Starting from the mean square value of a random function $f(t)$ definition:

$$f^2_{\mathrm{rms}}(t) = \lim_{T\to\infty} \frac{1}{T} \int_0^T f^2(t)\ dt$$

we obtain:

$$u^2_{\mathrm{rms},i}(x,t) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \left\{ \sum_{m=1}^{M} \sum_{n=1}^{N} [p_i^{m,n} \cos(\tilde{k}_j^{m,n} \tilde{x}_j + \omega_{m,n} t) + \right.$$
$$\left. q_i^{m,n} \sin(\tilde{k}_j^{m,n} \tilde{x}_j + \omega_{m,n} t)] \right\}^2 dt.$$

MDSRFG (cont.)

And after some mathematical manipulations:

$$\overline{u_i u_i} = \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{N} p_i^{m,n} p_i^{m,n} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{N} q_i^{m,n} q_i^{m,n}$$
$$= 2\int_0^{\infty} E(k)dk \;\approx\; 2\sum_{m=1}^{M} E(k_m)\Delta k_m,$$

hence,

$$\overline{u_i u_i} = \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{i=1}^{3}\left[\frac{4}{N}E_i(k_m)\frac{(r_i^{m,n})^2}{1+(r_i^{m,n})^2} + \frac{4}{N}E_i(k_m)\frac{1}{1+(r_i^{m,n})^2}\right]$$
$$= \frac{2}{N}\sum_{m=1}^{M}\sum_{n=1}^{N} E(k_m) = 2\sum_{m=1}^{M} E(k_m),$$

MDSRFG (cont.)

$$\boxed{\overline{u_i u_i} = 2\sum_{m=1}^{M} E(k_m)}$$

as $E(k_m)$ is a positive quantity for any k, the kinetic energy is represented by a divergent series. This causes a strong dependency of the turbulence intensity of the generated fluctuating velocities with the number of points M considered to discretize the target spectrum.

MDSRFG (cont.)

So, some modifications to the DSRFG method are proposed:

$$u_i(\mathbf{x}, t) = \sum_{m=1}^{M} \sum_{n=1}^{N} \left[p_i^{m,n} \cos\left(\tilde{k}_j^{m,n} \tilde{x}_j + \omega_{m,n} \frac{t}{\tau_0} \right) + q_i^{m,n} \sin\left(\tilde{k}_j^{m,n} \tilde{x}_j + \omega_{m,n} \frac{t}{\tau_0} \right) \right],$$

where $p_i^{m,n}$ and $q_i^{m,n}$ are modified according to the previous analysis and τ_0 is added in order to handle the time correlation of the series.

$$p_i^{m,n} = \mathrm{sign}(r_i^{m,n}) \sqrt{\frac{4c_i}{N} E_i(k_m) \Delta k_m \frac{(r_i^{m,n})^2}{1 + (r_i^{m,n})^2}},$$

$$q_i^{m,n} = \mathrm{sign}(r_i^{m,n}) \sqrt{\frac{4c_i}{N} E_i(k_m) \Delta k_m \frac{1}{1 + (r_i^{m,n})^2}}.$$

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Validation of the procedure

1) Same test as in the work of Huang et al., *Journal of Wind Engineering and Industrial Aerodynamics*, 2010): an inhomogeneus and anisotropic turbulent flow field.

- ▷ von Kármán models:

$$S_u(f) = \frac{4(I_u U_{\mathrm{avg}})^2 (L_u/U_{\mathrm{avg}})}{[1 + 70.8(fL_u/U_{\mathrm{avg}})^2]^{5/6}},$$

$$S_v(f) = \frac{4(I_v U_{\mathrm{avg}})^2 (L_v/U_{\mathrm{avg}})(1 + 188.4(2fL_v/U_{\mathrm{avg}})^2)}{[1 + 70.8(2fL_v/U_{\mathrm{avg}})^2]^{11/6}},$$

$$S_w(f) = \frac{4(I_w U_{\mathrm{avg}})^2 (L_w/U_{\mathrm{avg}})(1 + 188.4(2fL_w/U_{\mathrm{avg}})^2)}{[1 + 70.8(2fL_w/U_{\mathrm{avg}})^2]^{11/6}}.$$

- ▷ Turbulence intensity values: $I_u = 8\%$, $I_v = 16\%$, $I_w = 24\%$.
- ▷ Turbulence integral length scales: $L_u = 0.6$ m, $L_v = 0.3$ m, $L_w = 0.1$ m

Validation of the procedure (cont.)

To apply the procedure we must first obtain the c_i values for p_i and q_i:

$$u^2_{\mathrm{rms},1} = (I_u U_{\mathrm{avg}})^2 = 2c_1 \int_0^\infty S_u(k)dk$$

$$\approx 2c_1\ 0.2377\ \beta\left(\frac{1}{3},\frac{1}{2}\right) I_u^2 U_{\mathrm{avg}} \quad \Rightarrow \quad c_1 = \frac{U_{\mathrm{avg}}}{2},$$

$$u^2_{\mathrm{rms},2} = (I_v U_{\mathrm{avg}})^2 = 2c_2 \int_0^\infty S_v(k)dk$$

$$\approx 2c_2 \left[0.1189\ \beta\left(\frac{1}{2},\frac{4}{3}\right) + 0.3163\ \beta\left(\frac{1}{3},\frac{3}{2}\right)\right] I_v^2 U_{\mathrm{avg}}$$

$$\Rightarrow \quad c_2 = c_3 \approx \frac{U_{\mathrm{avg}}}{2}.$$

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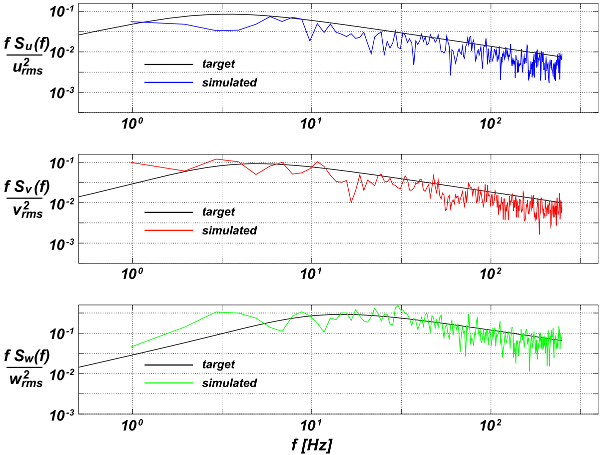
Validation of the procedure (cont.)

Comparison of the spectra by the MDSRFG method and the target spectrum:



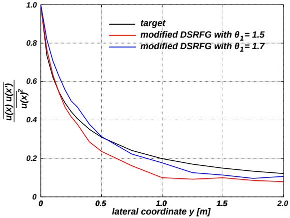
Validation of the procedure (cont.)

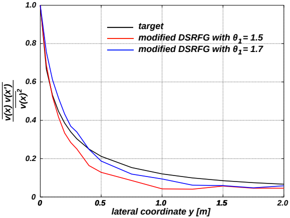
Comparison of the rms values of the simulated fluctuating velocities:

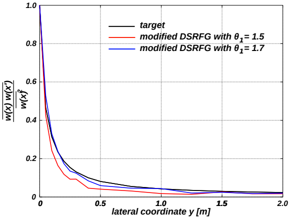
	σ_u	σ_v	σ_w
Scaling and transformation	0.9968	2.44	2.9956
Aligning and remapping	0.95	1.9987	3.08
MDSRFG approach	1.0527	2.1850	3.1123
target	**1.12**	**2.24**	**3.36**

Validation of the procedure (cont.)

Spatial correlation:







Validation of the procedure (cont.)

Time correlation: samples of temporal correlations from the MDSRFG and the DSRFG methods are shown in the next figures for each velocity component. Also, they are compared to the autocorrelation function of a random stationary process:

$$R_i(\tau) = e^{-|\tau|/T_i},$$

with time scales T_i $(i = u, v, w)$ computed as

$$T_i = \int_0^\infty R_i(\tau) d\tau \equiv \sum_{j=0}^{M_0} R_i(j\ \delta\tau)\ \delta\tau, \tag{1}$$

where $M_0 < M$. Low frequency fluctuations cause oscillations on the time correlation around the zero value as the time lag tends to infinite. Consequently, if equation (1) is approximated without an adequate upper limit of the sum, it will fail to estimate the scale. In this work the time scale is computed by setting M_0 to the first τ-axis crossing value.

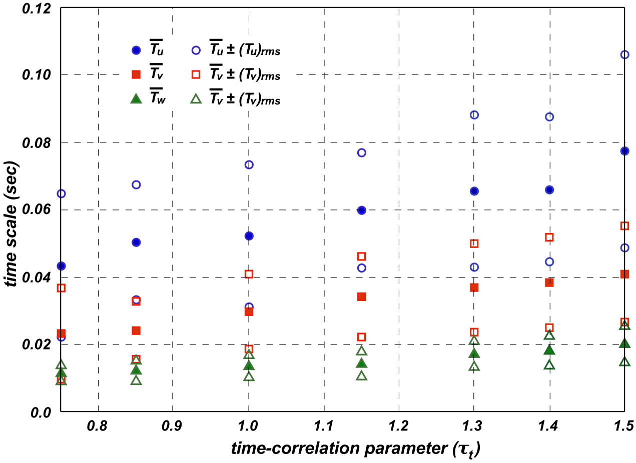
Validation of the procedure (cont.)

Time correlation: Time scale statistics comparison (sec).

	T_u	T_v	T_w
DSRFG approach	0.034 ± 0.028	0.022 ± 0.009	0.010 ± 0.002
MDSRFG approach	0.043 ± 0.021	0.023 ± 0.014	0.011 ± 0.002
target	0.043	0.021	0.007

Validation of the procedure (cont.)

Time correlation: Time scale statistics of the fluctuating velocity components as a function of τ_0 obtained by the MDSRFG method.



Some remarks

- ▷ For each node at the inlet section the cost at each time step is $O(MN)$. (Same as in the DSRFG method)
- ▷ The turbulence synthesis for some number of time steps (or the entire simulation process) can be done prior to the LES computations. (Same as in the DSRFG method)
- ▷ The anisotropic turbulence conditions at the inlet plane can be obtained by performing a previous RANS simulation or by experimental measurements. The two input parameters, L_s and τ_0 must be selected in order to reproduce the statistical properties of the flow under consideration. (Same as in the DSRFG method in the case of L_s)

Some remarks

▷ Is important to highlight the possibility to slightly modify the time scale with different τ_0 values in the MDSRFG method while the DSRFG method are limitated in this way. Note that even the target time scales, estimated by the Taylor's hypothesis, are in accordance with the values obtained by the DSRFG method, the application of the MDSRFG approach leads to a wider range of possible values by changing τ_0.

▷ Also, to validate the relation proposed for the equations of p_i and q_i, the influence of the frequency interval size Δf over the rms values of the time series was analyzed. What it is expected is that the synthetic turbulence generation provides a correct rms values as $\Delta f \to 0$, that is, as the discretization of the spectrum becomes finer the energy turbulence content in each frequency will be included in the time series generation.

Some remarks

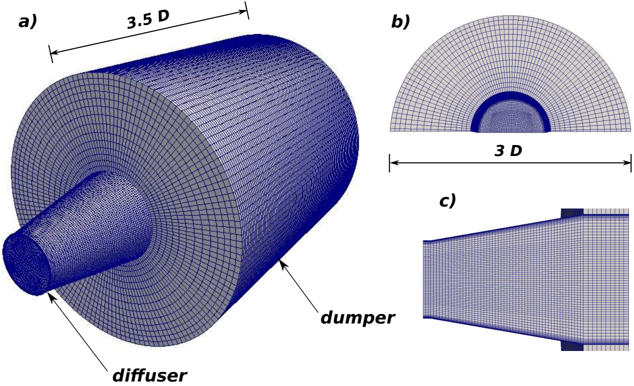
It can be seen that the method proposed in this work converges to the target values as Δf becomes smaller while in the case of the DSRFG method the values do not converge at all.

Δf	σ_u		σ_v		σ_w	
	DSRFG	MDSRFG	DSRFG	MDSRFG	DSRFG	MDSRFG
10	0.49	0.76	1.27	1.84	2.05	2.95
5	0.83	0.87	1.93	2.07	2.94	3.01
2	1.47	0.98	3.15	2.11	4.69	3.03
1	2.17	1.04	4.47	2.11	6.75	3.11
target	1.12		2.24		3.36	

▷ As a final observation we notice that the proposed approach, as any synthesized turbulence generation method, must be used as a turbulence initializer, i.e., a perturbation generator that "triggers" the transition to a fully developed turbulence state by LES. In this regard, it must be said that independently of the selected L_s value, the resolved scales are in concordance with the mesh (filter) size which is inherent to the LES conception.

Validation of the procedure (2)

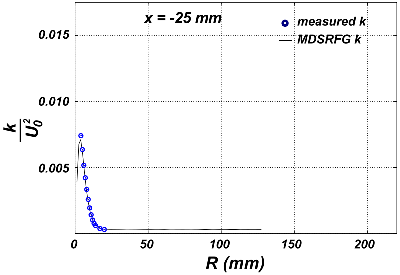
2) This test case consists of a swirling boundary layer developing in a conical diffuser that was experimentally studied by Clausen *et al* (1993).

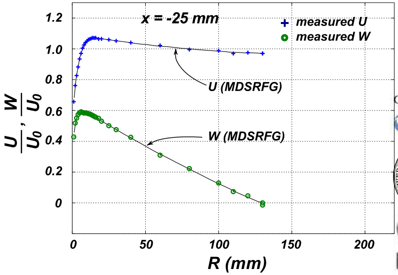


Reynolds number of the experimental test was 2.08×10^6 based on the diameter of the inlet section D, the mean axial velocity $U_x = 11.6$ m/s, the kinematic viscosity $\nu = 1.45 \times 10^{-6}$ m^2/s.

Validation of the procedure (2)

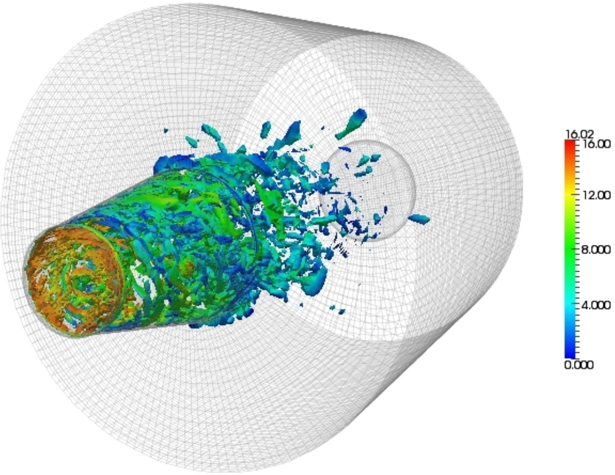
Inlet conditions by the MDSRFG method:





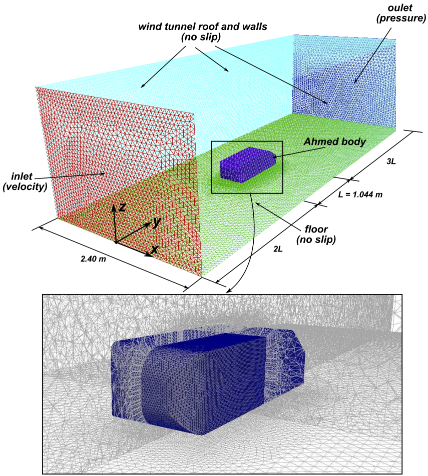
Validation of the procedure (2)

Q-vortex structures:

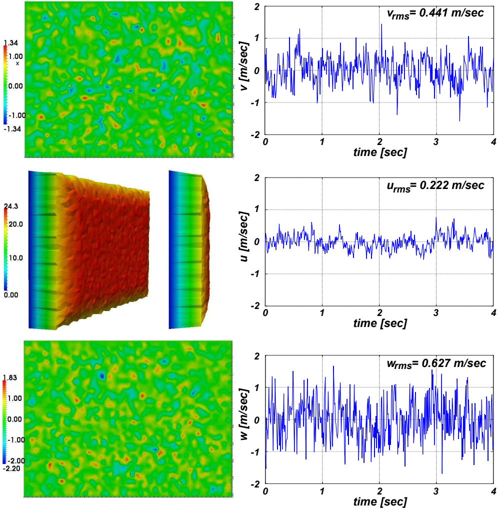


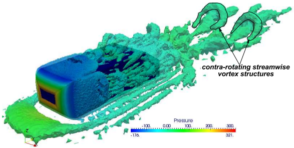
Validation of the procedure (3)

3) Simulation of a wind tunnel inlet flow field (Ahmed body test).



Validation of the procedure (3)





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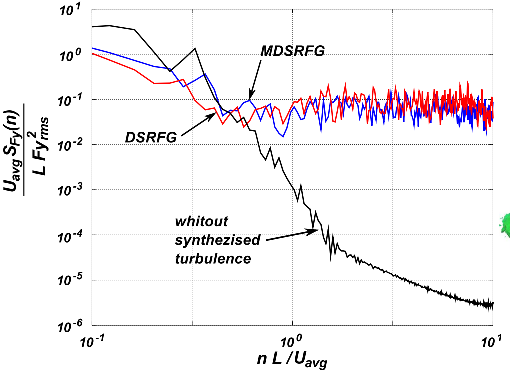
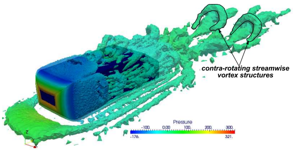
Validation of the procedure (3)

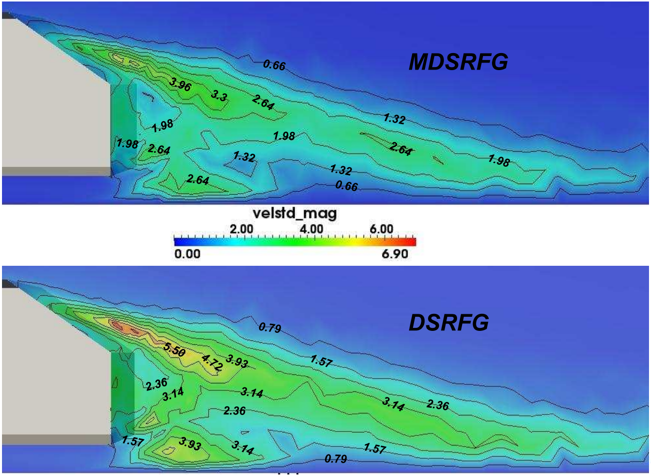
Drag force spectrum.





Validation of the procedure (3)

Standard deviation velocity magnitude.



Conclusions

▷ A general method for the generation of inflow synthesized turbulence was presented and evaluated. The method is based on the DSRFG method, preserving its main characteristics and advantages.

▷ The key point of the MDSRFG method presented in this study is that it preserves the statistical quantities that would be prescribed at the inlet of the domain independently of the number of samples M (number of points in the spectrum) considering in the computation of the factors $p_i^{m,n}$ and $q_i^{m,n}$.

▷ As each fluctuating velocity component is generated in each node independently of the others, the method is highly parallelizable. Furthermore, the generation of each nodal fluctuating velocity component can be done previously to the computation by LES, calling in each time step the corresponding nodal value.

Conclusions

▷ Analysing the swirling flow inside a diffuser problem, the MDSRFG method has shown that can set properly the inlet conditions for LES of turbulent flows.

▷ With regard to the simulation of the flow over the Ahmed body, it has been shown that the results obtained in the LES, i.e., force coefficients, level of unsteadiness in the wake and the back-light of the model, are very sensitive to the upstream inflow conditions.

::THANKS::