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The Mahler measure of linear forms as special values of solutions of algebraic differential equations

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Abstract

We prove that for each $n \geq 4$ there is an analytic function $F_n(x)$ satisfying an algebraic differential equation of degree $n+1$ such that the logarithmic Mahler measure of the linear form $\mathbb{L}_n = x_1 + \cdots + x_n$ can be essentially computed as the evaluation of $F_n(z)$ at $z = n^{-1}$. We show that the coefficients of the series representing $F_n(z)$ can be computed recursively using the n-th. symmetric power of a second order linear algebraic differential equation and we give an estimate on the growth of these coefficients.

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