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STRUCTURAL OPTIMIZATION OF STIFFENED SHELLS USING EVOLUTIONARY ALGORITHMS

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Abstract. *The optimum design of stiffened shell structures is the main objective of this paper. Combinatorial optimization methods and more specifically algorithms based on evolution strategies are implemented for the solution of the optimization problem. Three optimization types have been considered: sizing, sizing combined with shape and sizing combined with shape and topology. The efficiency of evolution strategies for solving optimization problems of real-scale stiffened shell structures under design codes is examined. For the discretization of the stiffened shell structures the TRIC (TRIangular Composite) and BEC (BEam Composite) elements have been used. TRIC is a simple but sophisticated 3-node shear-deformable isotropic and composite facet shell element suitable for large-scale linear and nonlinear structural behaviour of complex shell structures, while BEC is a 2-node isotropic, composite shear-deformable beam element in space.*

1 INTRODUCTION

During the last three decades many numerical methods have been developed to meet the demands of structural design optimization. These methods can be classified in two categories, the deterministic and the probabilistic ones. Mathematical programming methods are the most popular methods of the first category, while evolutionary algorithms are the most widely used class of methods of the second category. These evolutionary algorithms rely on analogies to natural processes and they are evolution-based systems maintaining a population of potential solutions.

The sizing optimization combined with shape and/or topology optimization of large-scale three-dimensional stiffened shell structures is a computationally intensive task. In shape-topology-sizing optimization of stiffened shells the aim is to minimize the weight of the structure under certain restrictions. The optimum design of shell structures has attracted comparably less attention. Only recently some studies have been presented dealing with this problem [1-4]. In this work the efficiency of Evolution Strategies (ES) belonging to the evolutionary algorithms for solving optimization problems is investigated for the optimization of real-scale stiffened structures under design codes.

The analysis of shells itself also presents a challenge, since their formulation may become cumbersome and their behaviour can be unpredictable with regard to geometry or support conditions. In this work, the robust and accurate TRIC shell element, proposed by Argyris and co-workers is employed. TRIC is based on the natural mode method [5] while its formulation is simple but sophisticated making it suitable for the analysis of thin and moderately thick isotropic or composite plate and shell structures. Significant saving in computational time is also achieved thanks to the analytic expression of the required quantities, while due to its natural formulation TRIC does not suffer from the various locking phenomena.

2 STRUCTURAL OPTIMIZATION

2.1 Formulation of the problem

Structural optimization problems are characterized by various objective and constraint functions which are generally non-linear functions of the design variables. These functions are usually implicit, discontinuous and non-convex. The mathematical formulation of structural optimization problems with respect to the design variables, the objective and constraint functions depend on the type of the application. However, all optimization problems can be expressed in standard mathematical terms as a non-linear programming problem. A discrete structural optimization problem can be formulated in the following generic form:

$$\begin{aligned} &\min \quad && F(s) \\ &\text{subject to} \quad && g_j(s) \leq 0 \quad j=1,...,m \\ & && s_i \in R^d, \quad i=1,...,n \end{aligned} \tag{1}$$

where F(s) and g(s) denote the objective and constraints functions respectively. R^d is a given set of discrete values, the design variables s_i (i=1,...,n) can take values only from this set.

2.2 Design codes

Shell structures, stiffened and non stiffened, are very common in engineering practice in order to cover long and/or wide span and column-free spaces or they are used as storage tanks of gas liquid or grain materials or in aeronautical structures. Shell structures are very efficient structures because they can accomplish the best performance in terms of the ratio of strength over the material volume.

The design codes adopted to perform the optimization are Eurocodes 1 and 3 [6, 7]. In order to comply with the Eurocode requirements, certain constraint functions are employed both for the shell structure and the stiffening beams. The loads imposed refer to the ultimate limit state according to Eurocode 1. Details on the load combinations are provided subsequently.

For the design of the stiffening beams the constraining functions are given by the formula of EC3 [7] regarding beams subjected to biaxial bending under compression.

$$\frac{N_{sd}}{Af_y/\gamma_{M1}}+\frac{M_{y,sd}}{W_{pl,y}f_y/\gamma_{M1}}+\frac{M_{z,sd}}{W_{pl,z}f_y/\gamma_{M1}}\leq 1.0 \tag{2}$$

The upper flange of the beam cross section is assumed to be rigidly connected to the shell part of the structure and therefore longitudinal buckling is not developed in the beams. N_{sd}, $M_{sd,y}$, $M_{sd,z}$ are the stress resultants, $W_{pl,y}, W_{pl,z}$ are the plastic first moment of inertia and f_y the yield stress and γ_{M1} is a safety factor which is considered equal to 1.10.

For the design of the shell part of the structure, stress constraints are imposed. The Von Mises yield criterion is employed in order to assess the value of an equivalent stress that will be compared with the yield stress f_y. Therefore the following expression should be satisfied for each triangular shell element:

$$\sqrt{\sigma_1^3+\sigma_2^3-3\sigma_1\sigma_2+3\tau^2}\leq f_y/\gamma_{M0} \tag{3}$$

where σ_1, σ_2, τ are the principal and the shear stresses in the triangle mid-surface. The safety factor γ_{M0} is considered equal to 1.10.

2.3 Shape-topology-sizing optimization

The problem of shape-topology-sizing optimization of stiffened shell structures with the objective to minimize the weight of the structure under certain behavioural constraints on stresses and displacements is considered. This aim is implemented in this work in three phases as follows: shape, topology and sizing. In the first phase the general shape of the structure is a predefined curved surface while the final shape is controlled by the inclination of the curved surface at the supports. In the second phase the position of the stiffeners per x and/or y dimensions in plan are defined. Finally, in the third phase the dimensions of the cross-sectional areas of the stiffeners and the shell thickness are considered as additional design variables of the combined structural optimization problem of the first two phases.

3. THE TRIC SHELL ELEMENT

An attempt to device a shell element with robustness, accuracy and efficiency has led to the derivation of the TRIC shell element [8]. The derivation of TRIC's stiffness matrix was established upon a rather physical approach based on the observation of the element's deformational modes and the accumulated experience of Argyris and co-workers obtained from previous shell elements that they developed using physical lumping procedures. The formulation is based on the natural mode finite element method [5], a method introduced by J. Argyris in the 1950's that separates the pure deformational modes-also called natural modes-from the rigid body movements of the element.

3.1 Kinematics of the element

For the multilayered composite triangular shell element four coordinate systems are adopted. Namely, the natural coordinate system that has the three axes parallel to the sides of the triangle, the local elemental coordinate system, placed at the triangle's centroid and the global Cartesian coordinate system where global equilibrium refers. Finally, for each ply of the triangle, a material coordinate system 1, 2, 3 is defined with axis 1 being parallel to the direction of the fibers. The use of these different systems makes TRIC suitable for modeling multilayer anisotropic shell structures and can degenerate as special case to a sandwich or a single–layer configuration.

Three total natural axial strains γ_t are measured parallel to the edges of the triangle. These strains are in the natural formulation the equivalent to the engineering strains of the Cartesian system. Similarly, the total natural transverse shear strains γ_s are defined for each of the triangle edges. The axial strains γ_t are related to the three in-plane local Cartesian strains γ' according to the expression

$$\gamma_t = B^t\gamma' \Leftrightarrow \begin{Bmatrix} \gamma_{t\alpha} \\ \gamma_{t\beta} \\ \gamma_{t\gamma} \end{Bmatrix} = \begin{bmatrix} c^2{}_{\alpha x'} & s^2{}_{\alpha x'} & \sqrt{2}s_{\alpha x'}c_{\alpha x'} \\ c^2{}_{\beta x'} & s^2{}_{\beta x'} & \sqrt{2}s_{\beta x'}c_{\beta x'} \\ c^2{}_{\gamma x'} & s^2{}_{\gamma x'} & \sqrt{2}s_{\gamma x'}c_{\gamma x'} \end{bmatrix} \begin{Bmatrix} \gamma_{x'x'} \\ \gamma_{y'y'} \\ \sqrt{2}\gamma_{x'y'} \end{Bmatrix} \tag{4}$$

where $c_{ix'}$ and $s_{ix'}$ is the cosine and the sine of the angle between the i side of the triangle and the local x axis. Similarly for the transverse shear strain:

$$\gamma_s = T_s\gamma_s' \Leftrightarrow \begin{Bmatrix} \gamma_\alpha \\ \gamma_\beta \\ \gamma_\gamma \end{Bmatrix} = \begin{bmatrix} c_{\alpha x'} & s_{\alpha x'} \\ c_{\beta x'} & s_{\beta x'} \\ c_{\gamma x'} & s_{\gamma x'} \end{bmatrix} \begin{Bmatrix} \gamma_{x'z'} \\ \gamma_{y'z'} \end{Bmatrix} \tag{5}$$

The constitutive relations between the natural stresses and the total natural strains are established by initiating the following sequence of coordinate system transformations

Material system → Local system → Natural system

With simple geometric transformations one can easily reach to an expression for the constitutive matrix in the natural coordinate system for both axial and transverse deformations. The corresponding natural stresses σ_c and the natural transverse shear stresses σ_s are obtained

$$\begin{Bmatrix} \sigma_c \\ \sigma_s \end{Bmatrix}_r = \begin{bmatrix} \kappa_{ct} & \cdot \\ \cdot & \chi_s \end{bmatrix}_r \begin{Bmatrix} \gamma_t \\ \gamma_s \end{Bmatrix}_r \tag{6}$$

alid for each layer r. Matrix κ_{ct} defines the constitutive matrices of axial and symmetrical bending while matrix χ_s corresponds to the anti-symmetrical bending and transverse shear modes. Additional information for the derivation of the natural constitutive matrix can be found in [8].

3.2 Natural modes and generalized forces and moments

The multilayered shell element TRIC has 6 Cartesian degrees of freedom per node. The natural stiffness is based only on deformations and not on associated rigid body motions. The element has 18 degrees of freedom but the actual number of straining modes is 12, schematically:

18 Cartesian d.o.f. – 6 rigid body d.o.f. = 12 straining modes

The element TRIC includes 6 rigid body and 12 straining modes grouped in the vector

$$\underset{(18x1)}{\rho'_e} = \begin{bmatrix} \underset{(6x1)}{\rho_0} \\ \underset{(12x1)}{\rho_N} \end{bmatrix} \tag{7}$$

Where ρ_0, ρ_N represent the rigid body and the straining modes, respectively. The natural modes ρ_N are related to the elemental Cartesian $\bar{\rho}$ via

$$\rho_N = \bar{\alpha}_N \bar{\rho} \tag{8}$$

and the total axial strains are related to ρ_N

$$\gamma_t = \alpha_N \rho_N \tag{9}$$

Matrices $\bar{\alpha}_N$ and α_N are always related to the current geometry of the element only. The local Cartesian elemental vector $\bar{\rho}$ is connected to the global Cartesian elemental vector ρ via

$$\bar{\rho} = T_{06}\rho \tag{10}$$

where T_{06} is a matrix containing direction cosines [8]. Using (10) we may write (8) as

$$\rho_N = \bar{\alpha}_N \bar{\rho} = \bar{\alpha}_N T_{06}\rho \tag{11}$$

3.3 Axial and symmetric bending stiffness terms

The natural stiffness matrix corresponding to the axial and symmetric bending modes can be produced from the statement of variation of the strain energy with respect to the natural coordinates, viz.

$$\delta U = \int_V \sigma_c^t \delta\gamma_t dV \tag{12}$$

Following a series of calculations, the expression for the natural stiffness matrix is derived. Transformations are subsequently initiated calculating the natural matrix first to the local coordinate system and then to the global coordinate system

$$\delta U = \rho_N^t \left[\underbrace{T_{06}^t \left[\underbrace{\bar{\alpha}_N^t \left[\underbrace{\int_V \alpha_N^t \kappa_{ct} \alpha_N dV}_{\text{natural coord. (12x12)}} \right] \bar{\alpha}_N}_{\text{local coord. (18x18)}} \right] T_{06}}_{\text{global coord. (18x18)}} \right] \rho_N \tag{13}$$

Details concerning the element’s full natural and Cartesian stiffness matrices are sited in [5].

4 THE BEC BEAM ELEMENT

The BEC beam was proposed by Tenek and Argyris [9] and was the outcome of an attempt to device a beam element suitable for modeling laminate beams. Its numerical efficiency and ability to overcome various parasitic phenomena make it a good choice even when it comes to isotropic beams. BEC and TRIC can be combined in a very neat way due to the similarities on their formulations, which again is based on the natural mode finite element method.

4.1 Kinematics

Four coordinate systems already described for TRIC also apply for the BEC beam element. Each node has six degrees of freedom; therefore each element consists of 12 local dof. The formulation is again based on separating the deformation modes to the six rigid body modes that leave the element unstrained and to six natural straining modes. Each mode refers to a certain straining situation. Therefore, an explicit relation between the strain and the natural mode vector can be established:

$$\begin{bmatrix} \gamma_{xx} \\ \sqrt{2}\gamma_{xy} \\ \sqrt{2}\gamma_{xz} \end{bmatrix} = \frac{1}{l} \begin{bmatrix} 1 & z & -3z\zeta & y & -3y\zeta & . \\ . & . & . & . & . & \frac{1}{\sqrt{2}}\left(\frac{\partial \Psi(y,z)}{\partial y} - z\right) \\ . & . & . & . & . & \frac{1}{\sqrt{2}}\left(\frac{\partial \Psi(y,z)}{\partial z} + y\right) \end{bmatrix} \begin{bmatrix} \rho_{N1} \\ \rho_{N2} \\ \rho_{N3} \\ \rho_{N4} \\ \rho_{N5} \\ \rho_{N6} \end{bmatrix} \tag{14}$$

or using tensorial notation $\gamma = a_N \rho_N$, where ζ is the non-dimensional natural coordinate along the element and Ψ(y, z) is the warping function that describes the behavior under torsion.

4.2 Natural modes and stiffness matrix

As already mentioned, BEC includes 6 rigid body ρ_0 and 6 straining modes ρ_N, grouped in the vector

$$\underset{(18x1)}{\rho'_e} = \begin{bmatrix} \underset{(6x1)}{\rho_0} \\ \underset{(6x1)}{\rho_N} \end{bmatrix} \tag{15}$$

The correlation between the natural straining modes and the local degrees of freedom is achieved by means of simple algebraic expressions.

$$\underset{(6x1)}{\rho_N} = \underset{(6x12)}{\overline{a}_N} \; \underset{(12x1)}{\overline{\rho}} \tag{16}$$

where $\overline{\alpha}_N$ is a transformation matrix. The stiffness matrix for each element in the global system is formulated through the expression:

$$k = T_{04}^t \left[\underbrace{ \overline{a}_N^t \underbrace{\left[\int_V a_N^t \kappa a_N^t \right]}_{natural\, k_N} \overline{a}_N }_{local\, k'} \right] T_{04} \tag{17}$$

where κ is the constitutive matrix that refers to the properties of the material employed and T_{04} the matrix of direction cosines.

5 EVOLUTION STRATEGIES FOR DISCRETE OPTIMIZATION PROBLEMS

Evolutionary computation encompasses methods of simulating evolution on computing systems. The first attempt in the field of evolutionary computation was focused in building a computer program that would simulate the process of evolution in nature. Evolutionary algorithms belong to evolutionary computation and represent the probabilistic category of optimization methods. Evolutionary algorithms have been found capable to produce very powerful and robust search mechanisms although the similarity between these algorithms and the natural evolution is based on crude imitation of biological reality. The resulting evolutionary algorithms are based on a population of individuals, which are subjected to processes of mutation, recombination/crossover and selection. Among the most widely used class of evolutionary algorithms are the Evolutionary Programming, Genetic Algorithms and Evolution Strategies.

In structural optimization problems, where the objective function and the constraints are highly non-linear functions of the design variables, the computational effort spent in gradient calculations required by the mathematical programming algorithms is usually large. In two studies by Papadrakakis et al. [10,11] it was found that probabilistic search algorithms are computationally efficient even if greater number of optimization cycles is needed to reach the optimum. These cycles are computationally less expensive than in the case of mathematical programming algorithms since they do not need gradient evaluation. Furthermore, probabilistic methodologies were found, due to their random search, to be more robust in finding the global optimum, whereas mathematical programming algorithms may be trapped in local optima.

The ES optimization algorithm starts with a set of parent vectors and if any of these parent vectors gives an infeasible design then this parent vector is modified until it becomes feasible. Subsequently, the offsprings are generated and checked if they are in the feasible region. The computational efficiency of the multi-membered ES is affected by the number of parents and offsprings involved. The ES algorithm for structural optimization applications can be stated as follows :

1. *Selection step:* selection of s_j ($j = 1,2,...,\mu$) parent vectors of the design variables
2. *Analysis step:* solve $K(s_j)u_j = f$ ($j=1,2,...,\mu$)
3. *Constraints check :* all parent vectors become feasible
4. *Offspring generation:* generate s_j, ($j=1,2,...,\lambda$) offspring vectors of the design variables
5. *Analysis step:* solve $K(s_j)u_j = f$ ($j=1,2,...,\lambda$)
6. *Constraints check:* if satisfied continue, else change s_j and go to *step 4*
7. *Selection step:* selection of the next generation parents according to $(\mu+\lambda)$ or (μ,λ) selection schemes
8. *Convergence check:* If satisfied stop, else go to *step 3*

6. NUMERICAL TESTS

Cylindrical shells are perhaps the most useful of the shell structures because they lend themselves to relatively easy construction, while they can span large areas with a minimum of material. They are very efficient structures because they use the arch shape to reduce stresses and thicknesses in the transverse direction. The curve of the cross section of the shell is usually a circle. However, any other shape may be used, such as the ellipse, a parabola, or a funicular curve which fits the thrust line of the applied load. Each curve has its particular structural and aesthetic qualities. Long shells have a span/radius ratio more than 5, while a short shell has a span/radius ratio less than 1. Shells between these limits are called intermediate shells. The numerical test of a long stiffened and non stiffened shell structure has been considered in this study to illustrate the efficiency of the proposed methodology in shape-topology-sizing optimization problems with discrete design variables. The purpose of this structure is to cover a span of $30\times60m^2$ having a circular cross section. The shape optimisation refers to the selection of the curvature shown in Figure 1, which represents the first design variable. In order to investigate the influence of the curvature in the optimum design, four different inclinations 0^o, 5^o, 10^o, 15^o and 20^o of the curved surface at the supports are considered. Topology optimization refers to the selection of the position and the number of the stiffeners, while sizing optimization refers to the selection of the stiffener cross section and the shell thickness.

The stiffeners are of arch shape in the transverse direction and of linear shape in the longitudinal direction, while their sections are to be selected from available AISC standard steel sections of wide-flange (W) shapes. The Young's modulus is taken 200 GPa (29,000 ksi) while the yield stress is F_y=250 MPa (36 ksi). To satisfy the practical fabrication requirements all the stiffeners of the structure are to be set in one group. Member cross sections are to be oriented as follows: (i) arched stiffeners lying on the X-Y plane are placed such that the strong bending axis takes place about the Z axis, (ii) straight stiffeners lying on the Y-Z plane are placed such that the strong bending axis takes place about the X axis. The discretization of the shell roof comprises of 3,422 TRIC elements with 10,080 d.o.f. and of 116 to 1,696 BEC elements, depending on the number of stiffeners used.

Initial Design	Optimum Design	No of Generations	No of FE Analyses	Optimum Volume (m^3)
Upper	20^o, 25mm	2	31	45.92
Random	20^o, 25mm	7	23	45.92

(a) non-stiffened shell

Initial Design	Optimum Design	No of Generations	No of FE Analyses	Optimum Volume (m^3)
Upper	15^o, 5mm, W_{10x12}/2m	59	203	11.35
Random	15^o, 5mm, W_{10x12}/2m	38	131	11.35

(b) Stiffened shell in one direction

Initial Design	Optimum Design	No of Generations	No of FE Analyses	Optimum Volume (m^3)
Upper	20^o, 7.5mm, W_{6x9}/4m	42	127	14.62
Random	20^o, 7.5mm, W_{6x9}/4m	17	62	14.62

(c) Stiffened shell in one direction in fixed positions every 4m

Initial Design	Optimum Design	No of Generations	No of FE Analyses	Optimum Volume (m^3)
Upper	15^o, 5mm, W_{6x9}/2m, W_{8x13}/30m	68	197	10.93
Random	15^o, 5mm, W_{6x9}/2m, W_{8x13}/30m	24	86	10.93

(d) Stiffened shell in both directions

Table 1: Cylindrical Shell - Performance of DES for the three test cases

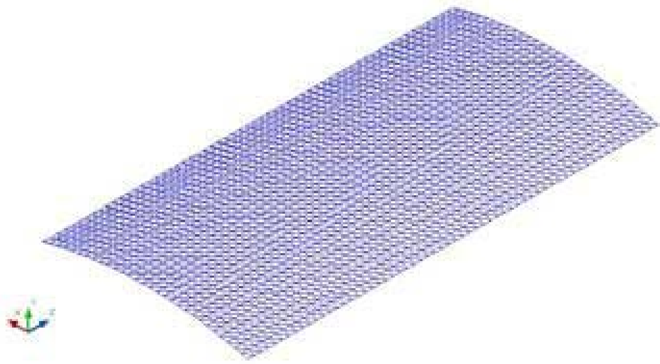

Figure 1: Cylindrical Shell and Finite Element mesh

The loadings on the structure are the following: (i) Uniformly distributed loading acting vertically in the negative Y axis direction of magnitude of 0.75 kN/m^2 that can be considered as snow loading. (ii) Uniformly distributed loading acting vertically in the positive Y axis direction of magnitude of 0.60 kN/m^2 that can be considered as wind loading and (iii) gravity uniformly distributed loading acting vertically in the negative Z axis. Three load combinations at their ultimate limit states are considered for the design of the structure as follows:

$$1.35\cdot(\text{iii})+1.5\cdot(\text{i})$$
$$1.35\cdot(\text{iii})+1.5\cdot(\text{ii})$$
$$1.35\cdot[(\text{iii})+(\text{i})+(\text{ii})]$$

All these load combinations define service loading schemes for which the stress related provision of the Eurocode design code are to be satisfied.

In applying ES, the (5+5)ES scheme is adopted and two different initial populations were used corresponding to the upper design values of the design set and to a randomly selected initial population. Four different cases of the cylindrical shell have been examined: (i) without stiffeners, (ii) with stiffeners in the transverse direction in varying positions, (iii) with stiffeners in the transverse direction in fixed positions and (iv) with stiffeners in both directions. In the first case each parent vector has two design variables corresponding to the inclination and the thickness of the shell. In the second case each parent vector has four design variables those of the first case and two others corresponding to the stiffeners position and their cross-section in the transverse direction. In the third case the design variables are the inclination, the thickness and the cross sections of the stiffeners, while in the fourth case each parent vector has six design variables those of the first two cases and two others corresponding to the stiffeners position and their cross-section in the longitudinal direction.

The optimum designs achieved for each case are depicted in Table 1. In the first case the optimum material volume is 45.92 m^3 and the optimum design corresponds to 20^o inclination and 25 mm thickness of the shell. In the second case the optimum material volume is 11.35 m^3 and the optimum design corresponds to 15^o inclination, 5 mm thickness of the shell with stiffeners of W8x13 cross section placed every 2 meters. In the third case, the following optimum design has been attained: the optimum volume is 14.62 m^3 and the optimum design corresponds to 20^o inclination, 7.5 mm thickness of the shell with stiffeners of W6x9 cross section having fixed positions placed every 4 meters. In the final case the optimum volume is 10.93 m^3 and the optimum design corresponds to 15^o inclination, 5 mm thickness of the shell, arched stiffeners of W6x9 cross section placed every 2 meters and straight stiffeners of W8x13 cross section placed every 30 meters. The optimum design computed for each of the four different cases examined is independent of the selected initial population. It can be seen that the optimum weight achieved in the cases of the stiffened shell is less than half the optimum weight achieved in the case of the non-stiffened shell, while the longitudinal stiffeners contribute to a small additional reduction of the material volume of the structure.

5 CONCLUSIONS

Evolution Strategies and in particular their discrete version can be considered as robust and efficient tools for practical design optimization of stiffened shell structures. With relatively few finite element analyses the evolution strategies implemented in this study can reach the optimum design irrespective of the type of optimization problem, whether it is sizing or combined sizing-shape-topology optimization. The finite element discretization of the shell structure and the stiffeners with the natural mode triangular shell element and the beam element BEC, respectively, further reduces the computational cost resulting in optimum designs of real-scale problems of the order of a few hundred of seconds CPU time on a Pentium III 1000MHz computer. Moreover, the presented results indicate the beneficial effect of the optimum layout of the transverse stiffeners to the performance of these types of structures resulting in a substantial reduction of material weight compared to the non-stiffened ones.

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