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SYMMETRY BREAKING FLOW TRANSITIONS AND OSCILLATORY FLOWS IN A 3D SOLIDIFICATION MODEL

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Abstract. *This study focus on the 3D character of flow and the loss of symmetry that produces some unsteadiness and perturbs the shape and the dopant distribution in configurations interesting directional solidification. Only hydrodynamic in the melt is analyzed. The critical stability limited for the onset of the natural convection under 2D assumption is extended to a preliminary 3D investigation. It is found that the initial steady symmetric flow becomes asymmetric for lower Ra number in comparison to the 2D approach. The loss of symmetry occurs first in the transversal plan. For relative low Ra it is observed that the heat transfer increases on the bottom active wall and decreases on the vertical acting walls without significant intensification in the global transfer. The classical spiral characterizing the 3D effect of the surface limiting the domain is also observed.*

INTRODUCTION

Natural convection has been subject of an intensive research due to its importance in various engineering, biological and geophysical systems. For instance, food product drying processes, solute intrusion in sediments in coastal environments, nuclear waste disposals, contaminant transport in ground water, chemical processes are few examples to mention. Double diffusive natural convection takes place also in solidification and melting of metal alloys. The mushy zone existing during the solidification of alloys consists of a fine mesh of dendrite growing from solid-liquid interface. The composition of the resulting solid is generally different than that of the melt. Hence, heat and mass transfer occur simultaneously in the mushy zone, which may be modeled as a double diffusion process in a saturated porous medium. In such process the thermal and concentration buoyancy forces either aids or opposes each other, depending on the type of alloy and process of heating [1].

In this study we focus on thermal natural convection. A vertical cavity corresponding to available 2D results under a destabilizing thermal gradient is considered [2, 3]. Flow bifurcation is expected and flow may become three-dimensional. Two dimensional flow patterns may be justified for a certain range of controlling parameters. It is found also that the flow structure may become unsteady.

MODEL AND NUMERICAL METHOD

The problem under consideration, as shown in figure 1, is cartesian configuration domain filled with liquid metal (Pr =0.01) and heated from below and cooled on the top [2, 3]. The left and right walls are heated on ¾ of the hight and adiabatic on the upper quarter. The front and the rear walls are supposed adiabatic. In the present work we will present only the steady state results.

Governing equations

A Direct Numerical Simulation, (DNS) is considered in the present study in 3D configuration. All properties are assumed constant except the density variations verifying Boussinesq approximation. The dimensionless governing equations based on the above assumptions are as follows:

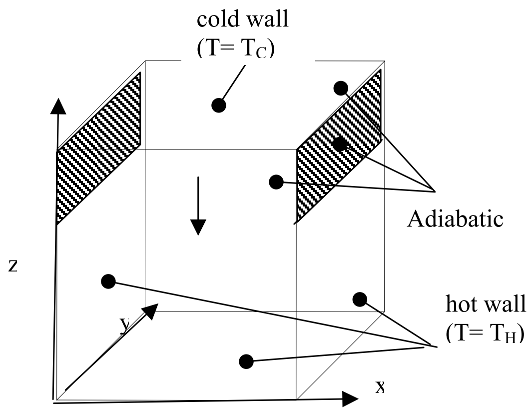

Figure 1: Schematic diagram of physical configuration

Continuity

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}+\frac{\partial W}{\partial Z}=0 \qquad (1)$$

X-Momentum

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}+W\frac{\partial U}{\partial Z}=-\frac{\partial P}{\partial X}+\nabla^2 U+\frac{Ra\theta}{\Pr}\cos(\varphi) \qquad (2)$$

Y-Momentum

$$\frac{\partial V}{\partial \tau}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}+W\frac{\partial V}{\partial Z}=-\frac{\partial P}{\partial Y}+\nabla^2 V \qquad (3)$$

Z-Momentum

$$\frac{\partial W}{\partial \tau}+U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}+W\frac{\partial W}{\partial Z}=-\frac{\partial P}{\partial Z}+\nabla^2 W+\frac{Ra\theta}{\Pr}\sin(\varphi) \qquad (4)$$

Energy

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}+W\frac{\partial \theta}{\partial Z}=\frac{1}{\Pr}\Delta\theta \qquad (5)$$

L_Z is used as a reference length for reducing the problem $u_T=\nu/L_Z$ a reference for the velocity and $\rho(u_T)^2$ a reference for the pressure, respectively. The dimensionless temperature is defined as $\theta = (T-T_C)/(T_H-T_C)$. The non-dimensional parameters in the above equations are Prandtl number $\Pr=\nu/\alpha$ and Rayleigh number $Ra = g\,\beta_T\,\Delta T\,L_Z^3/(\nu\alpha)$, where β_T, ν, and α are the coefficient of volumetric expansion, kinematics viscosity, and thermal diffusivity, respectively.

Velocity is assumed to be in the entire contour of the domain and the temperature boundary conditions are :

$\theta(X,Y,0)=1$
$\theta(0,Y,Z)=\theta(L_X/L_Z,Y,Z)=1$ for $0<Z\le 3/4$
$\partial\theta/\partial X(0,Y,Z)=\partial\theta/\partial X\,\theta(L_X/L_Z,Y,Z)=0$ for $3/4<Z<1$
$\theta(X,Y,1)=0$

Method of solution

Equations (l)-(5) are approximated by using a staggered, nonuniform control volumes grid, a third order accurate QUICK scheme [5] for the advection terms. ULTRA-SHARP [9, 10] flux limiter is used to remedy to non-physical oscillations. The SIMPLE algorithm is used to couple momentum and continuity equations. The momentum equations are solved by the implicit procedure (SIP) [12], which is extended here to handle 3D problems. The discretization of the pressure correction equation results a symmetric coefficient matrix that is solved by the conjugate gradient (CG) method [13]. On the other hand, the coefficient matrix of the set of equations resulting from the discretization of the energy equation is non-symmetric and solved iteratively by the BI-CGSTAB method [14]. SSOR preconditioning [13] is used for accelerating the convergence rates of both the CG and the BI-CGSTAB methods. To reduce the high computer times inherent in the solution of 3D natural convection problems, a full approximation storage (FAS) full multigrid (FMG) method [14] is applied to the problem. The equations are solved by a four level fixed V-cycle procedure starting at the coarsest grid and progressing to the finer grid level. For prolongation operations tri-linear interpolation is used for all variables. For restriction, the area weighted average procedure is used for all quantities defined on the control-volume surface such as velocities. The volume weighted average procedure is adopted for all quantities defined at the control-volume center such as pressure and temperature. For time dependant problems, second order accurate Euler scheme is used.

In this work 82×82×82 (and 102×102×102) irregular grids are used on the finest level. The non-uniform grids have denser clustering near the surface boundaries. The code was validated with the numerical results of Mukutmoni and Yang [13] for more details see [14].

RESULTS AND DISCUSSIONS

The effect of Rayleigh number on heat transfer on the bottom and on the top of the cavity is presented in table 1. The transfer is not symmetric between the top and the bottom because of the heating on the lateral walls. For the lower Ra the obtained *Nu* is conductive not equal to 1 because we still use the 1D diffusive transfer as heat reference. We can see that the average heat transfer on the lower surface increases with Ra because of the intensification of the flow-taking place inside the cavity. As we have two contra rotating main cells inside the cavity (figure 2-a), this bring cold fluid from top plate to the bottom one through the bulk (figure 2-b). This is the reason of the increases of the transfer on the bottom and decrease on the sidewalls. For higher Rayleigh numbers the global transfer increases.

Ra	Nu_{lower}	Nu_{upper}
10^0	0.258	2.10
10^1	0.258	2.10
10^2	0.260	2.10
$3x10^2$	0.265	2.10
10^3	0.281	2.10
$2x10^3$	0.305	2.11

Table 1: Nusselt value estimation for upper and lower walls

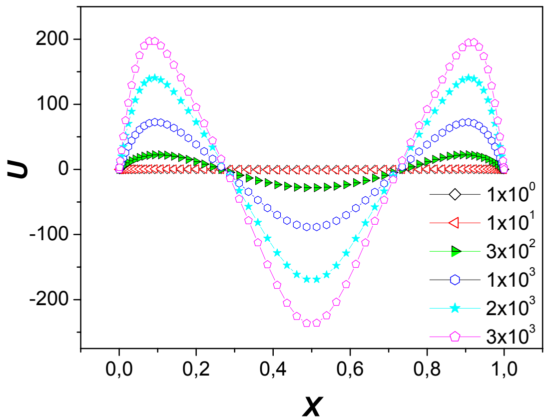

Figure 3: The vertical velocity component in the horizontal center median plan (X-Y)
for different Ra (Pr = 0.01, cubical cavity)

For 2D case we have a symmetrical solution until *Ra* = 5000 and well asymmetrical for *Ra* ≥7000. It becomes unsteady for *Ra* >30000 and periodical for *Ra*> 40000 [2, 3].

In 3D case the non-symmetrical and unsteady case appear for lower *Ra* in comparaison to the 2D approach. On figure 4 the local heat transfer on the bottom surface is represented for different Ra. For low Ra a diffusive solution is obtained were the maximum gradient is in the middle due to the imposed thermal boundary conditions. The increase in Ra number illustrates the effect of the lateral walls on the flow were the viscous effect affects the flow and the maximum transfer is in the core of the surface. For Ra $=3\times10^3$ we have a loose of symmetry in the transversal direction. The loose of symmetry appears earlier than the 2D approach and it is in the perpendicular plan than the obtained in 2D case.

CONCLUSION

In this work we present a simplified phase change problem in a cavity similar to fluid part of vertical Bridgman configuration heated from bottom is used. The effect of the Ra number on the flow structure and heat transfer distribution is analysed. First comparison with available 2D results is done and the main result concerns the symmetry breaking appearance for lower *Ra* and in the transversal plan. The flow becomes 3D for relative low Ra and the preliminary results illustrate the effect of the deep of the cavity on the global flow. This work follows on providing 3D characterisation for dynamic transitions on configuration very useful for directional solidification.

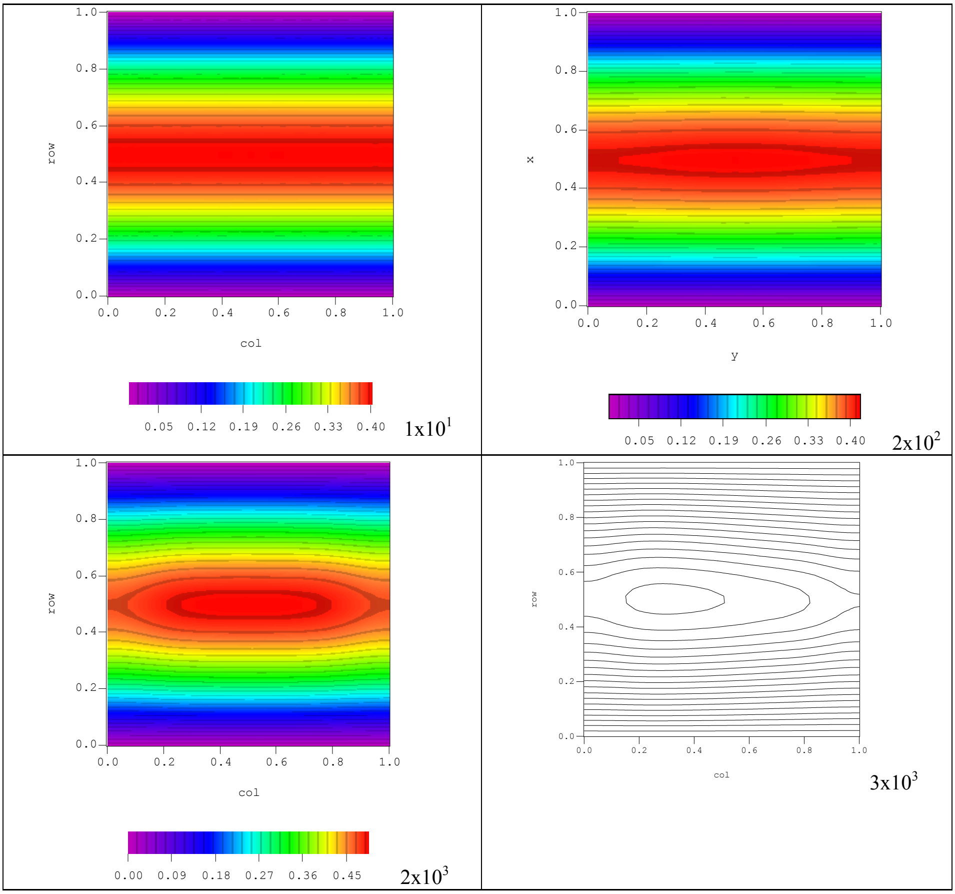

Figure 3: The local Nusselt number value on the hot (lower) surface for different Rayleigh number values

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