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THREE-DIMENSIONAL BEAM-SOLID CONTACT ELEMENT FORMULATION FOR ANALYSIS OF PILE INTERACTION

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Abstract.

This work aims to expand and improve capabilities in the field of geotechnical engineering for modelling the interaction between piles and soils. This includes the development and implementation of a new contact formulation for modelling the interface behavior between a pile and the surrounding soil. In this new formulation, the pile is modelled using a beam element and the soil is modelled using solid elements. The use of beam elements may allow for better characterization of the pile response while solid elements are better suited to model the soil. This poses the problem of how to characterize and model the interface between the pile and soil. This paper presents one possible formulation.

1 INTRODUCTION

In soil-pile interaction problems, pile behavior is best characterized using beam elements; specially to capture nonlinear deformation and force distributions. Soil response, on the other hand, is more effectively described using solid continuum based elements. The beam and solid combination poses a challenge for advanced modelling when frictional contact mechanisms and potential separation are to be considered. To date, conventional interface elements of both the thin layer type (Zienkiewicz et al. (1970), Pande and Sharma (1979), Desai (1981), Desai and Siriwardane (1984)) and constraint based type (Chan and Tuba (1971), Huges et al. (1976), Petersson (1977), Wriggers (2002)) have not been used effectively to connect mixed beam-solid pile-soil interaction models.

This research seeks to enhance soil-pile interaction modeling capabilities through the direct integration of beam-column and solid type models. The approach is based on 3D frictional node-to-surface contact. Contrary to traditional contact implementations, the contact surface is defined using a semi-analytical representation of the three-dimensional surface of a discrete 3D-beam finite element.

This new approach allows for geotechnical models to simulate nonlinear pile behavior through the characterization of complex cross sectional properties such as reinforcement or defects by means of an advanced beam element with fiber cross section. Moreover, it enables the incorporation of size effects due to frictional resistance against rotation, or the effect of closely packed arrays of piles.

The following sections provide a brief description of the new element with the beam centerline and three-dimensional interaction surface. A few examples showing the applicability of the contact element are also presented. More details on the formulation, implementation, and application of this interface element can be found in Petek (2006) and Petek et al. (2007)

2 ANALYTICAL DESCRIPTION OF THE BEAM SURFACE

2.1 General

Figure 1a shows a schematic of the new beam-solid contact element depicted between points $\mathbf{x}_a$ and $\mathbf{x}_b$. The beam surface, Γ, is shown in the figure. A node on the solid body is denoted $\mathbf{x}_s$ and its projected location on the beam centerline, $\mathbf{x}_c$, is also indicated. The beam is assumed to govern the contact behavior and is considered the *master* body. The solid is the corresponding *slave* body and the solid node $\mathbf{x}_s$ is hence referred to as the slave node.

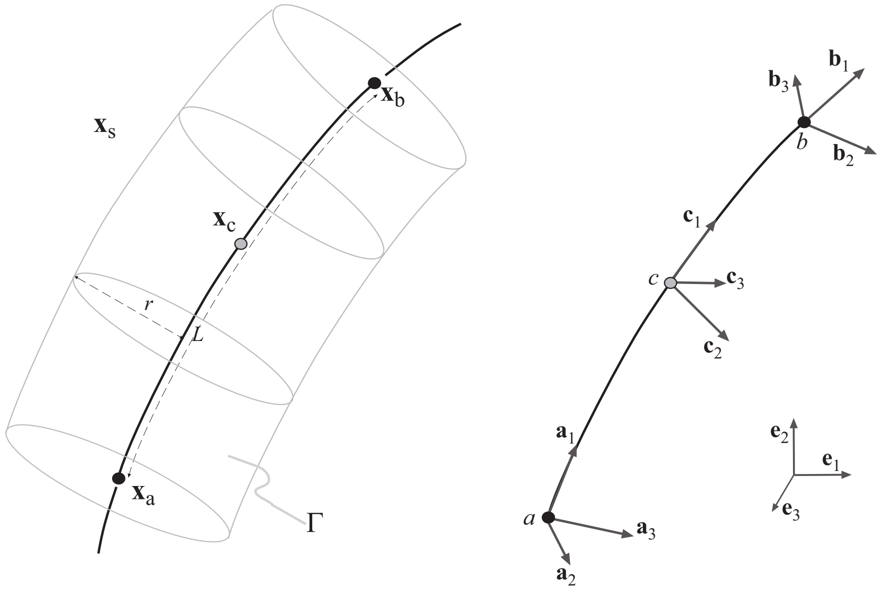
Figure 1b shows the tangent vectors to the beam centerline, $\mathbf{a}_1$ and $\mathbf{b}_1$, at ends a and b respectively, along with their corresponding orthogonal base vectors $\mathbf{a}_2$, $\mathbf{a}_3$ and $\mathbf{b}_2$, $\mathbf{b}_3$ defining the cross-section at these ends.

2.2 Projection of the Slave Node onto the Beam Centerline

The present contact formulation is developed using the projection, $\mathbf{x}_c$, of the slave node, $\mathbf{x}_s$, onto the beam centerline such that

$$\mathbf{x}_c = \boldsymbol{\varphi}_c(\xi_c) \tag{1}$$

where $\boldsymbol{\varphi}_c(\xi)$ is a function describing the beam centerline with $\xi \in [0, 1]$, and ξ_c is the local coordinate of the projected slave node.



(a) (b)

Figure 1: (a) Beam-solid contact element between $\mathbf{x}_a$ and $\mathbf{x}_b$, with slave node $\mathbf{x}_s$ and centerline projection $\mathbf{x}_c$, and with surface Γ (b) Centerline reference frames at a, b, and c.

2.3 Cylindrical Beam Surface, Γ

A cylindrical beam surface Γ is defined for the beam-solid contact element to describe interaction related to a pile with a circular cross section. For this purpose a second local coordinate, ψ, is used to describe the angular position around the beam centerline such that $\psi \in [0, 2\pi)$. Any point $\mathbf{x}^\Gamma$ on the surface, Γ, may be described using two local coordinates, ξ and ψ, such that

$$\Gamma : \{\mathbf{x}^\Gamma \in \mathbb{R}^3 \;|\; \mathbf{x}^\Gamma = \boldsymbol{\Gamma}(\xi, \psi) := \boldsymbol{\varphi}_c(\xi) + \mathbf{r}\,(\xi,\; \psi)\} \tag{2}$$

where $\mathbf{r}(\xi,\; \psi)$ is the radial vector extending perpendicular from the centerline location at ξ to the beam surface Γ.

2.4 Projection of the Slave Node on the Beam Surface

The projection $\mathbf{x}_c^\Gamma$ of the slave node $\mathbf{x}_s$ on the pile surface is defined using Equation (2) as

$$\mathbf{x}_c^\Gamma = \boldsymbol{\Gamma}\,(\xi_c, \psi_c) = \mathbf{x}_c + \mathbf{r}_c \tag{3}$$

where ψ_c is the angular coordinate of the slave node projection $\mathbf{x}_c^\Gamma$ on the beam surface.

2.5 Tangent Plane, $T\Gamma$

If a small deformation formulation is considered, differential movement at the beam-solid interface is confined to a tangent plane at the location of the projected slave node, $\mathbf{x}_c^\Gamma$ on the beam surface, Γ. Figure 2 shows a schematic of the beam surface and tangent plane. The tangent plane $T\Gamma : (d\xi, d\psi) \to \mathbb{R}^3$ is defined through base vectors $\mathbf{g}_\xi$ and $\mathbf{g}_\psi$.

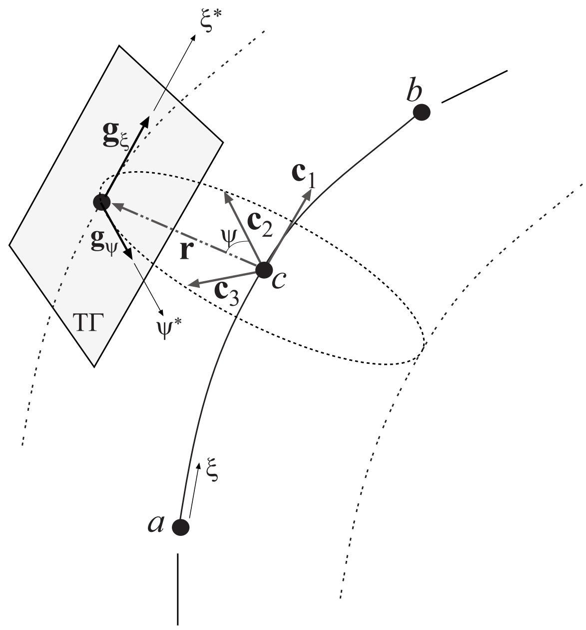


Figure 2: Pile surface and tangent plane

3 FRICTIONAL BEAM-SOLID CONTACT DESCRIPTION

3.1 General

The beam-solid contact formulation is based upon a geometric constraint that controls the interaction between two bodies. This formulation utilizes the Hertz-Signori-Moreau conditions for contact (Wriggers (2002)) which consider a geometric distance between the two bodies called a gap, denoted as g, and the corresponding normal contact force between the bodies, denoted as t_n. The Hertz-Signori-Moreau conditions are used in the form

$$g \geq 0 \,, \qquad t_n \geq 0 \,, \qquad t_n \cdot g = 0 \tag{4}$$

These conditions state that if the bodies are in contact and a positive normal contact force exists, the gap has to be zero. If the bodies are not in contact, the gap is greater than zero and hence no contact force is allowed. Inadmissible states include a negative gap, which implies penetration, and a negative contact force, which forces contact release.

For the beam-solid contact element, the gap is modified to account for the beam cross-section. For the circular (with radius r) cross-section case considered, the gap is calculated as

$$g = \mathbf{n} \cdot (\mathbf{x}_s - \mathbf{x}_c) - r \qquad . \tag{5}$$

3.2 Contact Virtual Work Formulation

The frictional contact formulation is developed using the contact constraint $g = 0$ and accounting for the differential movement between the bodies. Considering Figure 3, the virtual work for the contact system accounting for the contact constraint, $g = 0$, is written as

$$\delta W_{contact} \;=\; (\mathbf{N} - \mathbf{T}) \cdot \delta \mathbf{x}_s + (\mathbf{T} - \mathbf{N}) \cdot \delta \mathbf{x}_c + \delta t_n \, g \tag{6}$$

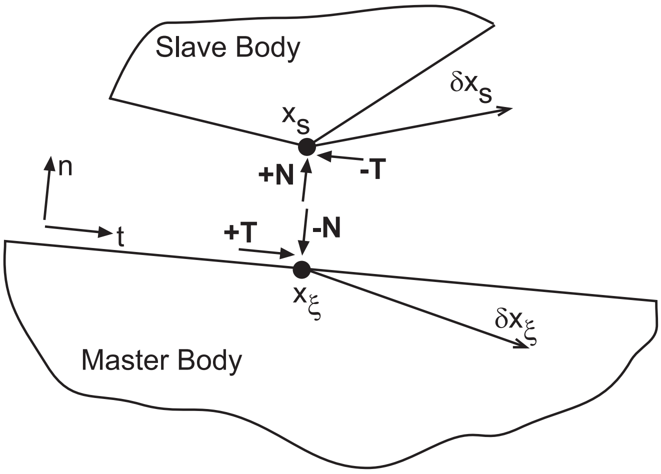


Figure 3: Two bodies in contact

where δt_n is again an arbitrary, independent variation. Using $\mathbf{N} = t_n\,\mathbf{n}$ and $\mathbf{T} = \mathbb{P}_n \cdot \mathbf{t}_s = \mathbf{t}_s$, with $\mathbb{P}_n = \mathbf{1} - \mathbf{n} \otimes \mathbf{n}$, Equation (6) can be rewritten as

$$\begin{aligned} \delta W_{contact} &= (t_n\,\mathbf{n} - \mathbf{t}_s) \cdot \delta\mathbf{x}_s + (\mathbf{t}_s - t_n\,\mathbf{n}) \cdot \delta\mathbf{x}_c + \delta t_n\, g \\ &= t_n\,\mathbf{n} \cdot (\delta\mathbf{x}_s - \delta\mathbf{x}_c) - \mathbf{t}_s \cdot \mathbb{P}_n(\delta\mathbf{x}_s - \delta\mathbf{x}_c) + \delta t_n\, g \qquad . \end{aligned} \tag{7}$$

Assuming that $\mathbf{n}$ is constant, the gap, g, is calculated using Equation (5) and slip, s, is determined as

$$\mathbf{s} \;=\; \mathbb{P}_n\,(\mathbf{x}_s - \mathbf{x}_c) \qquad . \tag{8}$$

Using Equations (5) and (8), Equation (7) can be rewritten as

$$\delta W_{contact} = t_n\,\delta g + \delta t_n\, g - \mathbf{t}_s \cdot \delta\mathbf{s} \qquad . \tag{9}$$

Following standard FE procedures expressions for the resisting force vector and the element tangent stiffness matrix are developed using 9 and its linearization, respectively. Consistent tangent operators have been developed for desirable quadratic convergence behavior (Petek et al. (2007)).

3.3 Kinematic Relations

Inclusion of frictional slip in the contact formulation requires special consideration. Slip is considered as the relative movement of a slave node on the beam surface. For simplicity and computational efficiency, incremental slip is assumed to be small and a tangent plane approximation is used in the time stepping procedure from steps n to $n + 1$. Details on the kinematic relations and implementation into a finite element code can be found in Petek (2006) and Petek et al. (2007)

4 PILE-SOIL INTERACTION EXAMPLES

Several examples related to pile applications were performed to ensure functionality and strong convergence behavior of the new beam-solid contact element. Test models considered pile frictional interface behavior in comparison to conventional design approaches. In these models, the pile was simulated using beam elements and was pulled up through a *donut* shaped

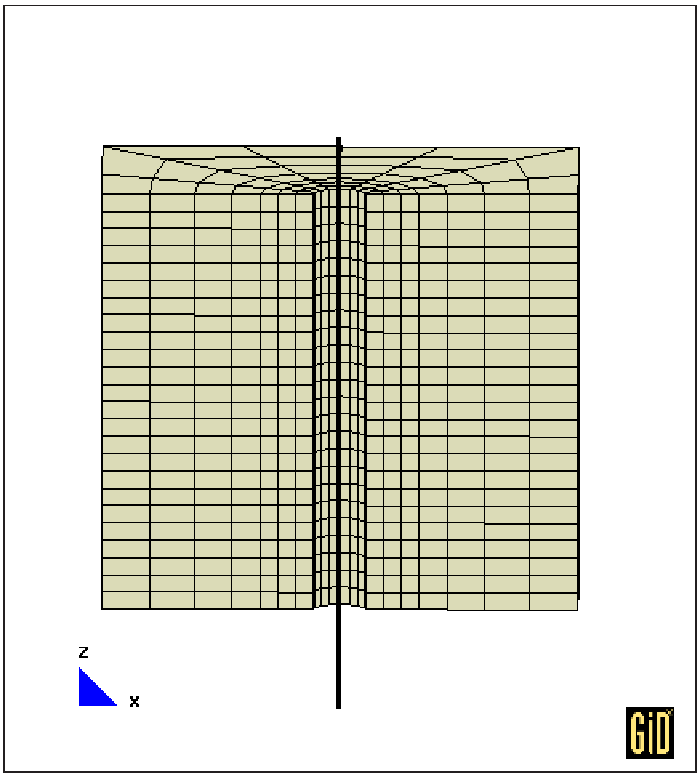


Figure 4: Model to test friction at pile-soil interfaces

Table 1: Results of friction pile tests compared to β-method

		Contact Element	β-Method	
μ	G	$Q_{contact} = \sum_{i=1}^{N} f_{s_i}$	$Q_s = \pi B \int_0^D \sigma_h(z) \tan\delta \, dz$	% diff
0.1	1000	141	135	-4.0
0.25	1000	344	335	-2.7
0.5	1000	666	670	0.6
1	1000	1240	1320	-6.0

body made of solid, elastic solid elements. Figure 4 shows the donut pile mesh. This model isolates the frictional resistance at the interface and excludes end-bearing effects.

The frictional element output is compared to results obtained using basic friction concepts. Table 1 compares the results between the finite element model and conventioanl frictional relations (β-method). The numerical and analytical solutions compare very well.

It is noted that the contact element forces shown in Table 1 correspond to the maximum frictional resistance in the model. Prior to reaching this level, the frictional resistance accumulates in *sticking* mode. Once the maximum frictional resistance is developed, elastoplastic *sliding* behavior occurs at the interface. Quadratic convergence behavior was maintained for both modes.

Other models considered the debonding and rebonding mechanisms for simulation of gap formation in laterally loaded piles. Figure 5a shows a deformed laterally loaded bridge bent model in elastic soil. The bent is pushed into the soil in the transverse direction and the piles lose contact with the soil on the opposite pile sides. Figure 5b shows contours of contact forces in the same deformed system. In this view, the deformed pile opening is clearly observed together with the development of large compressive contact forces near the pile surface.

5 CONCLUSIONS

In this paper beam-solid contact interface elements are introduced for the connection of beam elements with a circular cross section to solid elements. This development shows that it is possible to establish the position and change of a contact surface based on beam information at two nodes, and to allow the interaction with a slave node on a solid element. Examples are included that demonstrate the functionality of the contact elements in the context of pile foundation problems.

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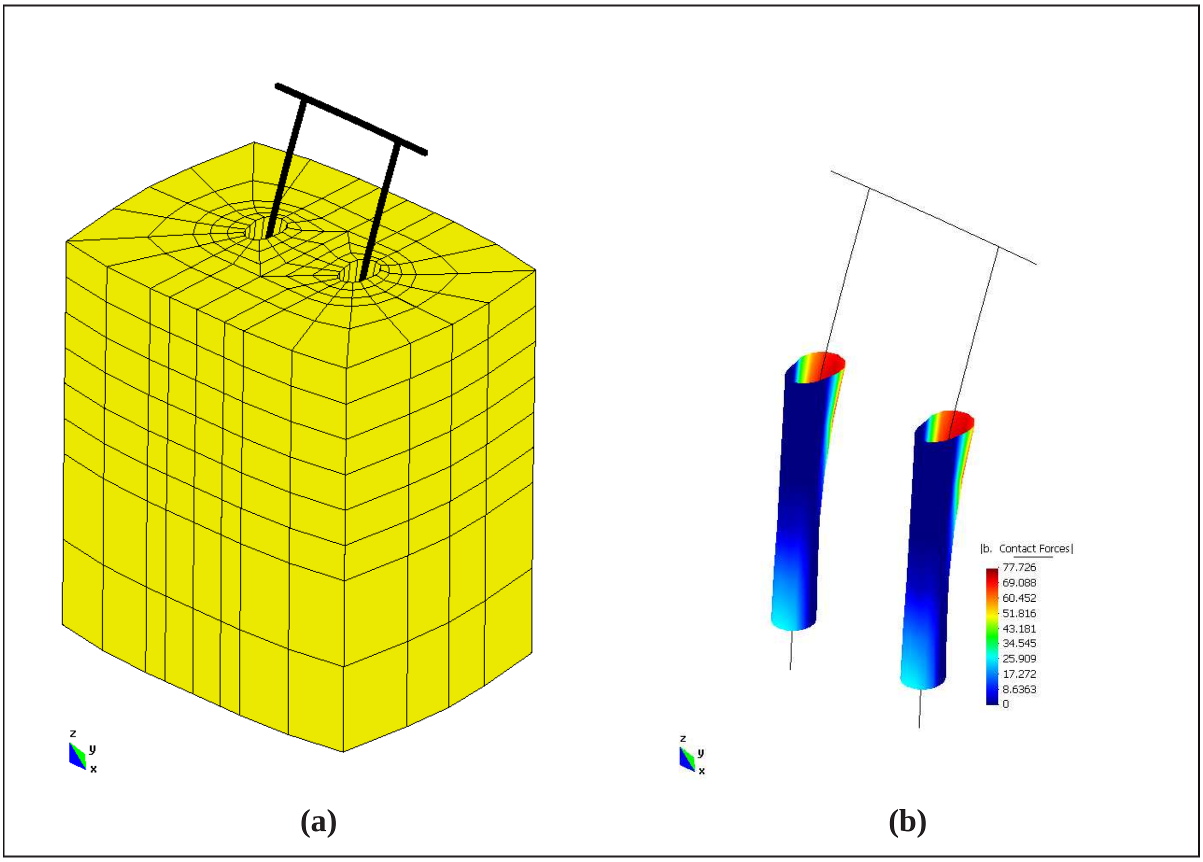

Figure 5: Bridge bent model showing (a) debonding, or gapping around the piles during lateral displacement, and (b)contact forces at the soil-pile interfaces.

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