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A NEURAL NETWORK APPROACH OF THE DYNAMICAL INCONSISTENCY PROBLEM OF DECISIONS

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Abstract.
The intertemporal inconsistency problem in decision making refers to those cases in which the individual has chosen in favor of those alternatives that ensure a delayed retribution, but in the interim he changes his choice to an alternative that is realizable early, even when the alternative available in long time is optimal for him.

This kind of inconsistencies over choices has been modeled as the interaction between two systems: a cool system which has an automatic and slow operation, and another hot system which operates in a fast and automatic fashion. The hot system makes its choices biased towards those alternatives with early realization while the cool system has a remarked preference for the delayed gratification alternatives. The individual decision results as the solution to the competition in operation between the two systems.

In this article, we model the dynamical mechanisms that govern both systems functioning and their interaction when they are engaged in decision making. These mechanisms are revealed by an analysis of a neurocaomputational approximation to the network operations. Then we consider the dynamical system that measures the activation of the minimal network when a stimulus input excites both systems.

As each system's activation is represented by nonlinear equations, the nullclines structure generates multiple equilibrium nodes which are found and classified given its local stability. As a final exercise, this work studies the relevance of each parameter in the dynamics of the system, and in the determination of the final decision.

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1 INTRODUCTION

In microeconomic theory the individual behavior is explained as the result of the individual's choices oriented to the pursuit of his goals. Any decision problem that the economic agent faces is characterized by a set of options and a preference relation that ranks the alternatives. Thus, the "decision problem" consists of choosing the available alternative that occupies the highest position in the preference ranking. A preference order satisfying completeness, transitivity and continuity, it is called a "rational" and it can be represented by an increasing monotone utility function. The solution to a decision problem is the alternative that maximizes this utility function. The Theory of Rational Choice was extensively applied to many economic problems with successful results in explanation and tractability (Mas Collel, Whinston and Green, 1995). For this reason, this approach turned into the cornerstone in economics and social sciences. The generalization of this theory to capture decisions where time is involved as a property of the goods implies more restricted preferences for a rational individual. Particularly, when the individual has to choose between two alternatives which are available in two different time periods; the utility of future alternatives has to be discounted by an "*invariable in time*" discount rate to make them comparable with the utility of present alternatives. Although this *discounting* assumption implies a restricted behavior for the agent, it does not generate any cost for the tractability property of the models.

The choices produced by a rational individual are not supported by the empirical evidence collected in laboratory experiments (Rabin, 1998; Camerer, 1995; Camerer et. al.,2005, Frederick et. al., 2002). A case where the traditional framework fails to find explanation refers the situation where the individual changes his choice from an option that is the most preferred in a time horizon but is delayed in gratification in favor to another alternative that is immediate in gratification. This bias is called the "*intertemporal inconsistency of choices*" and is associated to known pathological behaviors as addictions, procrastination and other impulsive behaviors.

To study this phenomenon, Metcalfe and Mischel (1999) proposed a theoretical framework where the intertemporal inconsistency of choices is the result of the competition between an emotional and a cognitive system which engage in the evaluation of the alternatives. The emotional or hot system operates in a simple, automatic and fast way while the cognitive or cool system is deliberative, reflective and slow. Although, this framework characterizes both systems, the interaction between them is presented by a static graph analysis without enlightening the dynamics mechanisms that govern the functioning of each systems and their interaction in the definition of a choice.

This work has the goal of understanding both dynamics mechanisms that govern the functioning both the hot and the cool systems and the competition between them in the study of the intertemporal inconsistency of choices. We present a model where both sub-systems are represented by a network. To capture the biological insights in the operations of the cool and the hot systems in the brain, the dynamics of both systems and their interactions are represented by a neurocomputational model. The participation of the cool and hot networks in the decision procedure is summarized in a dynamical system that represents the activation's evolution of the network's nodes. The study of this dynamical system let us understand the functioning of the dynamical mechanism that defines which systems is in charge of the decision.

The article is organized as follows. In Section 2, there are reviewed the key ideas developed in the economic literature about the intertemporal decision problem and the duality between emotion and cognition. Then, in Section 3 there is presented the insights that characterize the Cool-Hot System framework developed by Metcalfe and Mischel (1999) and the neural network model that represents the hot and cool systems. Section 4 analyses the dynamical system analysis of the neural network presented in the former section. A bifurcation analysis of the system's equilibriums is presented in the Section 5. Finally, the principal ideas of the article are summarized with the conclusions in the last Section.

2 BACKGROUND

To explain the inconsistencies in choices situated in different periods of times, the developments found in behavioral economic's literature has modified the utility function that represents the individual preferences. In these kind of models called *quasihiperbolic discounting approach*, incorporate a term in the discount factor which varies with the period of time considered. With this upgrade, the extended approach represents accurately the results collected in the experiments (Ainslie, 2002, Ainslie and Monterosso, 2004, Laibson D., 1997; Frederick, Loewenstein and O'Donoghue, 2002).

Other kind of economic models have represented the individual as an agent composed by two agents, each one with different preferences: one of them was more concerned with the long time while the other one had preferences biased to those alternatives which are available early (Thaler and Shefrin, 1981; Fudenberg and Levine, 2006). The negotiation problem between the two agents represented the willpower weakness of the agent when he faces the temptation for the early alternative. When the individual wanted to control his temptations, he could exert willpower control but he has to face a cost measured by a utility loss.

Another class of models that has appeared were inspired in developments and experimental evidence found in other disciplines like psychology and neuroscience. These studies propose a basic model as an explanation of the decision process, in which the choice is the result of the competition of two subsystems: one automatic system oriented to immediately alternatives or stimulus; while the other system operates under planned control and orientated his choices to alternatives that are optimal for the whole problem, included those decisions considered in the short time. While the automatic subsystem operates evaluating the emotional components that compose the decision problem, the controlled subsystem is in charge of those cognitive operations and rational judgments.

The cognitive-emotional duality argument was first applied in psychology to study the functioning of the working memory under posttraumatic stress situations (Metcalfe and Jacobs, 1996). Nevertheless, it was adapted to study the weakness of willpower when the individual faces emotional stimulus (Metcalfe and Mischel, 1999). This ideas and the supporting evidence discovered in neuroscience's experiments (McClure et. al., 2004) has guided the development of economic models of impulsiveness and the willpower optimal practice. In this kind of models, the optimization problem has two motivational functions to maximize that represent each cognitive and emotional system (Loewenstein G. and O'Donoghue T, 2004). The agent defines a plan of action that maximizes his utility function in the long term, but when he faces a temptation, he must exert a control over the emotional system which motivates the defection of the plan. If the inhibition of the emotional systems is not effective enough because to exert self-control is too expensive measured in utiles, he surrenders to the effect of the temptation and the emotions contributes to define the decision (Benhabib and Bisin, 2005).

The psychological model has depicted how the two systems compete between them to take control over the decision, but it lacks of a formal elaboration. On the other side, the economical models present a formal specification of the decision problem but the representation of the system's relations is not an adequately description of the dynamical mechanism that operate in the brain for the decision formation.

This work proposes a formulation of the self control regulation that the cognitive system exerts to control the human behavior. There is applied a model from mathematical neuroscience that explain how two populations of neurons can behave when one group is inhibiting the activity of the other.

3 THE COGNITIVE – EMOTIONAL APPROACH OF SELF-CONTROL REGULATION.

3.1 The Cool-Hot Systems Framework

The model we study in this paper is based on the framework presented by Metcalfe and Mischel (1999) to study several aspects of the self-control regulation problem. In this Section we present the main ideas of this approach.

According this framework two different systems, *cool* and *hot*, interact during the decision making process. The cool system is cognitive, deliberative, strategic, complex, reflective, slow processing and is where the self-control function takes place. In contrast, the hot system is emotional, automatic, simple, quick functioning and it is sensitive to stimulus effects.

As an illustration of how the cool system regulates the activity of the hot system let us consider the following example. An individual has committed himself to give up smoking. Whenever he faces the stimulus of being offered a cigarette he has two alternatives, to accept or reject the offer. Driven only by the emotional (hot) system, the individual would have a strong tendency to smoke. However, the cognitive (cool) system will "remind" the individual of his commitment, and his decision is expected to be deviated towards not smoking (rejecting the cigarette).

In Metcalfe and Mischel ‘s framework both systems are represented as directed graphs; i.e., a set of nodes and a set of edges. In their description the authors identified these graphs with neural networks. Even though there is no biophysical description of the participating elements, the hot and cool systems can be thought of as abstractions of the amygdala and the orbitofrontal cortex and hippocampus respectively.

As we mentioned in the introduction there is no dynamics associated with the participating elements (nodes and edges) of the graphs, and the network is used merely to illustrate the author's conclusions.

The assumptions of the M&M framework:

1) The cool nodes are highly interconnected while there are no connections among the hot nodes.
2) There is a cool node connected to each hot node. Note that there may be isolated cool nodes.
3) Activation of a node causes activation of all nodes to which it connects.

3.2 The Neural Network Model

The small neural network modeled in this section represents the crucial stage where the cognitive system inhibits the emotive system (Figure 1). The diagram shows the two inputs currents (S_C and S_H) incoming to each node in the cognitive (blue nodes) and emotional (red nodes) systems. When the hot node receives the stimulus, it activates itself generating an output current that innervates the final stage where there is evaluated if the output current is strong enough to execute an action or not (the brackets with the plus symbol represent an operator that translates the output current in an action if the current is positive or not in the other case). In the cognitive system, the excitatory node receives the input current from the stimulus and produces two output currents: one innervates the last stage which evaluate if the current is strong enough to produce an action; and the other current activates the inhibitor cognitive node which regulates the activation of the cognitive and the emotional nodes.

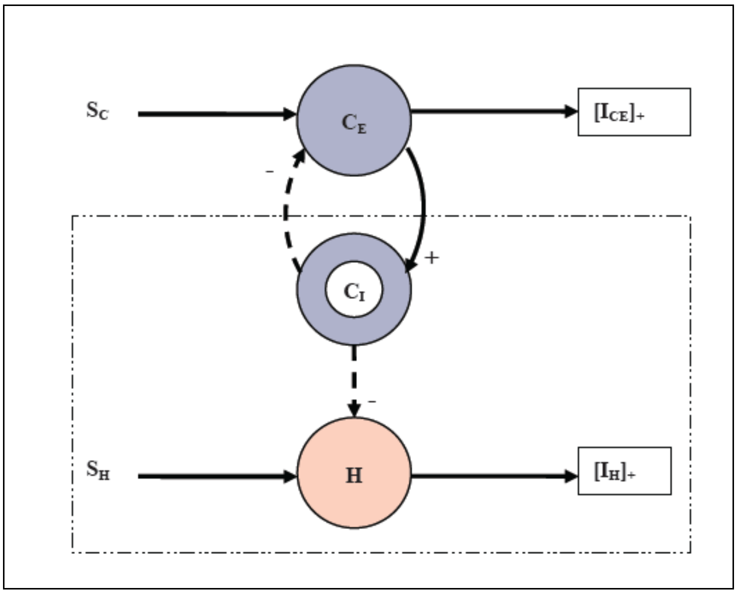

Figure 1. The Cognitive-Emotional Neural Network.
The blue node belongs to the cool systems and the red one belongs to the emotive system. The discontinuous square delimits the formal model presented in this work.

In the final stage of the network there is supposed to be an operator that evaluates the outputs currents from both cognitive and emotional systems. If those currents are positives, then the operator allows to being available the alternative associated to each node for a next evaluation by motivational functions like the ones proposed in Loewenstein and O'Donoghue (2004). So, the approach of neural networks suggests a constructive framework of these utilities functions.

The formal small model of the next section represents the evolution of the firing rate of the two nodes in the inhibition stage (in figure 1, the section of the network remarked by the discontinuous square, and figure 2). Making focus over these two nodes is justified because the reduction of dimensions to identify and visualize the main parameters that affect the role of regulation and self-control. The simplification reduces the excitation-inhibition relation between the cognitive nodes to an incoming input that activates the inhibitor cognitive node (B_C in Figure 2). The remainder components of the network stay without change.

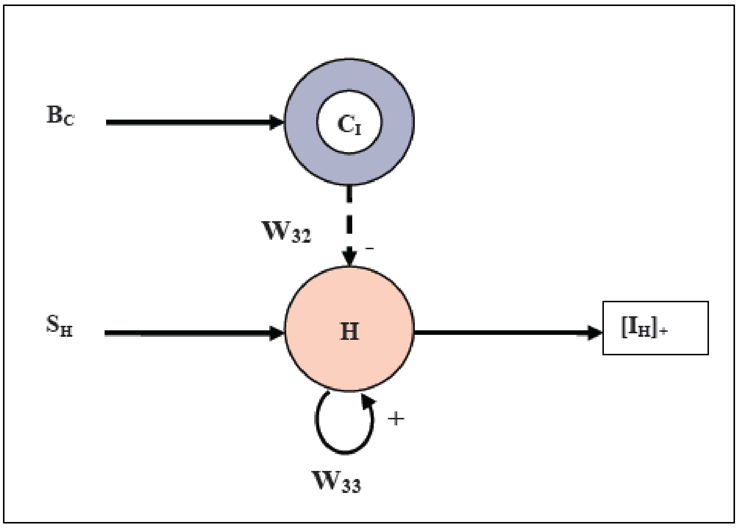

Figure 2. Diagram of the Small Neural Network.
The reduced model supposes that the inhibitor node fires when the incoming input from the excitatory node arrives by the B_C receptor.

4 THE FORMAL MODEL OF THE REDUCED NEURAL NETWORK

4.1 The Dynamical System Problem

The neural network sketched in the figure 2 is formalized by a model of the firing rates generated in both nodes. In mathematical neuroscience, the firing rate models measure the neuron's rate of spike's generation because the change in voltage in its soma; given its initial voltage level, the input incoming currents from the external stimulus and from the synaptic connections with other neurons.

The cognitive-inhibitor node of the model has a local action in the network. It only can be activated by the excitatory inputs from another cognitive node which transfers the effect of the stimulus to other nodes of the systems. Once the inhibitor nodes is active its outputs inhibits the nodes which it connects.

The hot-excitatory node, receives the input currents from the stimulus and the inhibitory current from cognitive node. Due to the change in its voltage, it is activated and fires an output current that innervates itself and which also is transferred to the final stage. In this first approximation to model the cool-hot framework, the hot node has the property to be in a rest or activated mode and the change in mode is conditional to initial parameters as the initial firing rate of the node, the strength of the synapses, and the magnitude of the stimulus. All this conditions defines the threshold level which if it is exceeded, then the node is activated.

The Firing Rate Model for the network is:

Cognitive-inhibitor node:

$$\dot{c}_I = -c_I + f(b_2) \text{ with } f(u) = \frac{1}{1+\exp(-u)} \tag{1}$$

Hot-excitatory node:

$$\dot{h} = b_3 + w_{32}c + w_{33}h - h^3 \quad (2)$$

The equation (1) expresses how the hot node changes its voltage given the initial level (c_I), when the node receives the input stimulus (b_2). The term *f(b₂)* is the activation function which transform the stimulus into a firing rate. The sigmoid form is commonly used in this kind of models because it is a monotone, continuous and differentially function that saturates in 1 as the arguments increases indefinitely.

Seemingly, the equation (2) measures the firing rate evolution of the emotional node. The first two terms in the right hand side are the inputs currents incoming from the stimulus source, the cool node and from the auto-synapses of the same hot node. The cubic term allows the existence of two modes: rest and activation states. The coefficient of this term has to equal (-1) in order for the systems to be bounded and an equilibrium point exist (Izhikevich E., 1996). The strength of the connections between nodes is measured by the weights w_{32} and w_{33}. The weights are positives when the node is excitatory and negative when it is inhibitory. For reason of tractability, it will be considered $b_3, w_{33} \in [0;1]$ and $w_{32} \in [-1;0]$. The weight w_{33} measures the capability of the hot node to remain near the excitability after it had fired and no stimulus is present.

In the dynamical models of neural networks, the equilibrium firing rate is determined when equations (1) and (2) equals zero. Since the cool node is uncoupled, then its firing rate level of equilibrium equals:

$$c_I^* = \frac{e^{b_2}}{1+e^{b_2}} \quad (3)$$

Then, when the incoming current from the excitatory cool node is positive and bigger enough the inhibitory node reaches in a fast way its activation.

Given the value of the cool node, the hot node could presents three possible cases were the equilibrium varies from one to three (Figure 3).

From the figure 3, it can be seen that in the case (a), the node is hyperpolarized in the equilibrium. Then for every stimulus, the node will continue deactivated. In the case (b) the equilibrium point correspond to an activated state, then, any stimulus, even the less, could activate the node. In the third case (c), both equilibriums are available and the external stimulus defines the final states of the node.

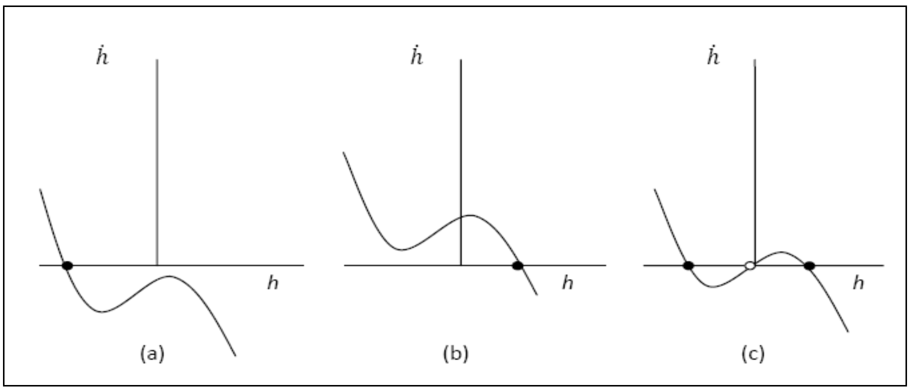

Figure 3. Possible equilibriums of the hot node
Case (a) The node is depolarized and it is a stable equilibrium. Case (b) In contrast, the node is in an stable and activated equilibrium. Case (c) There are two stable equilibriums and a third unstable that works as a threshold.

The possibility of three equilibrium points also appears in the general systems and it could be seen from the phase diagram sketched in the Figure 4. It could be seeing three equilibriums points. One situated in the origin is characterized by a saddle whose convergent paths delimit the basin of attraction of the other two stable node equilibriums.

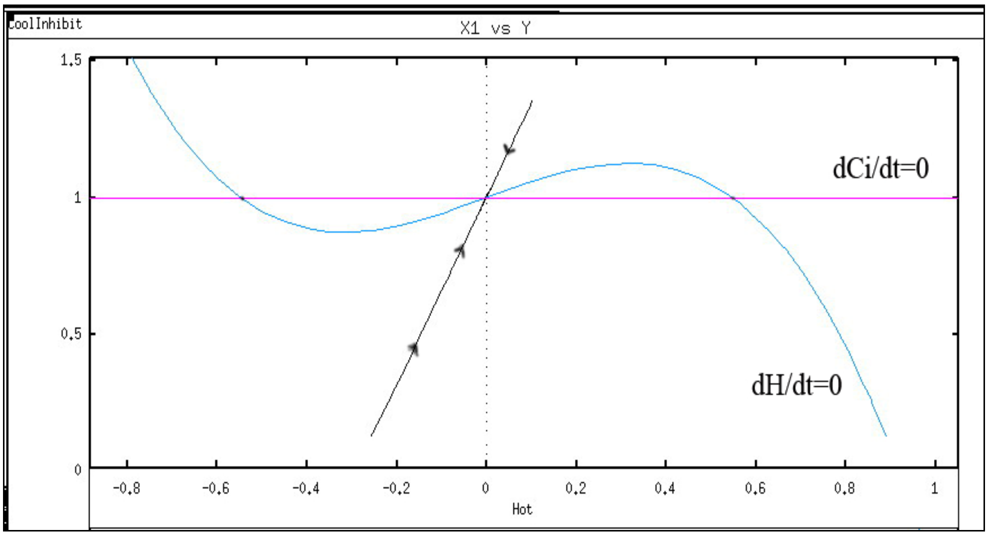

Figure 4. Nullclines of the Dynamical Systems

Example of nullclines determining three equilibrium points, with w_{32}=-0.5, w_{33}=0.3and b_3=0.5, b_1=100. It could be identified two stable equilibrium points: one is associated to complete inhibition of the emotional node; the other node belongs to the state where the node is active. A third unstable equilibrium point defines the basin of attraction of both stable equilibriums.

Proposition 1: *"The neural network that represents the inhibition relation between the cognitive systems and the emotional systems in the self-control problem presents three equilibriums points: two nodes separated by a saddle, where one each node is associated to a deactivation /activation situation of the emotional systems, and the saddle delimits the threshold level to go from one modality to the other."*

Proof:

As the ordinary differential equations in the dynamical system are continuous and nonlinear, and the systems has the characteristics decrypted (the cool activation function is monotone and the equation 2 is bounded), then there exists at less one equilibrium point.

The classification of the equilibriums into saddle and nodes could be checked by the determinant (Δ, equation 5) and trace (τ, equation 6) of the Jacobian matrix (equation 4) derived from the dynamical system (Strogatz, 1994).

$$Jacobian = \begin{bmatrix} -1 & 0 \\ w_{31} & -3h^2 + w_{33} \end{bmatrix} \tag{4}$$

$$\Delta = 3h^2 - w_{33} \tag{5}$$

$$\tau = (w_{33} - 1) - 3h^2 \tag{6}$$

$$Eigenvalues: \lambda_1 = -1 \; , \lambda_2 = -3h^2 + w_{33} \tag{7}$$

Because w_{33} cannot be bigger than one then the trace is always negative. As the eigenvalues of the linealized system takes real negative values; then the equilibrium could be a node if equation 8 verifies or a saddle in the opposite case.

$$h^* > \pm\sqrt{\frac{w_{33}}{3}} \tag{8}$$

So, the dynamical systems that represents the neural network of the section 2.1, presents a single stable node as equilibrium or three equilibriums where two are stables nodes and the third is a saddles between them.

When there three equilibriums, the nodes define two particular situations: one in which the emotional systems is turned off, particularly when the activation rate is negative; and one in which the emotional systems is active with a positive firing rate and commanding an order to the final stage of the network.

4.2 Bifurcation analysis

The system has a single stable node as solution and when the parameters are changed then two new equilibriums arise. This generation of new equilibriums is called as bifurcation in dynamical systems theory (Strogatz, 1994).

As the cool node is uncoupled and activated with a minimal stimulus, the parameter b_2 doesn't affect the number of equilibriums. The only parameters that count for this analysis are b_3, w_{33} and w_{32}.

Let consider a simplification replacement that allows us to make use of special graphs to analyze the changes in the parameters. The equation 2 now will be simplified to equation 9 where $z_3 = b_3 + w_{32} * c_I$; as $w_{32}<0$ and $c_I \in [0;1]$, so, $z_3 \in [-1;1]$:

$$g(h) = \dot{h} = z_3 + w_{33}h - h^3 \tag{9}$$

The bifurcation point is identified to happen when $g(h)=0$ and $g_h(h)=0$, indeed when

$$h = \pm\sqrt{\frac{w_{33}}{3}} \tag{10}$$

Proposition 2: *"The emotional systems can be perfectly inhibited only when the incoming current from the external stimulus is lesser than*

$$b_3 < \frac{(-2)\sqrt{3}w_{33}^{3/2}}{9} - w_{32}c_I \tag{11}$$

Proof:

From the equation (10), replaced in the equation (9), there can be obtained the equations of the two branches of the cusp curve in the parameters w_{33} and z_3 plane. The curve is

$$z_3 = (b_3 + w_{32}c_I) = \pm\frac{(-2)\sqrt{3}w_{33}^{3/2}}{9} \tag{12}$$

The condition of the equation (11) is derived from the left locus of the bifurcation curve (Figure 5 and Figure 6).

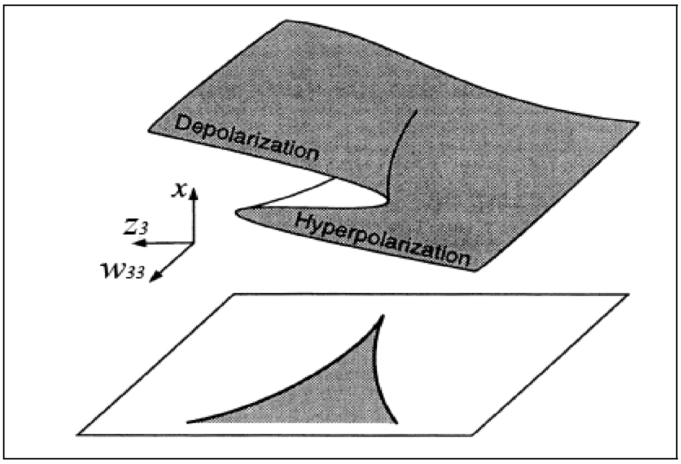

Figure 5. Cusp Catastrophe.
The surface folds over itself creating the bifurcation curve shown in figure 6 projected itself over the axes. (Adapted from Hoppensteadt and Izhikevich, 1997).

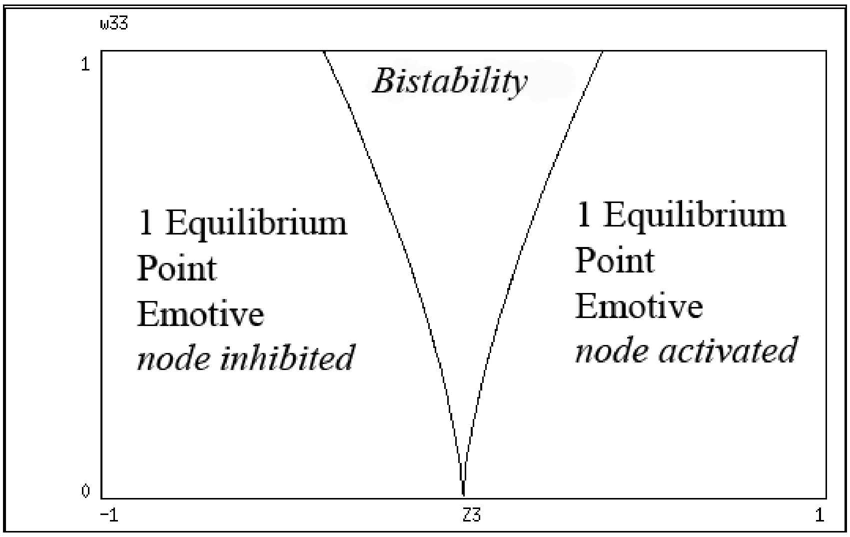

Figure 6. Cusp Bifurcation curve over the parameters plane.
Given the coordinates of the parameters, the point that they conforms falls in one of the three zones where there can be one or three equilibrium points.

The Figure 5 shows the zones delimited between the net input current (z_3) of the hot node and the excitability weight, where can be identified a single equilibrium point in two different qualitative zones and a third where there are the two stable nodes separated by the saddle. As the graphic demark in the left region are all the combinations of the parameters that make possible that the cognitive systems inhibit completely the emotional systems. The parameter $z_3<0$; what means that the stimulus is weaker that the inhibition current and the hot node is not stronger enough to generate the excitability by itself[1]. In the biestability zone, the combination of the values of z_3 with w_{33} reduces the threshold that is necessary to exceed for the hot node activation. Finally, in the right zone either z_3 or w_{33} are bigger enough to potentiate the activation of the emotional system.

[1] It must be remembered that w_{33} measures the predisposition of the systems to be active after the node have fired and there is no external stimulus.

5 FURTHER EXTENSIONS.

The formal model of the neural network presented in the former two sections, applies correctly to the self-control exercise of an agent that is trying to avoid temptations. But it is a very simplified version for the whole intertemporal inconsistency decision problem. The short version of the model is useful to visualize the results and analyze the relations between the parameters to understand how the regulation process works. Nevertheless, the complete model portrayed in the figure 1 includes the interaction between the two cognitive nodes, and the results could changes when the external stimulus perceived by the cognitive system is weak. Analyzing these complete model gains in realism but losses in tractability, especially in the resolution of the dynamical system.

The second issue that the modelization must treat is about the learning process that generates the weights considered in the network. An evolutionary approach of the behavior could determine what weights warranties an optimal decision process. An example in the study of the working memory for goal-directed behavior was modeled by Nakahara and Doya (1998). In his case, the more fitted agents presents values of the parameters near the saddle-node bifurcation; but in our case the suspect goes in the reverse direction.

Finally, the last stage of the networks where the firing rates produced by both systems are evaluated to define if the command is executed or not; could present a more inflexible operation like a threshold which limits the cases where each system can commands an order and generates a consecutive action.

6 CONCLUSIONS

The self-control regulation of the emotional system was represented by a small neural network. In face of an external stimulus both cognitive and emotional systems turn active. The excitatory cool node activates the inhibitory cool node that regulates its activity and the emotional node's firing rate. From the cognitive systems and action is commanded which is evaluated in the final stage. The emotional node, otherwise, has the characteristic to remain in an excitation mode after the stimulus was perceived. This property allows to the emotional systems to fires when perceives the stimulus, even when its initial condition is to be in a rest position. This operating form is agreed with the weakness of willpower to the control the behavior under the effect of temptations.

The described approach was formalized by a dynamical system that measures the generation of firing rates by the cool inhibitor node and the emotional hot node. The solution to ordinary dynamical equations was found as a stable node in lower levels of firing rates for the hot node given the values of the parameters. But while the parameters are changed; two new equilibriums appears: a saddle and another stable node for higher level of firing rates of the hot node. If the parameters are increased again; the lower node and the saddle disappears, only surviving the equilibrium where the emotional systems indefectible ends in an active sate.

The values of the parameters that delimit the zones in which the equilibrium are feasible determine the threshold value of the excitability weight so the cognitive system always inhibits the emotional node. The strategies that the individual exercises to override the effects of temptation, extensively depict in Metcalfe and Mischel (1999) could be identified to particular values of the model's parameters.

The objective of this work was to propose a basic structural model of the decision process that could be a constructive version of the other models presented in the introductory section. The extensions proposed goes in this direction. Finding the adequate representation of the individual cognitive process makes possible the objective to find a general benchmark to takes as reference for comparison of the behavioral models recently developed in economics.

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