

## GUYED TOWERS DYNAMICS USING KARHUNEN-LOÈVE EXPANSIONS

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**Abstract.** The dynamics of a guyed tower is herein analyzed. The finite element model of a plane lattice tower is made of two-node truss elements fixed at its base. Two inclined, symmetrically arranged cables are also modeled with truss elements considering the lack of compression capacity of these elements. The tower is subjected to a transverse load (distributed along the height) that may be static or dynamic with a cosine temporal variation. Due to the intrinsic nonlinear behavior of this structural system, the analysis is carried out with a mechanical event simulation modulus of a finite element package. The gravity loads on the tower and cables and the initial pre-stressing of the cables are first applied until static equilibrium stabilizes. Then the load, either static or dynamic, is activated. Once the time history of the tower displacements is obtained, a Karhunen-Loève (KL) decomposition (also known as Proper Orthogonal Decomposition, POD) is performed so as to obtain the proper orthogonal values (POV's or Karhunen-Loève energy contributions) and the proper orthogonal modes (POM's or Karhunen-Loève basis). The bases are optimal in the least square method sense. The qualitative analysis of these results can give clues on the dynamic behavior of the structural system. Furthermore, the KL bases are optimal to construct a low reduced model, i.e. its use in a Galerkin's approach is convenient since very few modes contain the larger information on the dynamics. As is known the proper orthogonal modes (POM's) match the eigenmodes in the linear cases. If dealing with nonlinear cases as the present one, they contain additional information.

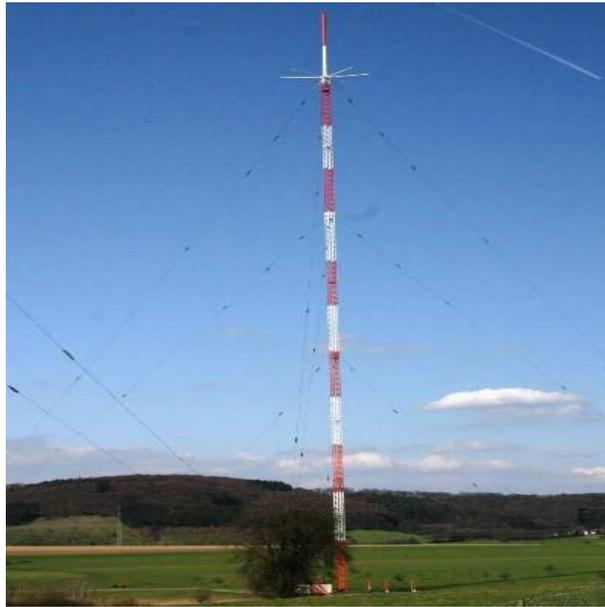


Figure 1: Typical guyed tower.

## 1 INTRODUCTION

Guyed towers are one of the most usual structural systems employed in the wireless, microwave communication industry. The last decades have been a popular design. Different aspects have been tackled by many researchers, among others, (McCaffrey and Hartmann, 1972; Madugula et al., 1998; Wahba et al., 1998; Kahla, 2000; Desai and Punde, 2001; Horr, 2004; Meshmesha et al., 2006; Shi, 2007; Preidikman et al., 2006). In particular, lattice towers are extensively used to support antennas of different sizes, shapes, and locations. A typical configuration comprises a lattice tower with triangular cross-section (three legs, horizontal and diagonal members) (see Figure 1). Their height is variable depending on the functions, but nowadays is not exceptional to see 300 m-height towers. The main structural characteristics are the large slenderness of the tower and the interaction with the taut guys. It is an example of a structure with nonlinear behavior even under service conditions. In this paper a simplified plane model of a guyed tower is studied. Its dynamic response is first found through a finite element procedure. Then a Karhunen-Loève decomposition, KLD (also known as Proper Orthogonal Decomposition, POD) is performed so as to obtain the proper orthogonal modes (POM's or Karhunen-Loève basis) and obtain a lower dimension representation. For references to these technique see, among others, Kreuzer and Kust (1996); Feeny and Kappagantu (1998); Lin et al. (2002); Lenaerts et al. (2001); Wolter et al. (2002); Kerschen et al. (2005); Ma et al. (2008). These modes may be afterwards used in a Galerkin's approach to solve another close problem. Other interesting application of the low-dimension model is the possible analysis of bifurcations and stability analysis.

The finite element model of the plane lattice tower is made of two-node truss elements and is supposed with nodes fixed at the base. Two inclined, symmetrically arranged cables are also modeled with truss elements considering the lack of compression capability of the cables. Due to the intrinsic nonlinear behavior of this structural system, the analysis is carried out with a mechanical event simulation modulus of a finite element package. This modulus has certain advantageous features as simultaneously tackling of motion, deformation and stresses for large-scale motions and an automatic time-stepping scheme together with an implicit time integration

method. First the gravity loads on the tower and cables and the initial pre-stressing of the cables are applied until static equilibrium stabilizes. Then the load, either static or dynamic, is activated. In particular, a constant, distributed along the height load is applied in three fashions, static and dynamic (time cosine variation with two different frequencies). Additionally, a lineal modal analysis is carried out. The obtained dynamic response can be written as a data ensemble in order to perform the KLD. This is a numerical technique that yields the “optimal” basis in the sense that it extracts the maximum energy content from the given data ensemble. This basis constitutes a lower dimension representation that allows the qualitative and quantitative analysis. In effect, one of the utilities is the qualitative analysis of the POV’s and POM’s to understand the energy contents of a given motion. A second one, the use of the POM’s as basis for a Galerkin-type approach to solve close cases (recall that the KLD approach is based on an ergodicity assumption). A third application is the analysis of bifurcation and stability through this lower dimension model. The first application is here in presented and the authors are working at the second one at present. The aim is to analyze variations at the configuration of the cable-tower systems solving them with the bases herein found. One possible utility is the study of guyed towers with concentrated masses modeling antennas by solving a simplified structural model. Since the location of the antennas vary in every tower, this tool could offer an economical way of finding a reliable approximated solution. Only few terms of the basis previously calculated would be sufficient to capture the complexity of the dynamic response.

## 2 TOWER-CABLE FINITE ELEMENT MODEL

The final aim of the work undertaken by this research group is to address the dynamics of a typical guyed tower with three-dimensional guy arrays. At this stage, however, a plane model is being tackled so as to study the influence of certain variables. The finite element model of the lattice tower is made of two-node truss elements. The tower is modeled as a 3D lattice with a square cross section and fixed at its base. But at this stage, only two inclined, symmetrically arranged cables contained in a plane, are considered. They are also modeled with truss elements considering the lack of compression capability of the cables. Only the motion in the cables plane will be accounted for. Due to the intrinsic nonlinear behavior of this structural system, the analysis is carried out with a mechanical event simulation modulus of a finite element package [ALGOR-Inc. \(2007\)](#). This modulus has certain advantageous features such as simultaneously tackling of motion, deformation and stresses for large-scale motions and an automatic time-stepping scheme together with an implicit time integration method. First the gravity loads and the initial pre-stressing of the cables are applied until static equilibrium stabilizes. This task is performed separately. Afterwards this static, equilibrated cable configuration is combined with the latticed tower. Cables and tower are subjected to initial forces, the first one with a tensile force and the tower with a compression one. Then the tower is subjected to a transverse load with three time variations:  $q(x, t) = q_0 \cos(\omega t)$ ,  $q_0 = 650 \text{ N/m}^2$  and

1.  $\omega = 0$  (static)
2.  $\omega = 1$  rad/s
3.  $\omega = 10$  rad/s

The tower-cables finite element model is depicted in Figure 2. It should be mentioned that the diagonals are in one direction in each side (the front and back faces are superimposed in the drawing). The height of the tower is  $L = 120$  m, the cables are fixed at 70 m from the base of

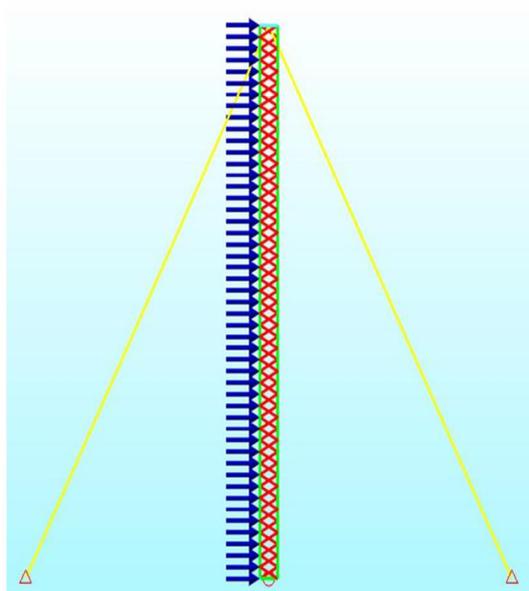


Figure 2: FEM of the guyed tower.

	vertical members	diagonals	cable
Area	$2.0910^{-3}m^2$	$1.110^{-3}m^2$	$2.68810^{-4}m^2$
Material	Steel AISI 1005	Steel AISI 1005	$E = 1.210^{11}N/m^2$
Initial Pretension	-28500 N	0	66000 N

Table 1: Structural data used in the FEM model.

the tower at each side, the tower section is 5 m times 5 m. the distance among the nodes of the vertical members is of 5 m.

Table 1 shows the data of the elements that compose the system.

In order to account for a real structure dissipation, a Rayleigh type damping was added adopting  $\alpha = 0.05$  and  $\beta = 0.05$ , being  $\alpha$  and  $\beta$  the coefficients related with the mass and the stiffness terms respectively. These values represent only very little energy dissipation. Also and to avoid the transient motion derived from the instantaneous combination of cables and tower, an axial dashpot with  $c = 10000$  N s/m is introduced at the top of the tower. Some of the output obtained from the dynamic analysis are depicted in the following figures. The tower subjected to the static load exhibits the typical time functions for the axial and transverse displacements at the top the tower (Figures 3).

Analogous plots are obtained for the dynamic loads with frequencies  $\omega = 1$  rad/s and  $\omega = 10$  rad/s respectively, which are depicted in Figure 4 and 5 respectively.

The typical cable motion for the three different loadings can be observed from a time plot of the motion at a point located at the midspan of the cable, as shown in Figure 6.

Also a natural vibration study was performed on the system. The nonlinear modal analysis feature of the MES software is in fact a linear modal analysis. The only function the nonlinear model analysis does that the linear modal analysis does not, is that the nonlinear modal analysis will examine nonlinear material properties and determine an initial "linear" stiffness to calculate the linear mode shapes and natural frequencies. This is still a small displacement modal analysis. Stiffness changes due to exaggerated mode shapes are not considered in the nonlinear

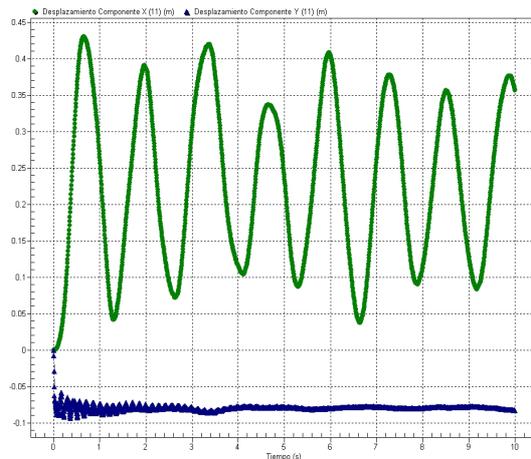


Figure 3: Axial and transverse motion at the tip of the tower . Static load.

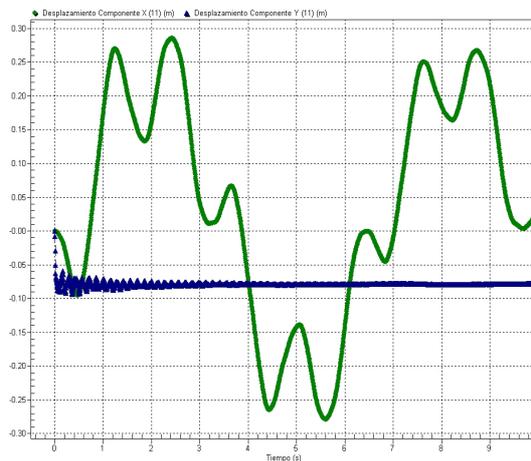


Figure 4: Axial and transverse motion at the tip of the tower . Dynamic load  $q(x, t) = q_0 \cos(t)$ .

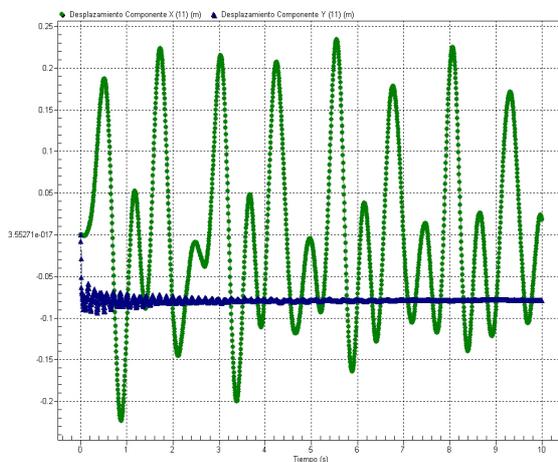


Figure 5: Axial and transverse motion at the tip of the tower. Dynamic load  $q(x, t) = q_0 \cos(10t)$ .

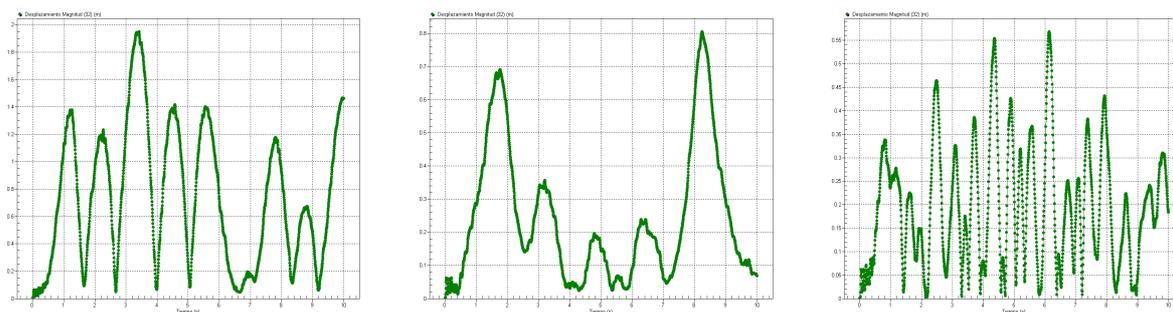


Figure 6: Time trajectory of center point of cable. a) static load; b) Dynamic load  $q(x, t) = q_0 \cos(t)$  and c) Dynamic load  $q(x, t) = q_0 \cos(10t)$ .

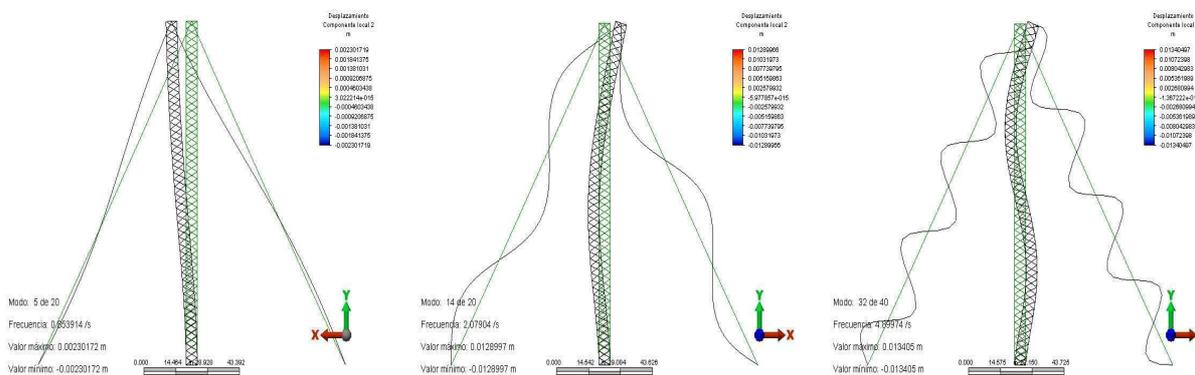


Figure 7: Three first bending-like natural modes of the tower found with linear modal analysis.

modal analysis. In the present case, since the material is linear, this is equivalent to the traditional lineal modal analysis. Figure 7 shows the first three bending-like natural modes and frequencies. The corresponding natural (linear) frequencies are:  $f_1 = 0.8539$  Hz ( $\omega = 5.365$  rad/s),  $f_2 = 2.079$  Hz ( $\omega = 13.06$  rad/s) and  $f_3 = 4.899$  Hz ( $\omega = 30.78$  rad/s). It should be mentioned that the linear modal analysis yields many lower frequency modes involving only the cables, not shown herein.

### 3 KARHUNEN-LOÈVE DECOMPOSITION (PROPER ORTHOGONAL DECOMPOSITION) FOR THE TOWER-CABLES SYSTEM

#### 3.1 Karhunen-Loève decomposition (Proper Orthogonal Decomposition) technique

A brief description of this technique is given in this section. Further details may be found in the related bibliography, see for instance Feeny and Kappagantu (1998); Wolter et al. (2002); Kerschen et al. (2005). As is known the three methodologies Karhunen-Loève decomposition, Proper Orthogonal Decomposition and Singular Value Decomposition (SVD) are related to each other, given certain conditions. In this work, a  $n \times m$  matrix contains the discrete information about the  $u(x_i, t_j)$  with  $i = 1, \dots, m$  (space) and  $j = 1, \dots, n$  (time). In the present work, they will represent axial or transverse displacements correspondingly. The following matrix U

Mode	Time steps					
	51		201		2001	
	Displacement					
	Axial	Transverse	Axial	Transverse	Axial	Transverse
1	85.707	99.654	89.148	99.652	90.282	99.653
2	14.176	0.338	10.704	0.340	9.565	0.339
3	0.082	0.003	0.103	0.003	0.108	0.003
4	0.022	0.002	0.029	0.002	0.029	0.002

Table 2: Influence of time step numbers in the energy distribution in % for the tower axial and transverse displacements. Static load.

contains the data,

$$\mathbf{U} = \begin{bmatrix} u_1(t_1) & u_2(t_2) & \cdots & u_m(t_1) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ u_1(t_n) & u_2(t_n) & \cdots & u_m(t_n) \end{bmatrix} \quad (1)$$

In order to simplify the algebra, the mean temporal value for each spatial node is subtracted allowing to work with a zero mean data generating the matrix  $\mathbf{V}$ . Consequently the correlation matrix is reduced to

$$\mathbf{R} = \frac{1}{n} \mathbf{V}' \mathbf{V} \quad (2)$$

Solving the SVD (or alternatively finding the eigenvalues and eigenmodes) of matrix  $\mathbf{R}$  one obtains the POV's and the POM's that condense the information given by the original data. There are two methods to apply KLD, the direct and the snapshot methods. Here the first one was employed since it is more convenient when the spatial data is scattered and the time intervals number is large.

### 3.2 KLD for the tower

The Karhunen-Loève decomposition (Proper Orthogonal Decomposition) was applied to the dynamic of the tower and the cables. Following the procedure described above, the POM's and POV's were found for the three transverse load cases, i.e. static load  $\omega = 0$ , and dynamic load  $\omega = 1$  rad/s,  $\omega = 10$  rad/s. The tower was divided into 48 spans to obtain  $m = 49$  spatial points. First and to adjust the number of time steps necessary to attain convergence,  $n = 51, 201$  and 2001 (time steps) were used and the energy contribution of each mode for axial and transverse displacements are depicted in Table 2 as a percentage of the sum of all the 49 POV's. As may be observed the axial displacement POV's show larger variations than the transverse displacement ones.

The study of the distribution of the energy in percentage corresponding to each POM provides an insight into the nonlinear effects of the dynamic response. As mentioned before, the number of time intervals considered to make the KL decomposition affects the results. Then in what follows, the reported results correspond to the case of 2001 time steps. Table 3 shows the variation of the percentage of energy varying with the frequency (three studied cases,  $\omega = 0$ ,  $\omega = 1$  rad/s,  $\omega = 10$  rad/s). Though further frequencies variation would be needed to make conclusions, these plots may give an indication of the strength of the nonlinear effects in the system and at the same time a reference to the dimension of the dynamics. Table 3 shows the values of

Frequency	Energy %	
	Axial displacement	Transverse displacement
Static case ( $\omega = 0$ )	90.282	99.653
	9.526	0.339
	0.108	0.003
	0.029	0.002
$\omega = 1\text{rad/s}$	92.411	99.935
	7.526	0.063
	0.053	0.001
	0.005	0.000
$\omega = 10\text{rad/s}$	87.314	91.684
	12.299	8.293
	0.365	0.017
	0.015	0.003

Table 3: Energy variation in % with forcing frequency for the tower axial and transverse displacements.

each POV as a percentage of the sum of the POVs, for both the axial and transverse displacements. As is observed the first POM retains a large part of the total energy of the system, more noticeable in the transverse motion. When the system is excited with a cosine function with  $\omega = 1\text{rad/s}$ , the first KL mode of the transverse displacement captures almost all the energy of the temporal response. As the frequency increases ( $\omega = 10\text{ rad/s}$ ) there is a transfer of energy (though not significant) from the first KL mode to the second. The third KL mode participation is irrelevant from the energy viewpoint for the transverse displacements. Regarding the axial displacement, the first KL mode participation increases for  $\omega = 1\text{ rad/s}$  and decreases and for  $\omega = 10\text{ rad/s}$  the second and third modes increase their energy.

Tables 4 and 5 depict the first three KL modes (POM's) for the axial and transverse displacements of the tower found with three transverse loads, one static and two dynamic ones respectively. As is observed there is a change when exciting the structure with  $\omega = 10\text{ rad/s}$ . In effect the first mode (the one that contribute with among 87% and 92 % of the total energy in the axial case and among 91% and 92% for the transverse motion) changes for both axial and transverse displacements. It was showed (Feeny and Kappagantu, 1998) that *in the case of free vibrations of an undamped, discrete linear system whose mass matrix is proportional to identity, the POM's converge to the mode shapes as the number of samples tends to infinity*. Another interesting feature (Kerschen et al., 2005) is that if the system is moving in resonance at a certain mode, the dominant POM is coincident with this mode shape, whatever the mass distribution. One may observe that when the forcing frequency is low (or null, static case) no changes are evident in the KL modes. Only for a higher frequency ( $\omega = 10\text{ rad/s}$ ) a modification of the first mode is present.

## 4 LOW REDUCED MODEL AND GALERKIN APPROACH

### 4.1 Simplified model of tower-cables system

At present the authors are working on a simplified structural representation of the cable tower system. The aim is to approximate the dynamics of the lattice tower by representing the tower as a beam-column. A shear beam theory is employed to account for the equivalent behavior of the truss (the stiffness of the diagonals in this case) by introducing a shear factor, calculated

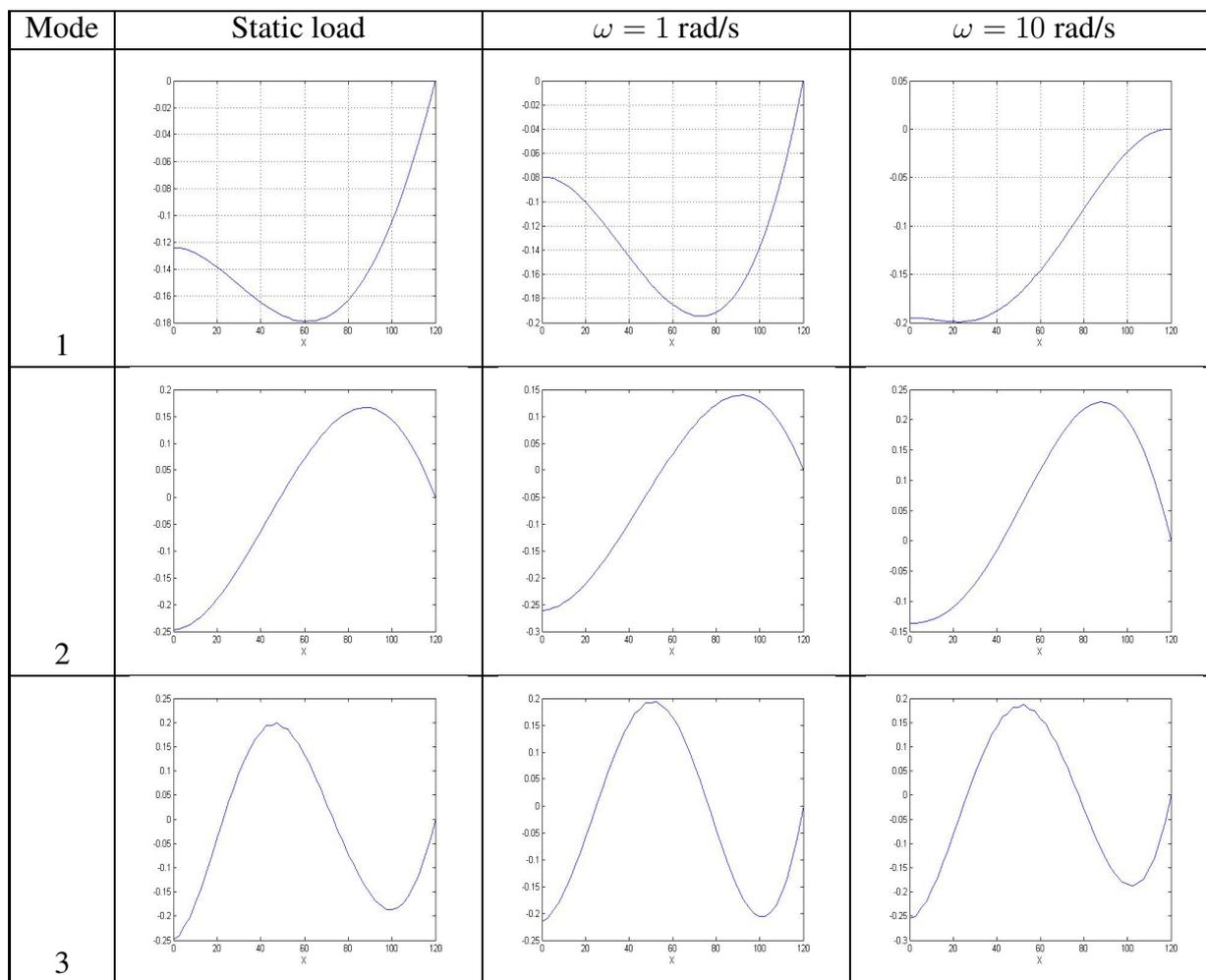


Table 4: First three KL modes (POM's) for the axial displacements of the tower for the three load cases

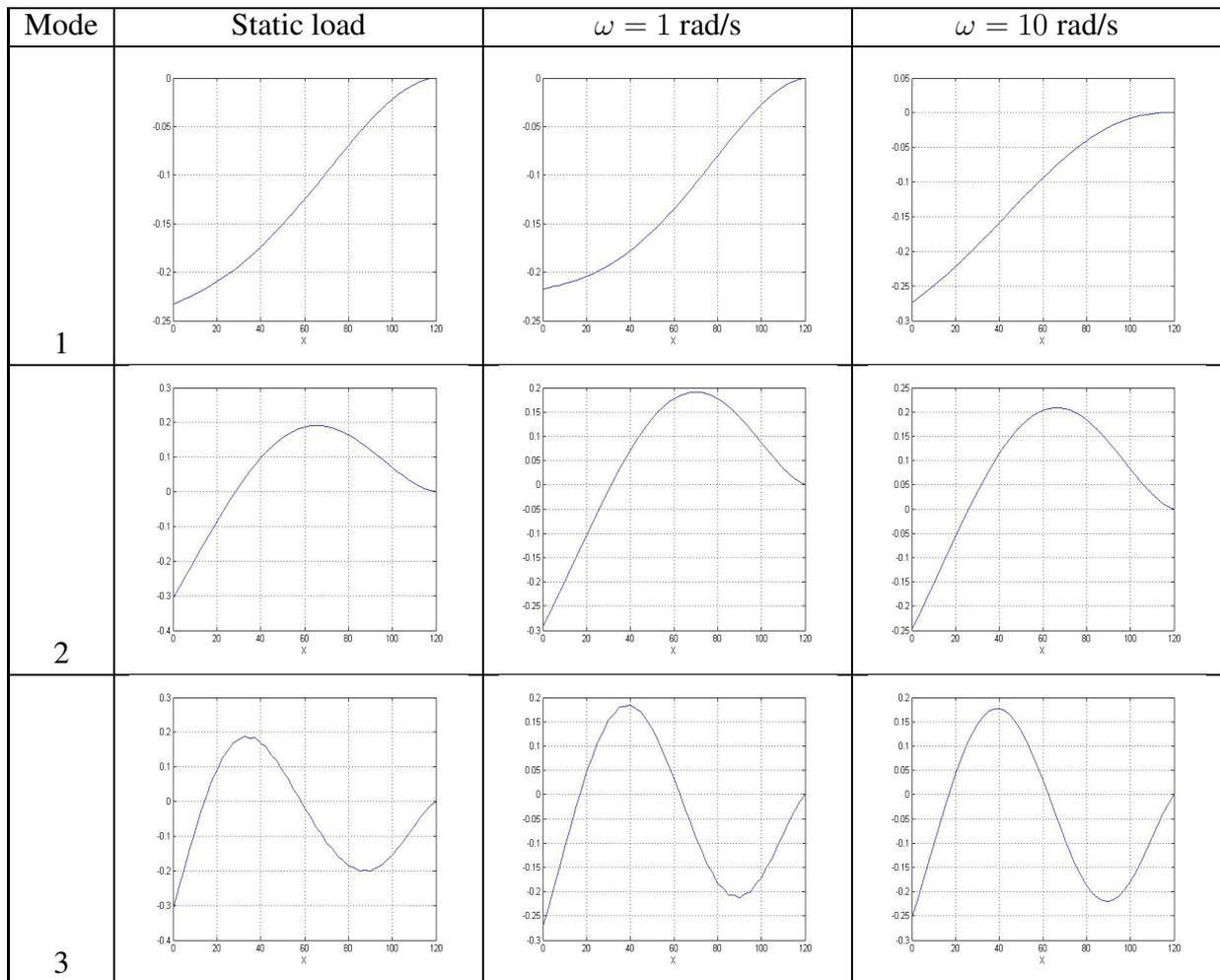


Table 5: First three KL modes (POM's) for the transverse displacements of the tower for the three load cases

from a limit case when the number of truss members is large. This factor is analogous to the one used to calculate the slenderness in latticed columns to verify buckling. No rotary inertia is considered. The second order effect due to the effect of the axial loads derived from the cables interaction and the tower weight.

A first approach is to consider a quasi-static behavior of the cables which has yielded successfully results when dealing with slack cables and low frequency dynamic loads, see for instance (Rosales and Filipich, 2006; Escalante et al., 2008). The authors have solved in other study the dynamic behavior of extensible cables stating the differential equations and solving them by various techniques being the more effective a finite elements approach and an iterative solution of a strongly nonlinear differential system (Escalante et al., 2008).

If one solves the cables separately by prescribing axial and transverse at the point where they are connected to the tower, the vertical and horizontal restoring forces can be found for a region of admissible displacements. This is very convenient since the cables are replaced by these forces and the inertia forces of the cables added at the cables center of gravity. Then the beam-column is combined with these forces and solved through a finite element software (FlexPDE). This approach was successful in a previous work when applied to (both inextensible and extensible) slack cables combined with a rigid platform and subject to dynamic loads of low frequencies content (Rosales and Filipich, 2006; Escalante et al., 2008). However, the first results of the dynamics when cables are taut, even with a static load, are not as satisfactory when compared with results of the “full” model (Figure 3). The interaction among the two cables and tower is not completely reproduced. At the present time, the simplified model of the beam-column is being combined with the cables, both modeled with a finite element method of the tower-cable system.

## 4.2 Future work on the Galerkin’s approach with KL basis

Once the simplified model is tested and adjusted, the KL basis of the tower will be introduced to perform dynamic analysis of cases with similar loading, and introducing some complexities as concentrated (small) masses as mentioned before. Also the KL basis of the cables can be found with the data extracted from the dynamic described in Section 2, which would give a complete reduced dimension model. Escalante et al. (2008) applies the KLD to the dynamics of an extensible cable.

## 5 FINAL COMMENTS

The first stage of a study of the dynamic of guyed towers has been presented. The dynamics in a plane of a simple latticed tower with two cables in the plane. The cables are taut. A finite element model was constructed within a nonlinear mechanical event simulation software. Both static and dynamic loads were studied as well as a linear mode analysis. A Karhunen-Loève decomposition was performed for the tower axial and transverse motions finding the POM’s and POV’s which show the influence of the nonlinearities and the dynamic content of the load. The application of these basis to a simplified model is under study.

## 6 ACKNOWLEDGMENTS

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