Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXVII, págs. 2605-2631 (artículo completo)
Alberto Cardona, Mario Storti, Carlos Zuppa. (Eds.)
San Luis, Argentina, 10-13 Noviembre 2008

SYNTHESIS AND OPTIMIZATION OF COMPLIANT MECHANISMS FROM A RIGID-BODY APPROACH

Martín A. Pucheta and Alberto Cardona

Centro Internacional de Métodos Computacionales en Ingeniería (CIMEC), INTEC (Universidad Nacional del Litoral-CONICET), Güemes 3450, S3000GLN Santa Fe, Argentina,
e-mails: {mpucheta,acardona}@intec.unl.edu.ar, http://www.cimec.org.ar

Keywords: Compliant-mechanism synthesis, Graph Theory, Pseudo-Rigid-Body Model, Bistable mechanisms, Optimization.

Abstract. A systematic approach for compliant mechanism synthesis starting from the problem requirements is proposed. The proposed method uses as topological solution space several atlases of compliant mechanisms specialized from rigid kinematic chains. Although these atlases -with rigid and flexible links and joints- configure a subspace of the possible compliant mechanisms, they are very useful concepts from the functional point of view, and additionally, they are suited for the systematization of the design process. Graph Theory is used to solve the type synthesis stage while the rigid analytical synthesis combined with a process called rigid-body replacement synthesis is used to synthesize the flexible members. Several industrial test problems were solved using this method: a two-degree-of-freedom switch, a multiple deflector for nozzles of a turbine engine, and a bistable actuator for landing gear retraction. The latter test will be illustrated throughout this paper. The best concept for each problem is optimized using commercial software. Part of the methodology may be extended for dealing with compliant mechanisms with arbitrary topologies different from those arising from rigid-kinematic chains.

1 INTRODUCTION

The many advantages of compliant mechanisms compared to their rigid-body counterparts have produced a growing interest in compliant mechanism synthesis methods. Compliant mechanism synthesis is currently a challenging area of development that has been addressed in several ways. Three main approaches can be highlighted:

i) *Rigid-body synthesis*: rigid-body replacement starting from rigid-synthesis results obtained by precision-positions methods (Berglund et al., 2000). The iterative analysis of Pseudo-Rigid-Body Models (PRBM) may also be used in combination with optimization techniques (Howell, 2001).

ii) *Continuum synthesis*: where topology and shape optimization are combined with linear and non-linear finite element analysis (Pedersen et al., 2001; Sigmund, 2001a). In Homogenization Methods (Bendsøe and Kikuchi, 1988) the densities of elements are taken as design variables; multiple materials were considered by Sigmund (2001b), then by Saxena and Ananthasuresh (2003), and recently, by Wang et al. (2005) using Level Set Methods.

iii) *Discrete synthesis*: the main difference with the previous approach is that the mechanism is represented by a network of truss or beam elements so that topological synthesis can firstly be solved using discrete algorithms; then, several sizing methods are applied for each enumerated topology. Some examples are: the Load Path Representation (Lu and Kota, 2006), the use of the Spanning Tree Theory (Zhou and Ting, 2005), and Graph Theory (Sauter et al., 2007). In these works, the design of sectional areas of nonlinear beam elements is achieved in several ways, using different degrees of complexity: fixed coordinates of nodes (Saxena and Ananthasuresh, 2001) or moving coordinates (taking element lengths as variables) with avoidance of elements overlapping (Lu and Kota, 2006; Pellegrino and Santer, 2007; Zhou and Ting, 2005); additionally, Sauter et al. (2007) considered curved beams with variable thickness.

In previous works (Pucheta and Cardona, 2007b; Pucheta, 2008), we modified an available solver for rigid mechanisms to computationally implement the kinematic synthesis of compliant mechanisms by means of *rigid-body replacement*. The rigid replacement solver can be used in two kinds of synthesis applications:

a) Replacement of rigid parts of a rigid mechanism by flexible members to obtain partially compliant mechanisms –"partially" referring to the presence of some rigid body or kinematic pair in the mechanism.

b) Design of bistable mechanisms, i.e. with two stable equilibrium positions; see, for example, the lid of a shampoo bottle in Figure 1.

In this paper, we present preliminary results for a new approach for the design of bistable mechanisms. The method is based on the solution of the initial and final unstrained positions of a compliant member composed of a clamped flexible link.

This paper is organized as follows: Section 2 continues the review of available compliant synthesis methods and introduces the scope of rigid-replacement synthesis. The adopted method is introduced in Section 3; the type synthesis method is explained in Subsection 3.1 and the rigid-replacement rules in Subsection 3.2. The design of bistable mechanisms is presented and illustrated in Section 4.

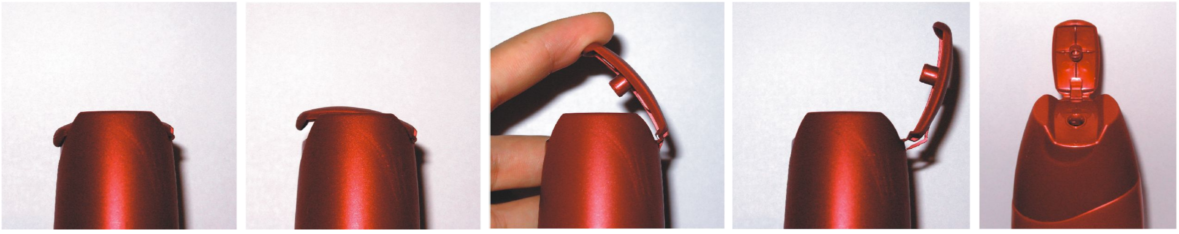

Figure 1: Bistable mechanism of a shampoo bottle lid: human intervention is needed to unlock and guide the lid until surpassing the angular unstable position, in the middle photograph; then, the internal strain energy stored in the flexible members drives the lid to the final stable position.

2 COMPLIANT SYNTHESIS METHODS

Synthesis methods consist of two traditional stages: topological synthesis and dimensional synthesis. Most of the topology optimization methods are based on the continuum approach in which the material is iteratively removed, either by reducing the density of an element or by eliminating the element completely. The *disconnection* and *gray areas* are the main drawbacks of continuum methods. Lu and Kota (2006) overcame these problems using their *load path representation* method: first, they enumerate topologies using a basic array of beam elements and retain those topologies for which there always exists a path between the input, output and grounded members, passing through intermediate members, then, they size each alternative by changing positions of nodes and sections of beams. Pellegrino and Santer (2007) also used this method with some improvements on the sizing, by incorporating the avoidance of overlapping elements[1] (or link crossings) by penalization. Zhou and Ting (2005) also proposed the enumeration of networks connecting input, output, support and intermediate nodes using the *spanning-tree theory*. Saxena and Ananthasuresh (2003) proposed a number synthesis method by which an optimal topology obtained by means of discrete synthesis is converted into a pseudo-rigid one.

On the other hand, Murphy et al. (1996) proposed the enumeration of compliant topologies from rigid kinematic chains in the form of atlases. The data base of stored atlases has a significant size. The use of atlases ensures to have *connected* topologies as design space but requires that the desired input and output members in the specifications be identified with members of each stored mechanism in all non-isomorphic ways. The use of rigid and compliant atlases that we proposed in (Pucheta and Cardona, 2007b,a) allows the designer to define prescribed parts, with fixations and the input/output relationship between their members, and then, automatically search them inside mechanisms of a selected atlas. This concept will be exploited in this paper in the topological synthesis stage. Then, the remaining problem is the sizing of each alternative.

Howell (2001) presented the Pseudo-Rigid-Body Model (PRBM) as a very practical tool to simplify the analysis and synthesis of compliant mechanisms. By using rigid-body components, the PRBM allows the designer to model and *analyze flexible members* that undergo large nonlinear deflections. The inverse design situation, called *Rigid-Body Replacement Synthesis*, is also very easy to make by identifying rigid-body components as the PRBM of *flexible members to synthesize*. The approach is useful for designing mechanisms to perform a traditional task of kinematic synthesis –path following, function generation and rigid-body guidance– without concern for the energy storage in the flexible members.

[1]The avoidance of overlapping elements is desired when the mechanisms must be manufactured in one layer.

3 THE PROPOSED METHOD

The method consists of the following stages: (i) convert the kinematic compliant problem into a rigid one defined by precision positions, (ii) apply rigid number synthesis methods to propose a valid topology, (iii) find the initial dimensions (link lengths and pivot positions) using Precision-Position Methods (PPM) (Sandor and Erdman, 1984), (iv) wherever possible, identify the resultant parts as the pseudo-rigid-body models of the compliant members to be synthesized. An additional optimization loop of *dimensional synthesis* must be applied to further refine the dimensions for minimizing the kinematic errors. These stages were implemented in a C++ language code, in a solver for type and initial sizing of mechanisms (Pucheta, 2008, chap. 8).

The type synthesis stage uses as topological solution space several atlases of compliant mechanisms specialized from rigid kinematic chains. These atlases -with rigid and flexible links and joints- configure a subspace of the possible compliant mechanisms. They are very useful concepts from the functional point of view, and additionally, they are suited for systematization of the design process.

3.1 Type synthesis using atlases of compliant mechanisms

Type synthesis is the primitive stage of conceptual design where topology, types of links and joints must be chosen to form mechanisms in all valid forms, without repetitions, and satisfying the structural requirements of the problem. Graph Theory has been traditionally used to model and solve this problem. In the literature we can find fragmented developments on type synthesis. Many researches were done on kinematic chains enumeration and atlas construction, but few works explain how to make use of theses atlases (Mruthyunjaya, 2003).

We propose the use of atlases by a previous enumeration, and then making a selection by comparison with the topology of the problem in hand, by a subgraph search. The method is composed by four steps:

T1) Give a graph description for the kinematic problem.

T2) Choose an *atlas of mechanisms*, also represented by graphs, for the desired solutions.

T3) Set the topological constraints to be fulfilled by the solutions.

T4) Find a subgraph occurrence of the *initial graph* inside the atlas.

In step **T1** we apply some rules on the initial description of the problem to construct a graph that we denominate *initial graph*, denoted as G_{ini}. This graph has connectivity properties, and types of links and joints of the prescribed parts.

Step **T2** supposes two previously developed steps:

T2-a) *Enumeration of all kinematic chains* for the desired degree of freedom.

T2-b) *Specialization of each kinematic chain* for the desired types of links and joints.

Kinematic chains enumeration was done following Tsai's proposal (Tsai, 2001). Each kinematic chain is represented by the adjacency matrix of its graph $G(E, V)$, where each link is represented by a vertex $v_i \in V$ and each joint $e_{ij} \in E$ is the connection between vertices v_i and v_j. The specialization of the link and joint types is formulated as an assignment problem and a colored graph representation is used. The adjacency matrix of the colored graph has integer

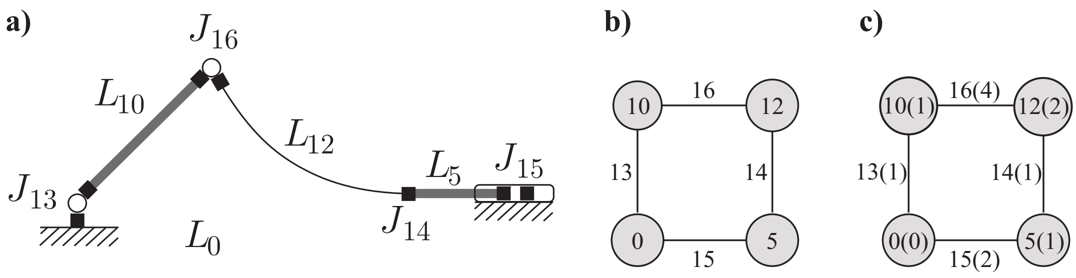

Figure 2: Mathematical models for a four-bar mechanism: a) FEM representation; b) Graph labelled with user's IDs; c) Graph with link and joint types colors (colored labelled graph).

entries representing link types on the diagonal entries and joint types on the outer diagonal ones. We call it *Type Adjacency matrix* $\boldsymbol{T}$, and it is defined as follows:

$$T_{ii}(v_i) = \begin{cases} 0 & \text{if } v_i \text{ is the ground,} \\ 1 & \text{if } v_i \text{ is a rigid link,} \\ 2 & \text{if } v_i \text{ is a flexible link,} \end{cases} \quad T_{ij}(e_{ij}) = \begin{cases} 0 & \text{no connection,} \\ 1 & \text{if } e_{ij} \text{ is a revolute joint,} \\ 2 & \text{if } e_{ij} \text{ is a prismatic joint,} \\ 3 & \text{if } e_{ij} \text{ is a flexible joint,} \\ 4 & \text{if } e_{ij} \text{ is a clamped joint.} \end{cases}$$

For example, the mechanism shown in Figure 2 has a type adjacency matrix:

$$\boldsymbol{T} = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 1 & 0 & 4 \\ 2 & 0 & 1 & 1 \\ 0 & 4 & 1 & 2 \end{bmatrix}.$$

Table 1 shows the number of specialized compliant 1DOF-mechanisms for the well known four- and six-bar basic kinematic chains (BKC). An alphabet-like coloring is considered: the links alphabet is $\{0 = \text{ground}, 1 = \text{rigid}, 2 = \text{flexible}\}$, and the joints alphabet is $\{1 = \text{revolute}, 2 = \text{prismatic}, 3 = \text{flexible\_hinge}, 4 = \text{clamped(fixed)}\}$.

	BKC			
suffix	Four bars	Watt	Stephenson	Total
R	211	50,267	52,507	102,985
RP	731	448,673	459,482	908,886
ROneP	506	178,845	183,623	362,974

Table 1: Compliant one-DOF linkage atlases (CompliantOneDof).

The atlas obtained with ground, rigid and flexible links, and revolute, flexible and clamped joints is denominated CompliantOneDofR. All non-isomorphic assignments include mechanisms inversions. The enumeration produced a very high number of mechanisms. When considering two types of kinematic pairs (revolute and prismatic) the atlas obtained is denominated CompliantOneDofRP. The number of alternatives is augmented almost 9 times because of the "combinatorial explosion", compare last column in Table 1. Typical kinematic problems often have one prismatic joint, so we also built the atlas CompliantOneDofRoneP to accelerate the search.

In step **T3** the user can restrict the number of links or joints of a determined type to some predefined values, filtering the number of explored mechanisms. The purpose of the filtering is to make the search more user friendly, since the exploration within the space of solutions satisfying the topological specifications can easily lead to thousands of feasible mechanisms.

Finally, step **T4** is automatically achieved. The initial graph is exhaustively searched inside each graph taken from the atlas. All non-isomorphic subgraph occurrences are detected and enumerated as feasible topologies; more details can be found in (Pucheta, 2008, chap. 3). The search is exhaustive and it finishes when a number of alternatives fixed by the user is reached.

After the type synthesis stage, an additional procedure decomposes each topology into single-open chains (SOCs): dyads and triads passing through the number of prescribed positions. These single-open chains are implemented as SOCs modules which have two functions: (i) evaluate the geometrical data to determine the multiplicity of solutions and the number of free parameters required to solve the geometrical loop-closure equations; or (ii) solve the equations once the geometrical data, free parameters and multiplicity are provided. Using the first function, all free parameters and their default bounds are determined and stored for each alternative.

A second stage of initial sizing for synthesis is launched for each feasible alternative as described in the next section.

3.2 Rigid-body replacement synthesis

We propose the use of a design space defined by the set of free parameters of single-open chains passing through a predefined number of prescribed positions. The user can change the values of their bounds, and a genetic algorithm is used to sweep the design space (Pucheta and Cardona, 2005; Pucheta, 2008). For every feasible individual, computed as rigid by PPM, rigid links are replaced by flexible ones following the graph description and special rules; also, the motion of the flexible joints are constrained to certain limits. The fitness function consists in the minimization of the size of the mechanisms together with three weighted constraints: minimal length of link dimensions, non-inversion of transmission angle (which includes the limit angle for flexible joints), and allowed space violation. Eventually, instead of considering non-inversion of transmission angle as a constraint, a full kinematic analysis is made for each individual to compute the fitness function.

3.2.1 Short review of Pseudo-Rigid-Body Models

A beam can be modeled by an articulated rigid body with the beam *characteristic length* and by torsional springs located on its *characteristic pivots* to emulate the beam stiffness (Howell, 2001).

The *characteristic length* is computed as γL where γ is the *characteristic radius factor* which is determined as function of the load case and boundary conditions, and L is the beam length. The spring stiffness $K(EI, L, \gamma)$ is used to reproduce the force-deflection relationship. It depends on the beam material properties E, the inertia of the cross-section I, the geometry L, and load case (direction of the applied force F and existence of end-moment loading M) through γ. The deflection is a function of the so-called *pseudo-rigid-body angle* Θ. See, for example, the parameterization of a beam of length L subjected to two different conditions in Figure 3-**a** and **b**. The boundary conditions as well as the load conditions are different. Both cases consider loads only at the end-points of the beams.

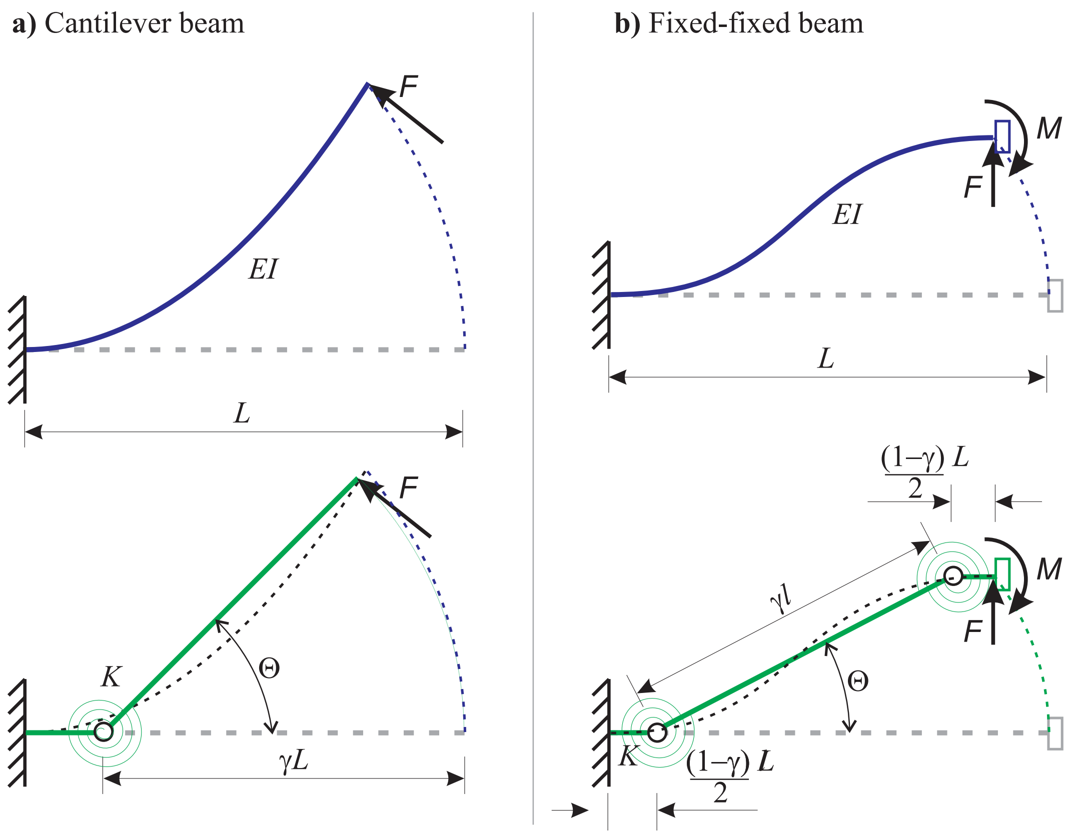

Figure 3: PRBM models for two beams (Howell, 2001).

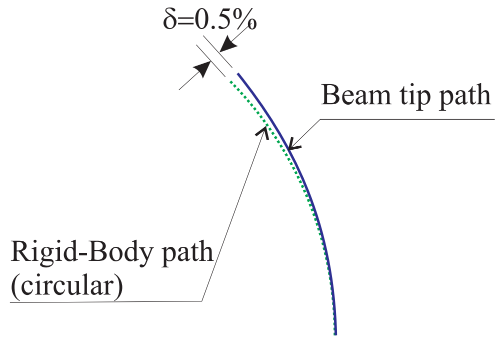

Figure 4: Error between tip trajectories: exact beam vs. its PRBM model (Howell, 2001).

There is an error with respect to the kinematic behavior of such simplified model. For example, in the cantilever beam, the error between the trajectories is shown in Figure 4. The difference of the trajectories between the exact and the PRBM model increases as the rotated angle Θ is increased. By defining an admissible path error between the exact beam and the PRBM, Howell computed a factor γ that maximized the deflection $\Theta_{\max}$ as function of the angle of the applied load F. The example shown in Figure 4 corresponds to a load applied perpendicularly to the beam for which a path error of 0.5% was obtained at $\Theta_{\max} = 73°$ and $\gamma = 0.8517$. Howell also determined that for a wide range of angles of the applied force F ([$135°$-$63°$]) the average of γ is 0.85. Additionally, for end-moment loading $\gamma_{\text{ave}} = 0.7346$ with $\Theta_{\max} = 124.4°$. For the model shown in Figure 3-**b** he found that $\gamma_{\text{ave}} = 0.8517$ with $\Theta_{\max} = 64.3°$.

For synthesizing a beam, the unknowns to be computed are its length and stiffness:

Beam length: Starting from the geometry of a rigid mechanism, a beam can be designed by identifying revolute joints of the rigid mechanisms as the characteristic pivots of the PRBM of the beam, then the beam length can be roughly estimated by proposing $\gamma = 0.8517$ for all beams in the mechanism.

Beam stiffness: For an initial design, Young modulus for steel E, and the values $I = \frac{L^4}{10^5}[\mathrm{m}^4]$ and $A = \frac{L^2}{100}[\mathrm{m}^2]$ (with $[L] = \mathrm{m}$) are heuristically defined. These parameters can be later optimized once the initial mechanism is found.

The spring of each characteristic pivot of the PRBM of a beam emulates the beam stiffness. However, a rigid mechanism does not support any moment at revolute joints. For the chosen beam length and stiffness, the major kinematic error in the path-deflection characteristic will arise in the resultant load-case when the beam is assembled with other pieces inside the mechanism. For example, the end-points of the beam would result loaded by traverse forces and end-moments which are different from that pure bending hypothesis considered by $\gamma = 0.8517$.

3.2.2 Replacement synthesis

We consider that, for each feasible closed-loop topology, each link has equal degree functionally and structurally (Howell, 2001). This means that structurally binary links are functionally binary, ternary links are functionally ternary, and so on. Additionally, all segments are assumed initially straight. A third assumption is that all links are homogeneous, i.e., rigid links are constituted only by rigid segments and flexible links, only by flexible segments. Another topological assumption is added: actuated links are considered rigid, and coupler links (floating) with imposed passing points are also considered rigid.

After computing the rigid dimensional synthesis, the set of minimal independent loops of a rigid mechanism topology is used to visit the links and analyze their feasibility to be transformed into their flexible equivalents. Links are analyzed in groups of three consecutive ones. When a flexible link is found, decisions about changing the coordinates of their end-points nodes are taken considering not only the type of the terminal joints, but also the type of the terminal links. Inside this analysis, ground and rigid links are equally considered as "rigid".

Let l_i be the considered flexible link, and let l_{i-1} and l_{i+1} be the adjacent ones. Also, let R be the length of the rigid link, and L the length of its flexible equivalent. The *characteristic ratio* $\gamma = R/L$ was taken to be fixed at a value $\gamma = 0.8517$. As it is shown in Figure 5, we synthesize two typical cases in the following way:

a) Fixed-Revolute (Revolute-Fixed) flexible link: the fixed node is relocated at a distance

$$L = \frac{R}{\gamma}$$

from the pinned end, measured along the link axis. This modification is applied if the link type of the preceding (or following) link l_{i-1} (l_{i+1}) is rigid. The other link can be either of rigid or flexible type.

b) Fixed-Fixed flexible link: both fixed nodes are relocated at a distance

$$L_{\frac{1}{2}} = \pm\frac{R}{2\gamma}$$

from the link midpoint along its axis. This modification is applied if the link types of the preceding and the following links, l_{i-1} and l_{i+1}, are both rigid.

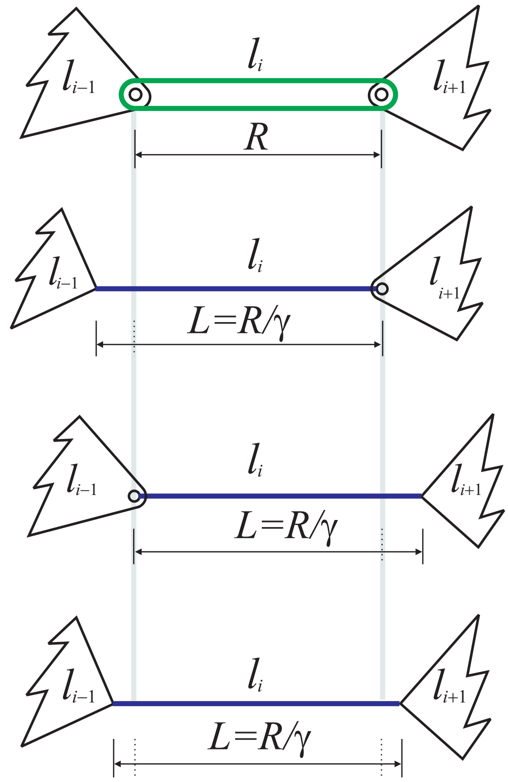

Figure 5: Rigid-body Replacements: a) Fixed-Revolute, a') Revolute-Fixed and b) Fixed-Fixed.

With these simple modifications, the characteristic pivots are located at the same location as the pin joints. Other cases are not replaced. Some cases are for now ignored and still have to be investigated.

The angle rotation in the neighborhood of clamped joints does not have to surpass certain limits. After rigid synthesis we have the information about rigid links rotations for a number of passing points, thus we can measure the maximum rotation between two links on the revolute joint which will be the characteristic pivot of the PRBM of the flexible link. This does not exactly reflect what occurs in flexible links, but it is useful to develop a new constraint: *limit angle for clamped joints* denoted as $\Theta_{\max}$.

In Figure 6-a we can see the initial and final position for a rigid link to be replaced. The maximum rotation, θ_{rigid}, is measured on the revolute joint. True rotation at the clamped end of the flexible link, θ_{clamped}, and the desired maximum angle, $\Theta_{\max}$, are shown in Figure 6-b. Note that the maximum angle performed by the rigid link in the rigid mechanism is a rough but useful approximation of the flexible link endpoints rotations in the transformed mechanism. This constraint was added into a solver for rigid mechanisms, a detailed explanation can be found in (Pucheta, 2008, chap. 6, chap. 8).

Some advantages of including compliant segments (while replacing kinematic pairs) into traditional mechanisms are well-known. Under the scope of planar linkages, these advantages are:

- Increase in precision by backlash elimination.

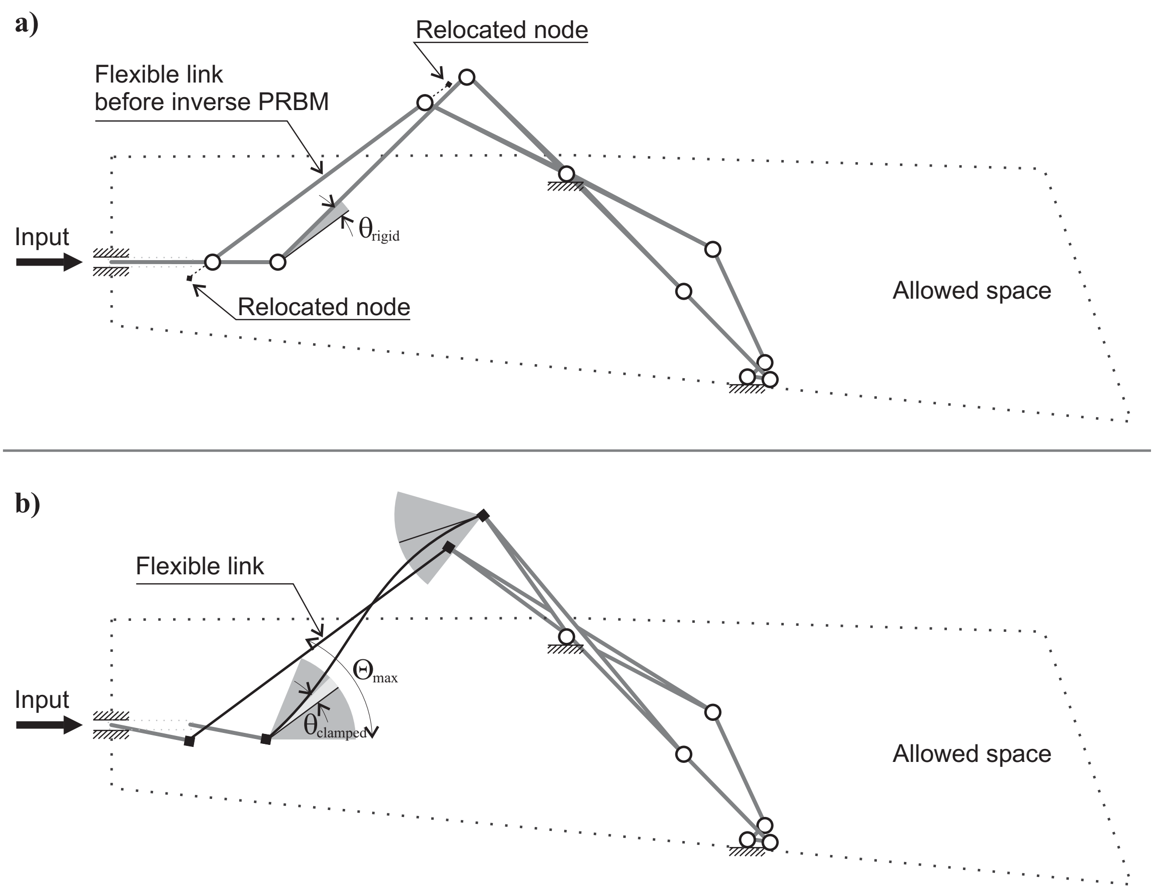

Figure 6: Example of replacement of a flexible segment and its limit angle for clamped ends.

- Maintenance reduction by elimination of wear and lubrication needs.
- Reduction of parts compared to rigid designs.
- Reduction of manufacturing and assembly time costs.
- Increase in the possibility of miniaturization and one layer manufacturing.

We can add

- Capabilities of nonlinear motions similar to that performed by articulated mechanisms including bistable behavior.

Fatigue life reduction is a common disadvantage for all compliant mechanisms. Among the disadvantages of the replacement method we can remark:

- After links transformation, flexible links are larger than their rigid counterpart, thus they might produce a worse fulfilment of the *allowed space* and *minimal link lengths* constraints (see Figure 6).
- Links which fully rotate cannot be replaced by flexible beams or rigid links with flexible joints.

4 DESIGN OF BISTABLE MECHANISMS

A bistable mechanism has two stable equilibrium positions within its range of motion. Power requirements may be greatly reduced by using bistable mechanisms, which require energy only to switch states, while requiring no energy to maintain state (Howell, 2001; Jensen and Howell, 2003, 2004). The design of bistable mechanisms is a synthesis problem where the energy storage characteristics are specified.

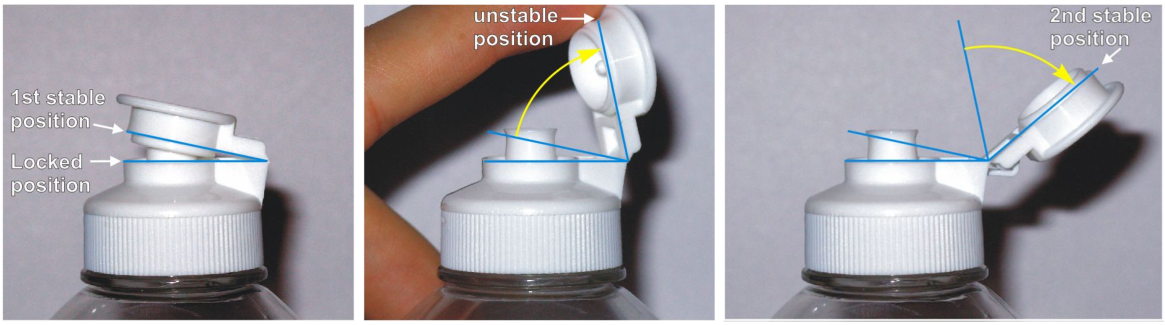

Figure 7: Bistable mechanism of a bottle lid: left and middle photographs show two positions driven by the human; then, the lid is driven by releasing the internal strain energy until the second stable position is reached.

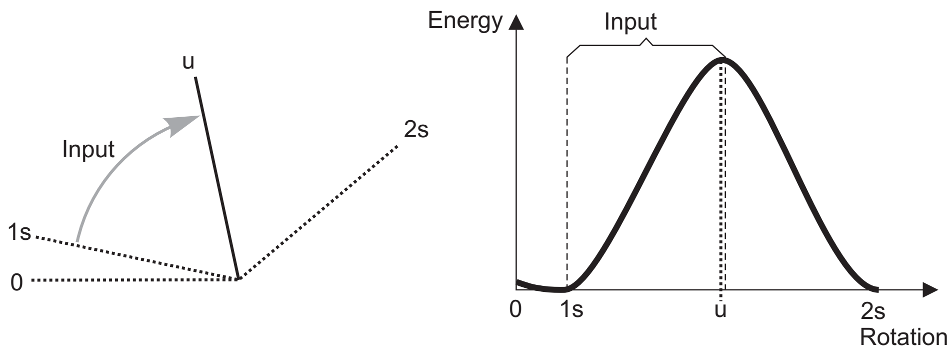

Figure 8: Energy characteristic for the opening operation.

In Figure 7, a sequence of photographs shows several actions required to open the lid of a bottle. Figure 8 shows the energy characteristic where the angular positions or time are related to the energy stored in the compliant members. The mechanism is detailed in Figures 9 and 10. This mechanism is functionally identical to the shown in Figure 1. It is a deformable triangle where one link is rigid, another link is the ground and the remaining member is a spring. The stored energy can be easily designed by choosing the shape of the triangle, since the summation of the lengths of the rigid links determines the deformed length of the spring. This triangle is widely applied in windshield wipers, several bottle lids, garage and car doors, among other applications.

Various flexible members, which have been used as parts of bistable mechanisms, have the *functionally equivalent* kinematic behavior of a spring, some of them are illustrated in Figure 10: (a) a flexible body with living hinges, (b) a pinned-pinned beam, and (c) a rigid-body segment clamped to a pinned-beam. Note that the, apparently unnecessary, complexity of the flexible

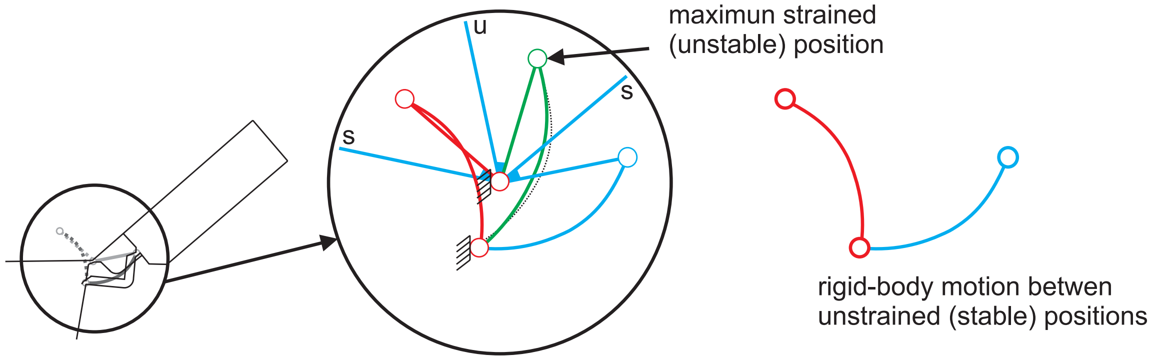

Figure 9: Functioning principle: the flexible member of the mechanism is unstrained in the initial and final positions (Jensen, 1998; Howell, 2001) while it suffers a maximum deformation in an intermediate position.

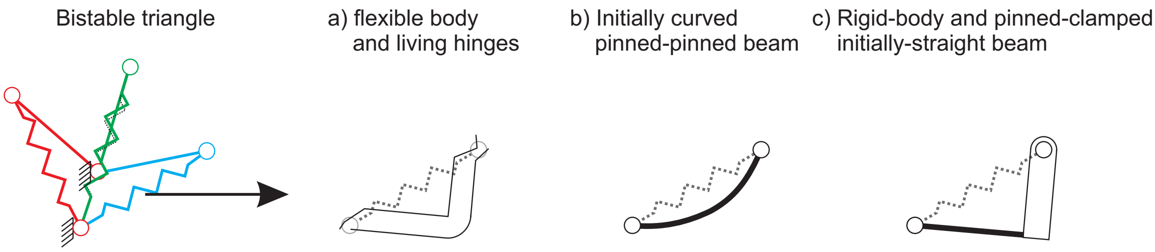

Figure 10: A flexible triangle with one spring is the simplest bistable mechanism; the spring can be replaced by other equivalent members.

member shown in Figure 9 has an aesthetic reason. A pinned-pinned beam has the advantage of supporting compression and extension loads on the line connecting the pins while a spring supports extension but it needs a prismatic joint between pins to avoid buckling if the load is compressive. The last design (c) will be used in the examples given later.

Figure 11: Bistable hinge of a shampoo lid: the human hand actuates the lid from the first stable position up to the unstable position is surpassed.

Other bistable design for a shampoo lid is shown in Figure 11. It consists in a single compliant hinge with shape of bow; the external borders of this bow suffer the maximum strains. It is said that the bow has two stable shapes rather than positions.

More complex energy specifications can be required. For instance, a non-null energy may be required at the stable positions. This requirement can be obtained with the aid of kinematic limitations or geometric constraints. For example, in the cellular phone shown in Figure 12, a little energy must be applied for opening the lid. In the second stable position, a remanent

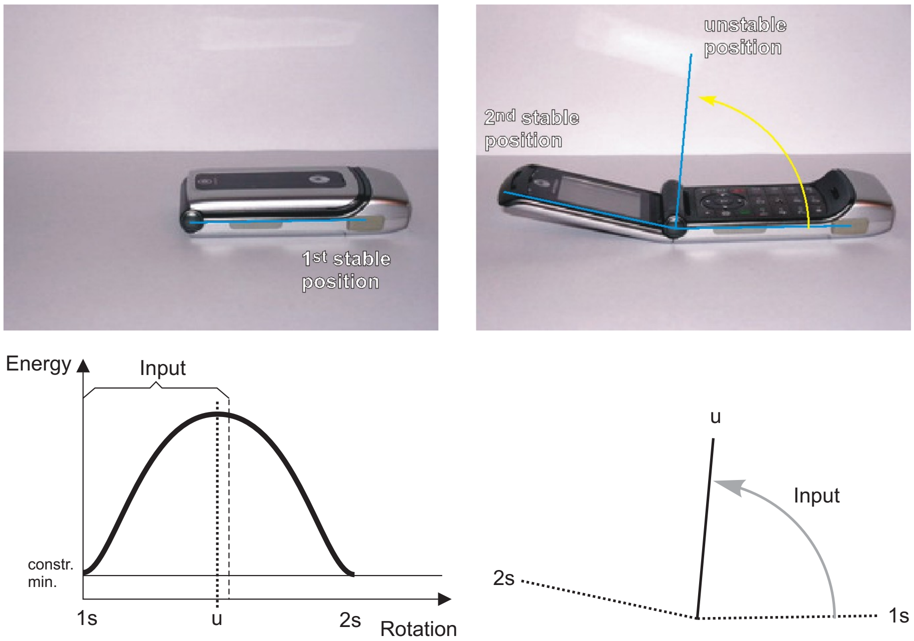

Figure 12: Energy characteristic for non-null local minima in the stable positions.

moment avoids an accidental closure. Thinking the energy as the "ball-on-the-hill analogy" (Howell, 2001; Anderson, 2005), this is an example of two constrained equilibrium positions, see Figure 13.

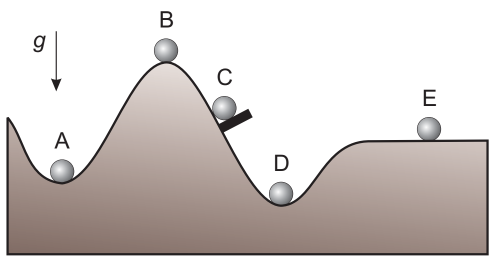

Figure 13: Equilibrium positions of a ball subjected to a gravity field g: A) and D) are stable; B) is unstable; C) is externally constrained stable; E) is neutrally stable.

For compliant linkage mechanisms, it is known "a priori" that compliant links or compliant joints have free of energy or unstrained positions. From a rigid-body point of view, a new approach based on the unstrained positions of the compliant joints is exploited to design bistable mechanisms with one flexible member.

4.1 Bistable actuator for a landing gear mechanism

In previous works, a four-bar mechanism was synthesized for the retraction/deployment of a landing gear mechanism (Pucheta and Cardona, 2005; Pucheta, 2008). The animation of the simplest result is the folding linkage shown in Figures 14 and 15.

A new study is made to analyze the feasibility of actuating the mechanism providing a bistable behavior. Four-bar compliant linkages can be designed to provide a bistable behavior (Howell, 2001; Jensen and Howell, 2003, 2004).

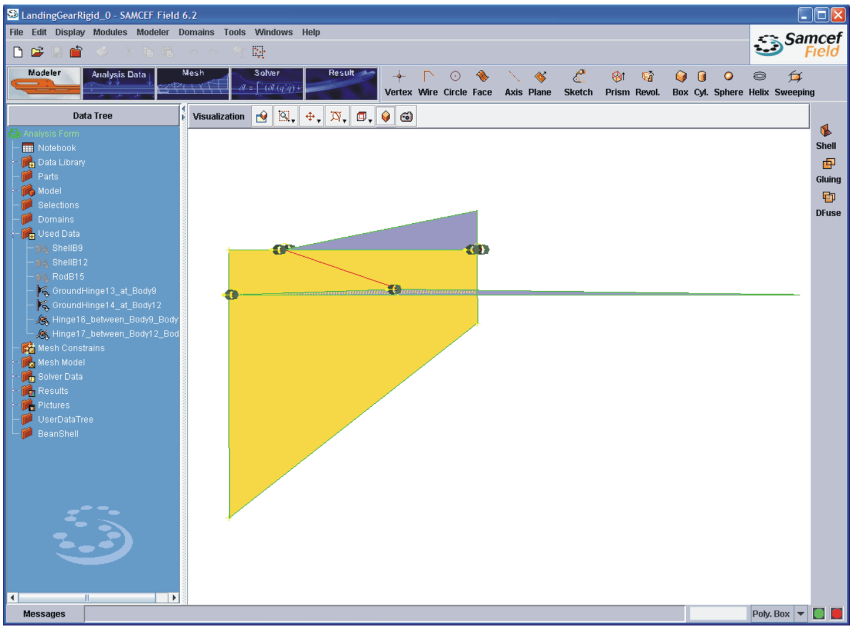

Figure 14: Landing gear mechanism.

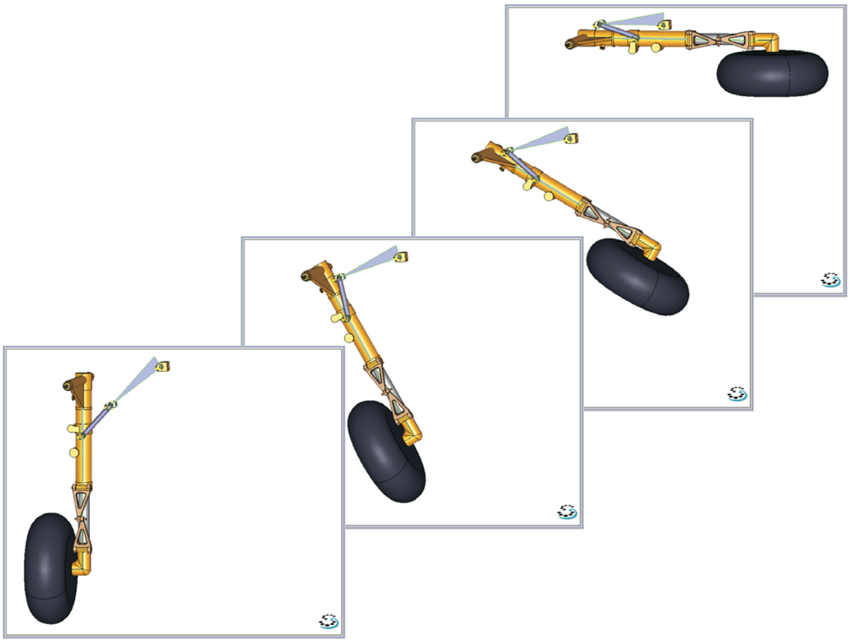

Figure 15: Deployment of a landing gear mechanism passing through four precision positions.

The description of the task shown in Figure 16 is defined over a previous solution shown in Figure 15. A new *allowed space* constraint is defined to locate the actuator of the new mechanism. Also, a rigid lumped mass is defined over a point connected with the ground by means of a hinge. This hinge will be the actuator and its parameters, angles denoted with αs, must be computed by the dimensional synthesis procedure. Since the landing gear mechanism is rigid, the problem consists in finding all the feasible bistable mechanisms to coordinate the motion between the new rotational input and the existing four-bar mechanism. The angle necessary to rotate the leg (L_{23}) $\frac{\pi}{2}$rad is γ_2, then a value for γ_1 is proposed to be a half of γ_2, i.e. $\gamma_1 = \frac{\gamma_2}{2}$. The CAD environment after the task definition is shown in Figure 17.

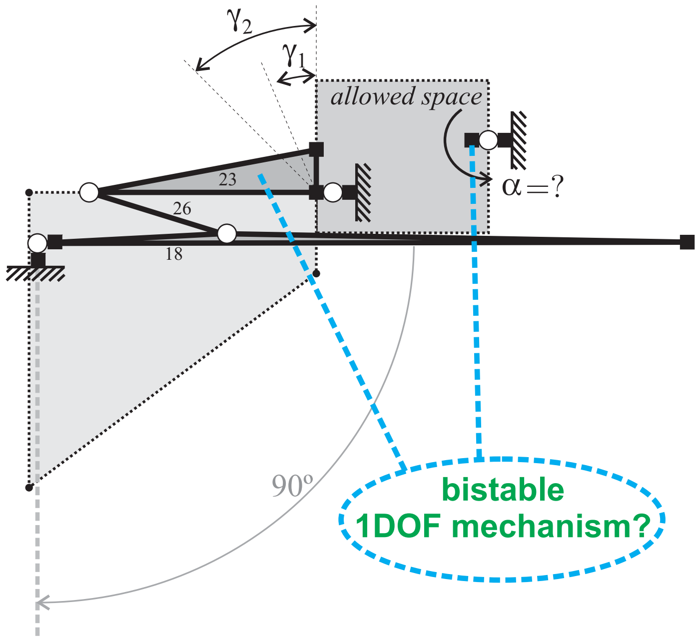

Figure 16: Required bistable task.

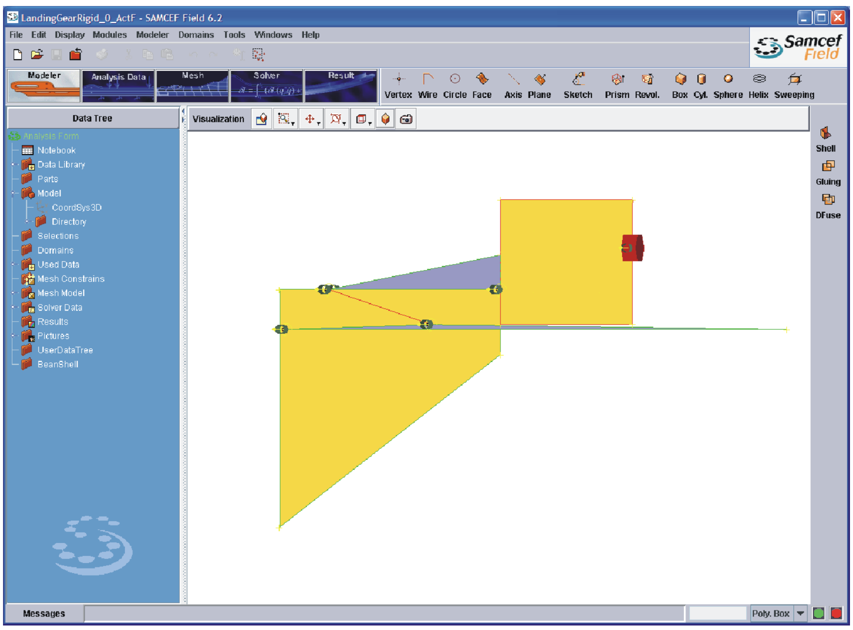

Figure 17: Definition of the bistable task using a CAD environment.

In order to obtain a compliant mechanism, we use two conditions:

Topological condition: At least one member able to store energy (beam, spring or flexural hinge) must be present in the mechanism. Since replacement synthesis is applied for beams, the use of a clamped-revolute beam or a clamped-clamped beam is considered[2].

Dimensional condition: The angle between two members connected by a clamped joint must be the same for the starting and final position.

The atlas of compliant linkages CompliantOneDofR, which contains mechanisms with clamped and flexural joints, is selected as topological design space so that the topological condition can be satisfied. The solver admits defining several structural constraints computed in two different instances:

- After a graph G_{A} is taken from the atlas. For this example test and the considered atlas, flexural hinges are not allowed.
- After the initial graph is found as subgraph of the graph taken from the atlas, i.e. $G_{\mathrm{ini}} \subseteqq G_{\mathrm{A}}$, see the subgraph occurrence shown in Figure. These constraints are given in terms of the minimum/maximum degree allowed to take the links. For this example, bodies of the landing gear, L_{18} (leg) and the intermediate body L_{26}, are constrained to be binary. The ground (link 0) is forced to be ternary to avoid the addition of new pivots.

4.1.1 Type synthesis execution

The execution of the type synthesis solver returns the alternatives shown in Figure 18. New elements (joint J_{34}, the flexible link L_{33} and joint J_{35}) are added in different ways without repetition. The alternatives found are a bit trivial and can easily be found by hand. However, the solver can enumerate solutions with higher order of complexity, with more loops and pivots, where the combination of link types and joint types is not an easy work.

4.1.2 Dimensional synthesis for Alternative 0

Alternative 0 has a pinned-pinned beam which can but does not store energy, it moves as a rigid mechanism. The actuation of the joint J_{32} is computed so that the whole mechanism is moved passing exactly through the three prescribed positions.

4.1.3 Dimensional synthesis for Alternative 1

In Figure 18, the Alternative 1 presents a flexible link L_{33} constrained by a revolute joint J_{34} with body L_{23} and by means of a clamped joint J_{35} with link L_{27}. The problem is solved using the precision-point method as shows Figure 19-a. Then, using the replacement rules explained in Section 3.2.2, the coordinates of joint J_{35} are changed as shows Figure 19-b where the original mechanism is shown in grey color.

The decomposition of the topology into single-open chains results in one triad passing through tree positions. The identification of significant dimensions for links L_{27}, L_{33} and L_{23} results in three complex numbers denoted as $\boldsymbol{Z}_0$, $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$, respectively. These three complex numbers, which are used to represent the triad shown in Figure 19-a, rotate the angles α's, β's and γ's, respectively. Angles γ's are data of the previous synthesis problem, while the other

[2]The term "revolute" or "pinned" are used in an interchangeable form for planar mechanisms. In pinned-pinned beams actuated at pins, compressive loads can produce buckling in initially-straight beams or store energy if the beam is initially curved. The *snap-through* effect of initially-curved pinned-pinned or initially-curved fixed-fixed beams is useful for many other engineering applications.

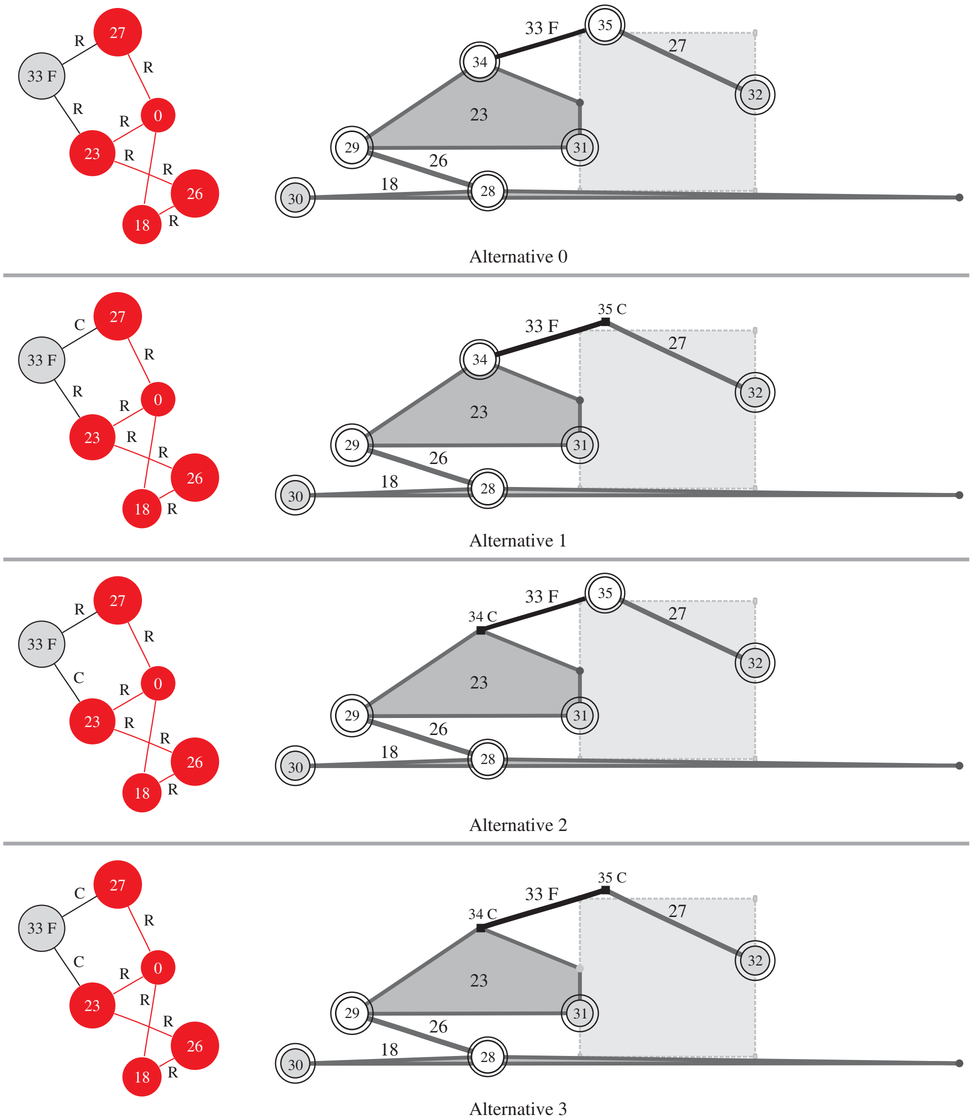

Figure 18: Outputs of the type synthesis solver and their corresponding physical sketches for **Alternatives 0** to **3**. References for joint types in graphs are: R=revolute, C=clamped. In sketches, flexible links have a letter “F”, other links are assumed to be rigid.

angles, α’s and β’s, are identified as “free parameters” by the synthesis solver. The bounds for these parameters must be proposed by the user. We choose the bounds to obtain a bistable behavior.

In Figure 19-b we show the complaint mechanism after rigid replacement. Following the replacement rules, the size of the rigid link $|\boldsymbol{Z}_1|$ is stretched to be $\frac{|\boldsymbol{Z}_1|}{\gamma}$ in the flexible version.

In order to obtain an unstrained final configuration, the relative angle between the clamped bodies must be the same; this angle is denoted as θ_{clamped}. Since the free parameters handle the design space for the rigid model, we must only propose equal values for angles α_2 and β_2 to obtain θ_{rigid} in the starting and final positions; thus, links $\boldsymbol{Z}_0$ and $\boldsymbol{Z}_1$ will develop a rigid-body motion between the initial and final positions. The deformation in the beam can be controlled by proposing the angles α_1 and β_1.

A triad passing through three positions with imposed offset has a nonhomogeneous system

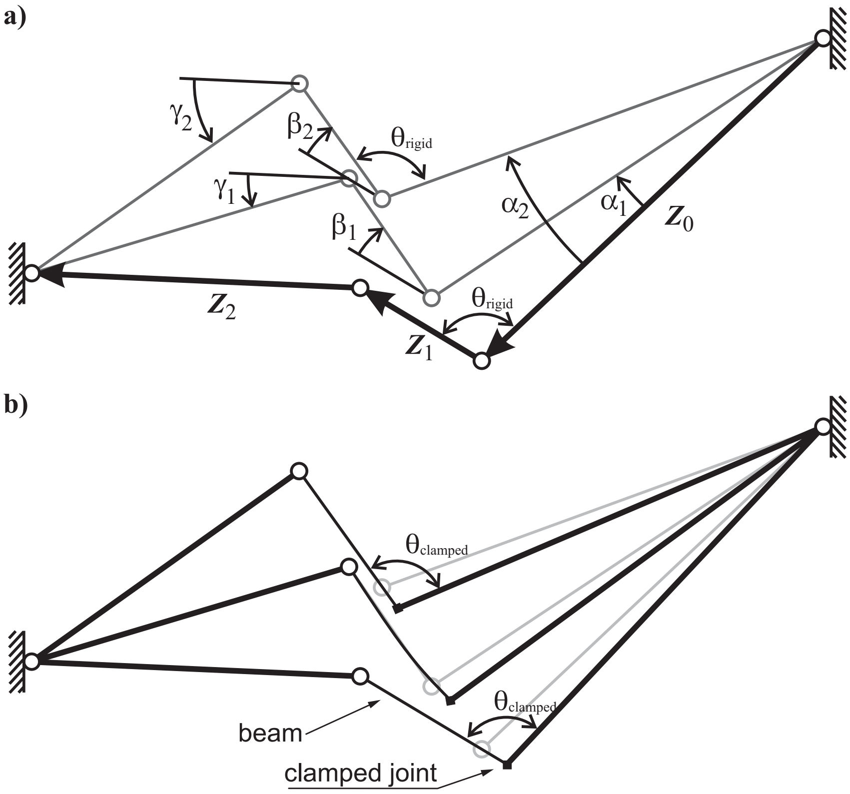

Figure 19: Dimensional synthesis of triad using rigid-body replacement.

(Pucheta, 2008, chap.4)

$$\begin{bmatrix} (e^{i\alpha_1}-1) & (e^{i\beta_1}-1) & (e^{i\gamma_1}-1) \\ (e^{i\alpha_2}-1) & (e^{i\beta_2}-1) & (e^{i\gamma_2}-1) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{Z}_0 \\ \boldsymbol{Z}_1 \\ \boldsymbol{Z}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\delta}^1 \\ \boldsymbol{\delta}^2 \\ \boldsymbol{r} \end{bmatrix}, \tag{1}$$

or briefly as

$$\mathbb{C}^{\text{off}}\mathbb{Z} = \mathbb{D}^{\text{off}}, \tag{2}$$

where $\mathbb{Z}$ are the unknowns, $\boldsymbol{\delta}^1$ and $\boldsymbol{\delta}^2$ are functions of the end-point displacements of the triad, here they both are null; $\boldsymbol{r}$ is the complex number which closes the triad, i.e. a vector going from one pivot to the other one. This system of equations has solution iff $\det(\mathbb{C}^{\text{off}}) \neq 0$.

The solution for Alternative 1 shown in Figure 20 satisfies both energy and kinematic requirements. The rotation measured on the leg of the landing gear shows an accurate value of $90°$. The difference with 90° in the leg rotation is of 6.E-4°; see Figure 21-left.

After rigid-body replacement, the *stored energy vs. time* characteristic has the desired bistable behavior so that no further optimization is needed, see Figure 21-right. The energy at time 0s is 0.0064138J, the maximum reached is 7.82605J at time 1.013s and is 0.0142488J at the final time.

4.1.4 Dimensional synthesis for Alternative 2

In Figure 18, the Alternative 2 presents a flexible link L_{33} constrained by a revolute joint J_{35} with body L_{27} and by means of a clamped joint J_{34} with link L_{23}.

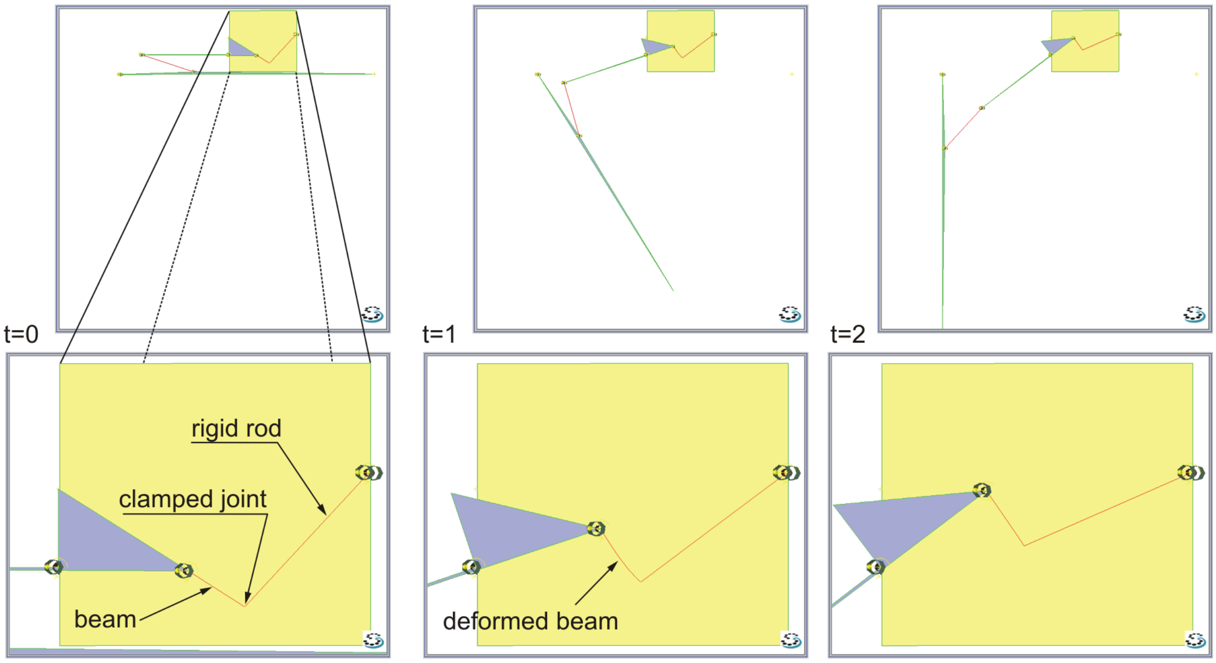

Figure 20: Dimensional synthesis of Alternative 1 using rigid-body replacement.

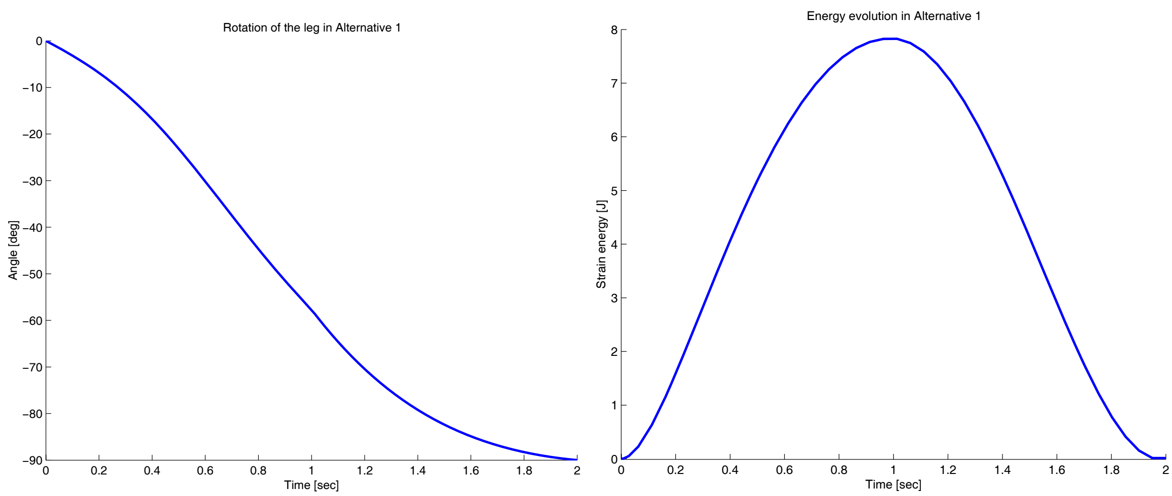

Figure 21: Rotation of the leg (left) and stored strain energy vs. time (right) for Alternative 1.

Figure 22 shows the diagram for the identified triad. In order to obtain an unstrained final configuration of the flexible element, complex numbers $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ in the rigid design (a) must develop a rigid-body motion between the initial and final position, preserving the angle denoted as θ_{rigid}. This can be obtained by proposing a value for β_2 equal to γ_2. The deformation in the beam can be controlled by proposing the angles α_1 and β_1.

After rigid-body replacement, the resulting model is simulated as shown in Figure 23. The difference with 90° in the leg rotation is 5E-4°, see Figure 24-left. The *stored energy vs. time* characteristic, shown in Figure 24-right, has the desired bistable behavior so that no further optimization is needed. The energy at time 0s is 1.0E-028J, the maximum reached is 23.5913J and is 0.00347787J at 2s.

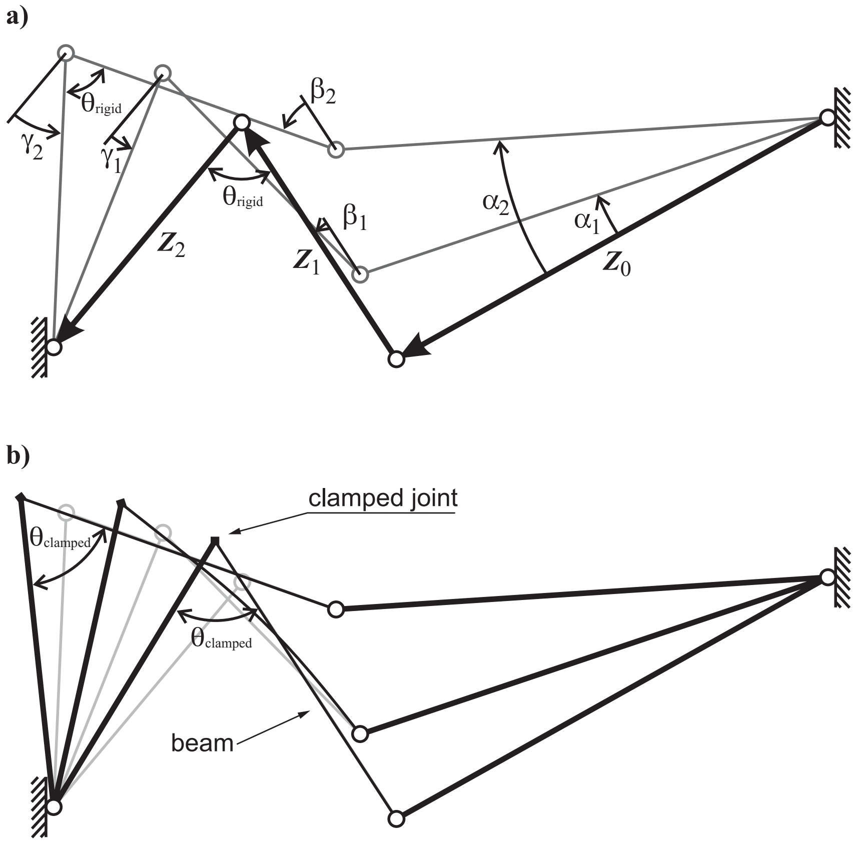

Figure 22: Dimensional synthesis of a triad using rigid-body replacement.

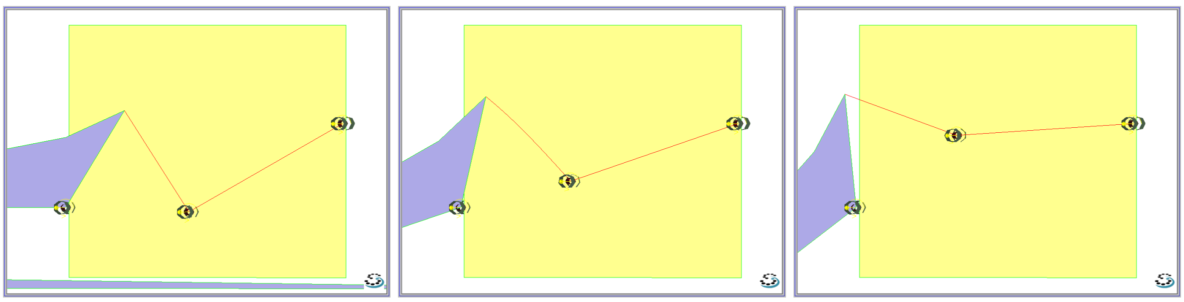

Figure 23: Simulation for Alternative 2 at times 0, 1 and 2 sec.

4.1.5 Dimensional synthesis for Alternative 3

The flexible member in Alternative 3 has two clamped ends. If we call θ_{rigid} the angle between the flexible link and the driver link, and θ'_{rigid} the angle between the flexible link the driven link of a four-bar mechanism like those shown in Figures 19 and 22, it is easy to realize that we cannot set up conditions to preserve both angles at the final configuration, except for the trivial case $\gamma_2 = \alpha_2 = 0$, in which the allowed motion at driver and driven links can only be oscillatory.

Non-trivial motion conditions to obtain a bistable behavior of a clamped-clamped beam can however be defined on (i) a triad if one of the ends is not a pivot, and (ii) on a single open-chain with four or more complex numbers. Its application will be investigated in future research.

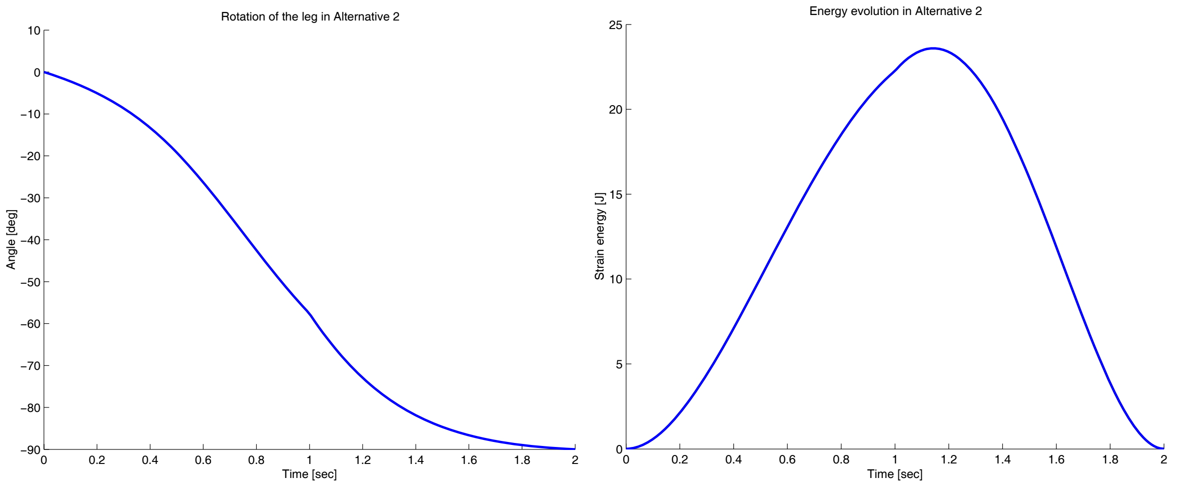

Figure 24: Rotation of the leg (left) and stored strain energy vs. time (right) for Alternative 2.

Nevertheless, we show a synthesis result in Figure 25 by setting an unstrained position in one of the clamped ends of the beam, which is interesting from the kinematic point of view.

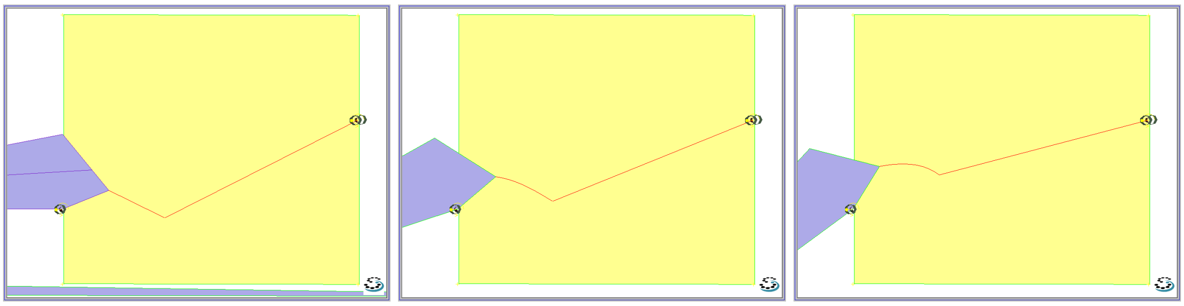

Figure 25: Simulation for Alternative 3 at times 0, 1 and 2 sec.

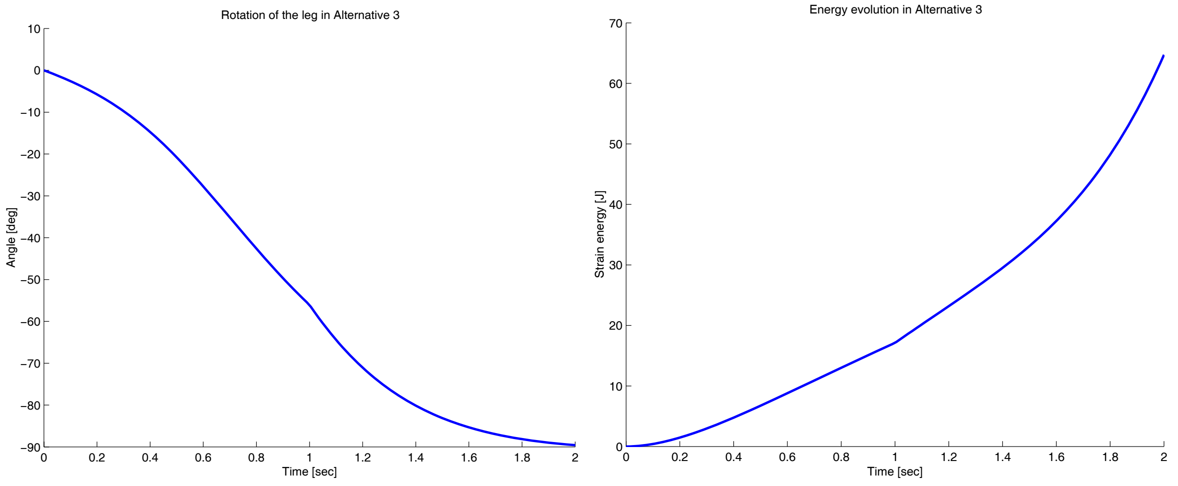

Figure 26: Rotation of the leg (left) and stored strain energy vs. time (right) for Alternative 3.

The kinematic error in the leg rotation is of 0.4250° so that the replacement synthesis helps again to find a good approximation of the required motion, see Figure 26-left. The stored energy, shown in Figure 26-right, is increased monotonically as the leg rotates, so it does not match the final energy requirement.

4.1.6 Bistable mechanisms derived from change-point rigid mechanisms

A change-point mechanism can pass through a dead-center position, i.e. the driver and coupler links are collinear; see Figure 27-**a** in the second position. This configuration reached by the mechanism is also called a change-point position. Even if the driver link returns to the initial position or it continues its rotation, the driven link has an oscillatory motion. The driven link returns to its initial position when the positions of the driver and coupler links configure one of their two geometric inversions. A change-point mechanism converted by means of replacement synthesis can be designed to provide a bistable behavior (Howell, 2001, chap.11), see Figures 27-**b** and **c**.

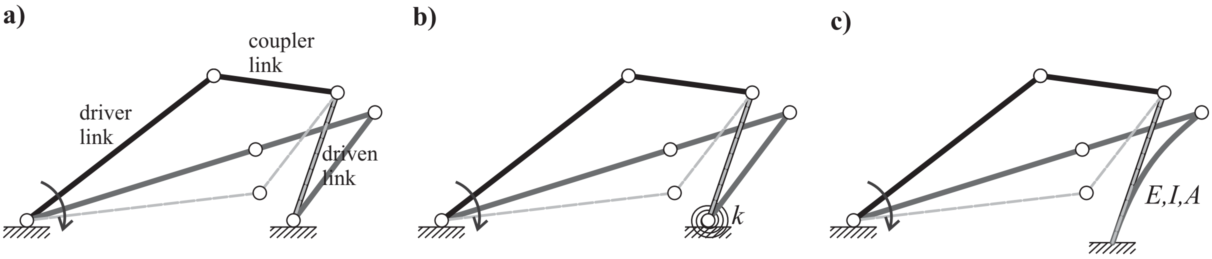

Figure 27: Change-point mechanisms: a) rigid, b) bistable pseudo-rigid, c) partially compliant obtained by replacement. The initial position is shown in black color, the second (change-point) position in dark grey color, and the final in dashed line with grey color.

As we can see in Figure 27-**c** this kind of mechanisms requires a clamped joint connected to the ground. This situation was not obtained in the alternatives shown in Figure 18 because a revolute joint J_{32} which attaches the rigid body L_{27} to ground was specified in the initial description, see Figure 16.

We change the initial specifications of the landing gear problem in the following way: the mechanism is actuated on joint J_{31} of the folding mechanism and a clamped point is only defined where the rotational input was firstly located. This allows, for example, a clamped connection with ground, see Figure 28. The first feasible topological solution has a grounded beam which corresponds to the complex number $\boldsymbol{Z}_0$ in Figure 29. Then, a three-position triad is designed for obtaining a change-point mechanism setting the angles of the complex number $\boldsymbol{Z}_0$ in the following way: $\alpha_1 = \frac{\pi}{6}\text{rad}$ and $\alpha_2 = 0\text{rad}$. The angle θ_{rigid} to be preserved is measured between an imaginary line joining the pivots and the direction of $\boldsymbol{Z}_0$. The angle α_1 limits the motion of the body represented by $\boldsymbol{Z}_0$ which is replaced by a clamped-revolute beam. The angles γ_1 and γ_2 are data and angles β_1 and β_2 are the free parameters.

The synthesis results in a good agreement in both kinematic and bistable behaviors, these results can be observed in Figure 32-left. The simulation of the bistable mechanism is shown in Figure 30.

For the second alternative a replacement of a clamped-clamped beam was made. The synthesis showed a good kinematic behavior (Figure 31) but was unable of fulfilling the energy requirement, see Figure 32-right.

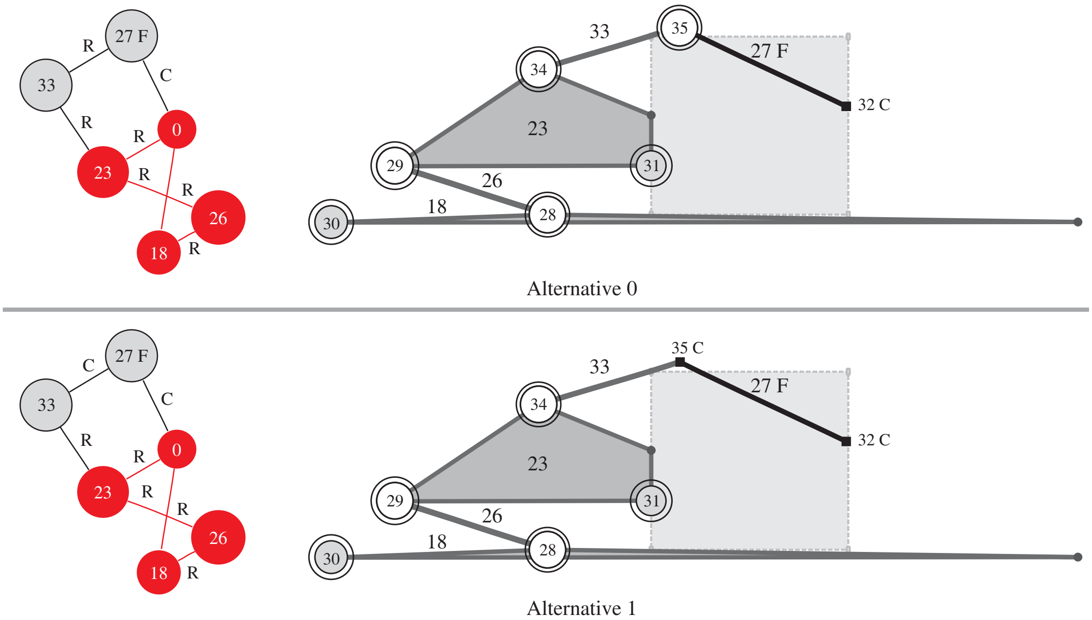

Figure 28: Topological solutions for new specifications.

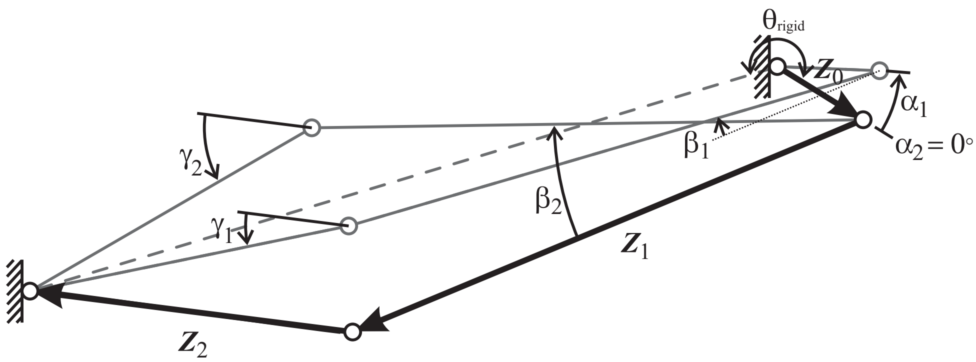

Figure 29: Dimensional synthesis of triad using rigid-body replacement.

4.2 More results

Several industrial applications were solved in the frame of the SYNCOMECS project: a flex-

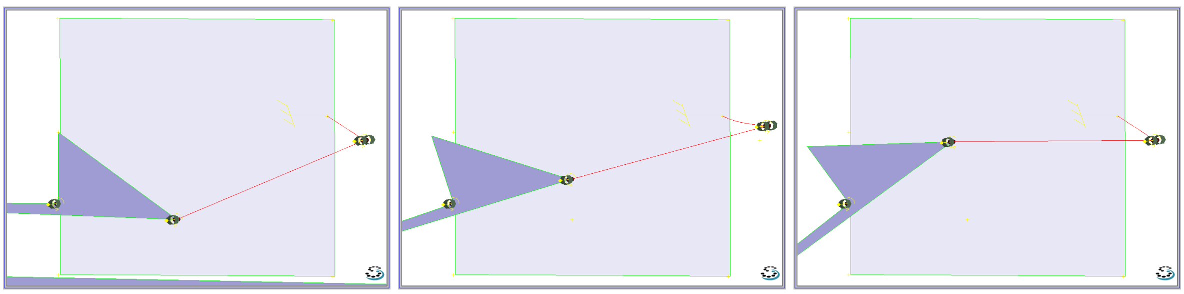

Figure 30: Simulation for a bistable mechanism derived from a change-point mechanism with a revolute-clamped beam.

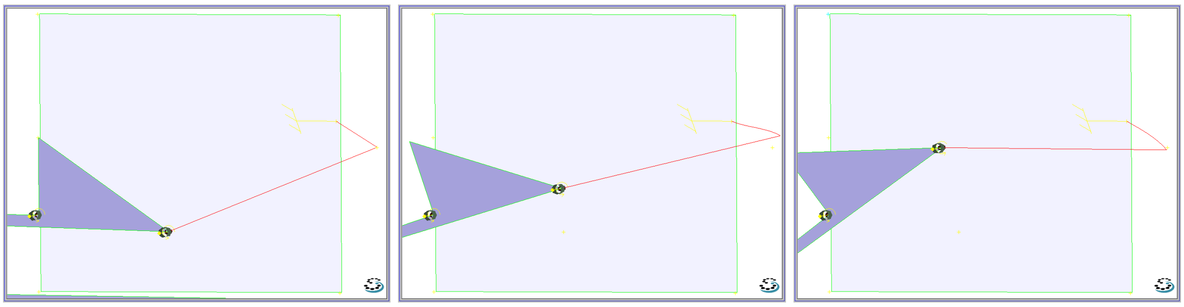

Figure 31: Simulation for a bistable mechanism derived from a change-point mechanism with a clamped-clamped beam.

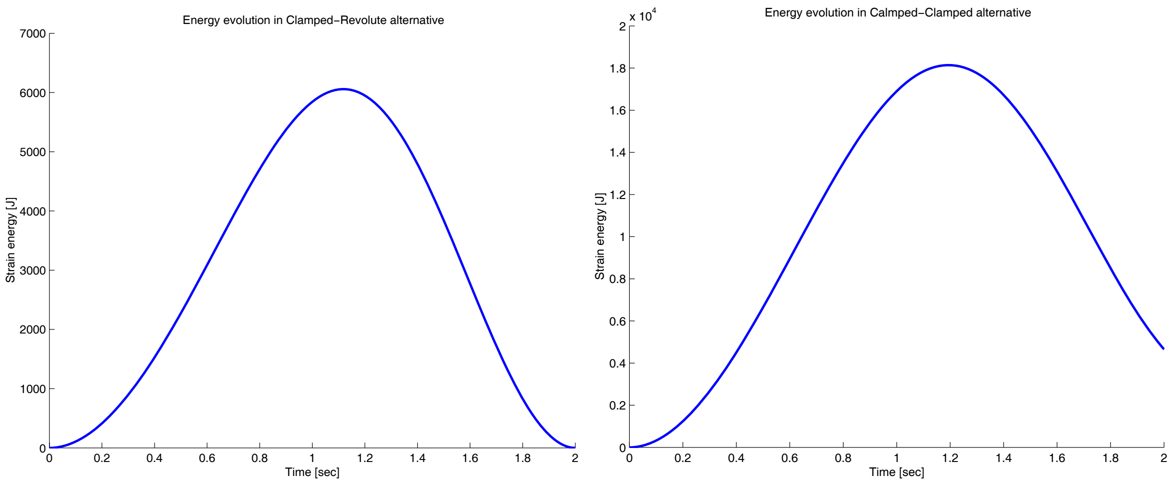

Figure 32: Stored strain for a change-point mechanism with a revolute-clamped beam (left) and a clamped-clamped (right) beam.

ible path generator, a two-degree-of-freedom switch, a single and multiple deflector for nozzles of a turbine engine and the herein presented bistable actuator for landing gear retraction. The best concept found for each problem was solved by optimization using commercial software. Details about the optimization results for bistable landing gears were presented in (Cugnon et al., 2007).

From the numerical experiments developed, we can identify some advantages and disadvantages with respect to other approaches inspired on structural optimization.

Advantages:

- The rigid-body replacement method combined with the exploration of atlases of compliant mechanisms enables the designer to find quick-design concepts for complex tasks. Some advantages like the low number of variables and the proximity to optimal solutions of the initial guess -mechanism topology with initial dimensions- provided by precision-points synthesis may help its convergence.

- For bistable requirements, motion and energy can be designed conjointly using revolute-clamped beams and the precision-point method traditionally used for rigid synthesis.

- It is easy to set up bi-stability problems as a three-position kinematic task. All mechanisms for opening/closing operations are examples where the task can be adapted for three positions.

- Without applying replacement rules, the design technique is useful for the design of rigid-link mechanisms with springs.

Disadvantages:

- Several difficulties found in the design of rigid mechanisms using precision-position synthesis such as poor behavior between passing points and circuit and branch defects are translated to the replacement synthesis.

- In some situations the replacement produces a mechanism which changes the kinematic behavior drastically compared to its rigid version. Therefore, several approximations made between the rigid and the compliant mechanisms obtained by replacement synthesis lead to big kinematic errors and their causes must be investigated.

5 CONCLUSIONS

A new study of the kinematic design of compliant mechanisms using the precision-position method and replacement synthesis was presented. The study was limited to replacement of binary initially-straight beams. The use of a unique characteristic ratio ($\gamma = 0.8571$) was useful –from the kinematic point of view– for designing the three replacement cases so that the leg of the landing gear was guided with little deviation from the target value of 90°. The problem was reformulated in order to design change-point mechanisms which also fulfilled the kinematic task. In the four-bar solutions shown, the actuated link was rigid and the driven link was rigid or flexible in the case of the change-point mechanism.

The type synthesis part of the methodology may be extended for dealing with compliant mechanisms with arbitrary topologies different from those arising from rigid kinematic chains. Using replacement synthesis, the obtained mechanisms with a clamped-revolute flexible beam have developed the required bistable behavior with very little error. The loop-closure equations of four-bar mechanisms with a clamped-clamped flexible beam offer insufficient conditions to define the bistable behavior.

The replacement rules and the satisfaction of bistable requirements by means of geometric considerations using precision-position synthesis are important contributions towards the automated conceptual design of compliant mechanisms.

6 FUTURE RESEARCH

In the bistable test problems shown in Figures 19 and 22, the bounds of the variables were set "by hand" to obtain the equalities and relationships between the angles. However, these relationships can be easily programmed to work automatically by implementing the proper rules.

Further research would incorporate synthesis rules for designing non-homogeneous links, initially curved segments, automated segment identification and force-deflection analysis. Specifically related to the latter topic, we are able to do a static analysis of forces and moments after rigid dimensional synthesis in order to choose the right pseudo-rigid-body models with their proper parameters: characteristic ratios, spring constants, etc. as function of load cases.

The study can be extended to the design of multi-stable mechanisms and micro-mechanisms.

ACKNOWLEGMENTS

This work has received financial support from *Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET)*, *Agencia Nacional de Promoción Científica y Tecnológica (ANPCyT)*, *Universidad Nacional del Litoral (UNL)* from Argentina, and from the *European Community* through grant *SYNCOMECS (SYNthesis of COmpliant MEChanical Systems)* project UE FP6-2003-AERO-1-516183 http://www.syncomecs.org.

REFERENCES

Anderson J.K. *Piezoresistive sensing of bistable micro mechanism state*. Master's thesis, Brigham Young University, Provo, Utah, USA, 2005.

Bendsøe M.P. and Kikuchi N. Generating optimal topologies in structural design using a homogenization method. *Computer Methods in Applied Mechanics and Engineering*, 71(2):197–224, 1988.

Berglund M., Magleby S., and Howell L. Design rules for selecting and designing compliant mechanisms for rigid-body replacement synthesis. In *Proceedings of the 2000 ASME Design Engineering Technical Conferences, DETC2000/DAC-14225*. Baltimore, Maryland, 2000.

Cugnon F., Cardona A., Selvi A., and Paleczny C. Synthesis and optimization of flexible mechanisms. In C. Bottasso, P. Masarati, and L. Trainelli, editors, *Multibody Dynamics 2007*, ECCOMAS Thematic Conference on Multibody Dynamics, pages 156–157. Politecnico di Milano, Milan, Italy, 2007.

Howell L. *Compliant Mechanisms*. John Wiley & Sons, New York, 2001.

Jensen B. and Howell L. Identification of compliant pseudo-rigid-body four-link mechanism configurations resulting in bistable behavior. *ASME Journal of Mechanical Design*, 125(4):701–708, 2003.

Jensen B. and Howell L. Bistable configurations of compliant mechanisms modeled using four links and translational joints. *ASME Journal of Mechanical Design*, 126(4):657–666, 2004.

Jensen B.D. *Identification of macro- and micro- compliant mechanism configurations resulting in bistable behavior*. Master's thesis, Brigham Young University, Provo, Utah, USA, 1998.

Lu K.J. and Kota S. Topology and dimensional synthesis of compliant mechanisms using discrete optimization. *Journal of Mechanical Design*, 128(5):1080–1091, 2006.

Mruthyunjaya T.S. Kinematic structure of mechanisms revisited. *Mechanism and Mechanism Theory*, 38(4):279–320, 2003.

Murphy M., Midha A., and Howell L. The topological synthesis of compliant mechanisms. *Mechanism and Machine Theory*, 31(2):185–199, 1996.

Pedersen C.B.W., Buhl T., and Sigmund O. Topology synthesis of large-displacement compliant mechanisms. *International Journal for Numerical Methods in Engineering*, 50(12):2683–2750., 2001.

Pellegrino S. and Santer M.J. Topology optimization of adaptive compliant aircraft wing leading edge. In *48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*. Honolulu, Hawaii, 2007.

Pucheta M. *Computational methods for design and synthesis of planar mechanisms*. Ph.D. thesis, Universidad Nacional del Litoral, Santa Fe, 2008.

Pucheta M. and Cardona A. Type synthesis and initial sizing of planar linkages using graph theory and classic genetic algorithms starting from parts prescribed by user. In J. Goicolea, J. Cuadrado, and J.G. Orden, editors, *Multibody Dynamics 2005*, ECCOMAS Thematic Conference. Madrid, Spain, 2005.

Pucheta M. and Cardona A. An automated method for type synthesis of planar linkages based on a constrained subgraph isomorphism detection. *Multibody System Dynamics*, 18(2):233–258, 2007a.

Pucheta M. and Cardona A. Kinematics synthesis of compliant mechanisms using rigid-body replacement. In C.L. Bottasso, P. Masarati, and L. Trainelli, editors, *Multibody Dynamics 2007*, ECCOMAS Thematic Conference on Multibody Dynamics, pages 156–157. Politecnico di Milano, Milan, Italy, 2007b.

Sandor G. and Erdman A.G. *Advanced Mechanism Design: Analysis and Synthesis*, volume 2. Prentice-Hall, New Jersey, 1984.

Sauter M., Kress G., Giger M., and Ermanni P. Complex-shaped beam element and graph-based optimization of compliant mechanisms. *Structural and Multidisciplinary Optimization*, 2007. (Available on-line DOI 10.1007/s00158-007-0182-7).

Saxena A. and Ananthasuresh G. Topology synthesis of compliant mechanisms for nonlinear force-deflection and curved path specifications. *ASME Journal of Mechanical Design*, 123(1):33–42, 2001.

Saxena A. and Ananthasuresh G. A computational approach to the number of synthesis of linkages. *ASME Journal of Mechanical Design*, 125(1):110–118, 2003.

Sigmund O. Design of multiphysics actuators using topology optimization - part I: One-material structures. *Computer Methods in Applied Mechanics and Engineering*, 190(49-50):6577–6604, 2001a.

Sigmund O. Design of multiphysics actuators using topology optimization - part II: Two-material structures. *Computer Methods in Applied Mechanics and Engineering*, 190(49-50):6605–6627, 2001b.

Tsai L. *Mechanism Design: Enumeration of Kinematic Structures According to Function*. CRC Press, Boca Raton, 2001.

Wang M.Y., Chen S.K., Wang X.M., and Mei Y.L. Design of multimaterial compliant mechanisms using level-set methods. *ASME Journal of Mechanical Design*, 127(5):941–956, 2005.

Zhou H. and Ting K. Topological synthesis of compliant mechanisms using spanning tree theory. *ASME Journal of Mechanical Design*, 127(4):753–759, 2005.