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OPERACIÓN ÓPTIMA DE CENTRALES HIDROELÉCTRICAS DE BOMBEO CONSIDERANDO UN MERCADO OLIGOPÓLICO.

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Palabras Claves: Mercados Eléctricos, Equilibrio de Nash-Cournot, Inecuaciones Variacionales, Función de Nikaido-Isoda.

Resumen: En este trabajo se presenta un método que resuelve el problema de coordinación hidrotérmica considerando una estructura del mercado oligopólica.
En los últimos quince años, la industria de la energía eléctrica en la Argentina ha cambiado de un escenario completamente regulado a uno competitivo en el cuál existe un pequeño número de empresas de generación que satisfacen la mayor parte de la demanda del sistema. Para emular este tipo de estructura del mercado se desarrolló un modelo de Mercado oligopólico. Para resolver el problema de coordinación hidrotérmicas bajo estos supuestos, no solo se consideran las características operativas del sistema, sino también los aspectos económicos del mercado.
En el presente trabajo se estudia como el precio y la configuración de las distintas unidades de generación del sistema es afectada por el uso no solo de plantas hidroeléctricas, sino también por unidades de bombeo. El comportamiento en conjunto del sistema se modela a través de una ecuación variacional, la cuál es resuelta por un método que se basa en un algoritmo de relajación y utiliza la ecuación de Nikaido-Isoda para encontrar el equilibrio de Nash-Cournot.
El método numérico desarrollado fue probado en una red eléctrica reducida que emula en diferentes aspectos al sistema argentino. Esta red se desarrolló sobre una de las redes standard de pruebas presentadas por la IEEE considerando la inclusión de plantas de generación hidroeléctrica que posean unidades de bombeo.

Abstract: This paper introduces a numerical method to solve the hydrothermal scheduling problem in oligopolistic markets.
The electric power industry in Argentina has shifted from a scenario in which operation schedule is fully regulated to a new competitive deregulated scenario in which there is a small number of generation companies which satisfy a large amount of the total power demand of the system. To emulate this kind of market structure, an oligopolistic market model has been developed. To solve the hydrothermal coordination problem under these assumptions, both operative restrictions, and market economical aspects must be considered.
In the present work we study how the price and configuration of the different units of the system are affected by the use of hydroelectric plants and also by pumped storage units through a mathematical market model represented by a variational inequality. The solution method is based on a relaxation algorithm, and the Nikaido-Isoda function is used for the calculation of Nash-Cournot equilibrium.
The numerical method developed was tested in a reduced electricity network which emulates the different aspects of the Argentinean system. This network was developed on the base of the IEEE 30-bus system considering the inclusion of hydroelectric plants and pumped storage units.

1. INTRODUCTION

The electricity generation market used to be a government driven monopoly where a national company owned every segment of the electric system. In the last two decades there has been a worldwide trend which leads to the deregulation of these markets, introducing different kinds of competition in some segments of this industry. To conclude, competitive market concepts are used nowadays in a traditionally monopolist industry (Rubiales et al., 2007).

Argentina is not outside this worldwide trend and begins its own deregulation process in the early 1990s, changing the vertically integrated structure of its electricity market to a segmented one not only vertically but also horizontally (Moitre, 2002). In this country, the electricity market includes financial and commercial agreements signed by agents who participate in the market by means of different kinds of contracts. These contracts take place in the spot market, where electricity is valuated and traded.

In order to model monopolistic electric systems, only technical aspects of the system must be considered. On the other hand, when oligopolistics electricity markets are modeled, economical aspects of pool agents must also be taken into account. These ones strongly influence the electricity price because power generation is distributed between the market actors (Moitre et al., 2005a). The energy spot price is an essential instrument which must be known in this kind of models. It represents the cost of the next Megawatt (MW) of charge to be provided, considering restrictions on several aspects, such as, transport and maintenance of security and quality of service of the system.

In the present work, an algorithm to solve the optimal scheduling of pumped storage units in an oligopolistic pool is presented. To represent this kind of markets, a Nash-Cournot model is defined whose equilibrium points are found through the Nikaido-Isoda function and the relaxation algorithm that transforms the equilibrium problem in an optimization one. Under some hypothesis, the hydrothermal coordination problem is solved using dynamic programming in combination with the relaxation algorithm previously mentioned.

2. FORMULATION OF THE MODEL

The proposed model considers the optimal programming operation at the pumped storage plant maximizing its profit.

Pumped storage units are a special kind of hydroelectric units which allow a more rational use of the hydraulic resources of a country. This kind of plant, has two different reservoirs at different levels connected by a penstock and a reversible turbine. When the system demand reaches its maximum level (peak hours), water flows from the upper to the lower reservoir through the hydropower plant generating electricity, as a conventional hydropower plant. Conversely, when the demand is lower (off-peak hours), the second reservoir refills the upper one by pumping water back; by means of this procedure, the plant has more water to generate electricity during periods of peak consumption. This operation allows the plant to flat the load of the system, increasing the load in off-peak hours and supplying power in peak hours.

The main problem when scheduling hydroelectric plants is to obtain the best strategy to manage the available water to generate electricity. Hence, the capacity storage units have to move energy blocks on time, spending water now should introduce more thermal generation in the future. The aim of this type of optimization is to find the point in which the future profit plus the actual profit become maximum.

The hydroelectric plant operator must compare the profit of spending a larger amount of water to generate electricity with the future profit of storing it. The Immediate Costs (IC) function measures the thermal generation costs during a specific time. Figure 1 shows the fact that these costs increase when more water is stored and decrease when more water is spent at the present stage. On the other hand, the Future Costs (FC) function illustrates the expected costs of generating and rationing from the next stage to the end of the planning horizon. FC function behaves in an inverse manner to the IC function, i.e., future costs decrease when the volume of water stored increases, because more hydroelectric power will be available in the future.

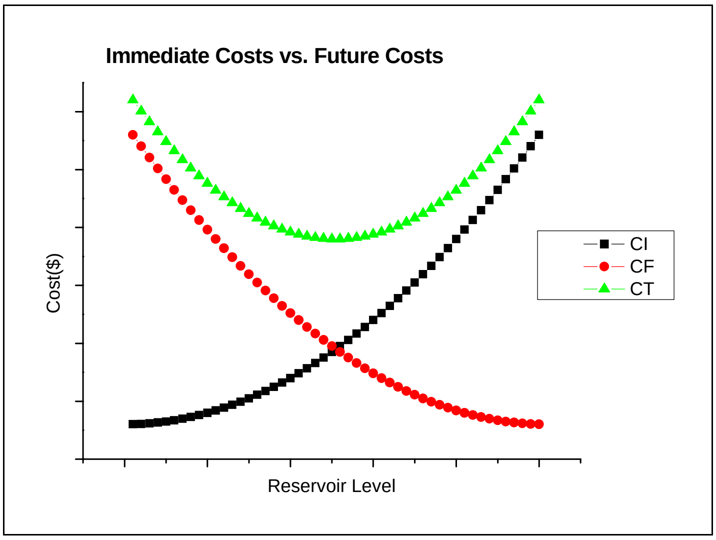

Figure 1: Immediate Costs vs. Future Costs.

To obtain the optimal decision, it is necessary to minimize the operation total costs (IC plus FC). In the optimum point, the derivative of the total cost function (FC + IC) is zero; therefore, the optimum level of water stored in the reservoir is when the absolute value of the derivatives of both functions (IC and FC) are equal. This value is called value of water, which represents the cost of unavailability of water in the future (Vega and Villena, 2006).

Hydroelectric plants with reservoirs have the ability to administrate the usage of water between different periods. Hence, thermal generation during the present period, and the possibility to store water so as to spend it in the future and vice versa must be considered. Therefore, the optimization problem is coupled through different periods. Figure 2 shows the static coupling between thermal agents in columns while the dynamic coupling of the hydroelectric agent between different periods is shown in the first row.

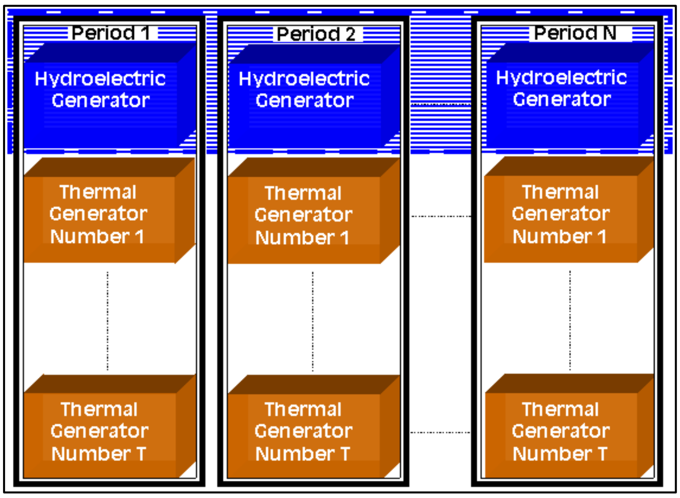

Figure 2: Static coupling vs. dynamic coupling.

The model introduced in this paper is used with a short horizon of analysis, though it is impossible to conclude about strategic use of water in a large horizon; what does not allow this model to determine the optimal volume of water to be spent in the planning horizon. In literature, there are two methods of solving this problem (Wood and Wollenberg, 1996). One of these methods considers that the total amount of water in the reservoir is available in the short term, but a value to the amount of water that is not spent is assigned to motivate hydroelectric plants to keep water beyond the horizon of analysis. The other method considers that a known fixed volume of water is available to be used in the planning horizon (obviously less than the total volume of water of the reservoir) as a result of long-term programming. This plan has been previously calculated by an algorithm whose planning horizon goes beyond the scope of this investigation. In this work, the second approach is adopted. The problem of how to consider the value of water in a long term is not taken into account because the amount of water available for each period is fixed, as a result of a previous optimization method which considers the strategic value of water. It is suppose that the hydroelectric plant must spend the total amount of water available in the planning horizon. The water stored available to be used during the planning horizon is represented by the total power that the plant can generate with it. On the other hand, the model does not present temporary dependency in the thermal units programming because of not considering on-off restrictions in the thermal unit operation.

To conclude, the problem was mathematically defined as follows:

$$Maximize \sum_{t\in T} Ben(P_t^H), \qquad (1)$$

where

T	Time periods.
P_t^H	Power generated [MW] hydroelectrically in period t.
Ben	Profit [$/h] of the pumped storage plant producing P_t^H .

The profit function is the sum of the incomes gained by selling the produced energy in the spot market and the outcomes spent by buying electricty to pump water for each period. The market behavior is modeled by a Cournot oligopoly and is solved finding a Nash equilibrium where thermal agents compete (static scenarios). An equilibrium problem should be solved for each period of programming. That is:

$$Ben(P_t^H) = P_t^H \, p_t^H (P_t^H), \qquad (2)$$

where $p_t^H(P_t^H)$ is the spot price defined by the Nash equilibrium given by thermal generators considering a hydroelectric generation of P_t^H

We have to consider the following restrictions

$$\begin{gathered} V_{t-1} - q\left(P_t^H\right) = V_t; \forall t \\ V^{MIN} \le V_t \le V^{MAX}; \forall t \\ P^{HMIN} \le P_t^H \le P^{HMAX}; \forall t \\ \sum_{t\in T} P_t^H = P^{HGEN}; \forall t , \end{gathered} \qquad (3)$$

where,

V_t	Water volume stored in reservoir in time t.
V^{MIN}	Reservoir minimum content limit.
V^{MAX}	Reservoir maximum content limit.
P^{HMIN}	Hydro unit minimum limit.
P^{HMAX}	Hydro unit maximum limit.
P^{HGEN}	Total power to be generated hydroelectrically during planning horizon.

For the sake of simplicity, every volume value was translated into MW. The electricity generated is higly affected by the dum's net head and its variations impact in the electricity generated by the turbine. Because of the short term horizon of the optimization problem, the net head does not change considerably, thus, it is not taken into account in the power generated calculus.

As it was previously mentioned, competence between the market agents is based on an oligopolistic cournot model, and the result of this model should be interpreted as a higher limit of price obtained in this market. Competence between thermal generators for each period is modeled as an n-players game.

As it was stated in (Contreras et al., 2004), an n-players game is a formal representation or a mathematical model of a situation in which a number of players (electricity companies in our case) interact in a set of strategic interdependence.

This means that the player own's action and that of the other participants in the game affects its profit.. An n-player game is defined as a three-tuple $\left\{N,\left(X_i\right),\left(\phi_i\right),i\in N\right\}$, where $N=\left\{1,2,...,n\right\}$ is the set of players, X_i, is the strategy space of player i; and ϕ_i, $i\in N$ is the profit function of player i that assigns a real value to each element of the Cartesian product of the strategy spaces $X_1\times X_2\times ...\times X_N$.

A vector x_i represents the individual action that a player i may take. Whenever all the players act together, they will take a collective action determined by a vector $\mathrm{x}=\left(x_1,...,x_n\right)$. Where X_i is an action set of player i, $\phi_i : X_i \rightarrow \mathbb{R}$ their payoff function, and X the collective action set.

Being $\mathrm{x}=\left(x_1,...,x_n\right)$ and $\mathrm{y}=\left(y_1,...,y_n\right)$ elements of the collective action set, an element $\left(y_i\middle|x\right)\equiv\left(x_1,...,x_{i-1},y_i,x_{i+1},...,x_n\right)$ of such collective action set may be considered as a set of actions where the ith player plays y_i while the other agents are play x_j, $j=1,2,...,i-1,i+1,...,n$.

A point $x^*=(x_1^*,...,x_n^*)$ is called the Nash equilibrium point if, for each i,

$$\phi_i\left(x^*\right)=\max_{(x_i|x^*)\in X}\phi_i\left(x_i\middle|x\right), \tag{4}$$

Notice that x^* solves the game $\left\{N,\left(X_i\right),\left(\phi_i\right),i\in N\right\}$ as follows: at x^* no player can improve their individual payoff by a unilateral (i.e., their own) action. To compute the Nash equilibrium the Nikaido–Isoda function is introduced; transforming an equilibrium problem into an optimization one. Let ϕ_i be the payoff function of player i, then the Nikaido–Isoda function is defined as

$$\Psi(x,y)=\sum_{i=1}^{n}\left[\phi_i\left(y_i\middle|x\right)-\phi_i\left(x\right)\right], \tag{5}$$

Each summand of the Nikaido–Isoda function represents the improvement in profit that a player will receive when the action is changed from x_i to y_i, while all the other players go on playing according to x. Thus, one player changes their action while the others do not. Hence, the function represents the sum of these improvements in the profit, being that the maximum value of this function always non-negative for a given x. Also, the function is non-positive for all feasible y when x^* is a Nash equilibrium, since no player can improve their payoff at equilibrium. Consequently, each summand can be at most zero at the Nash equilibrium.

In conclusion, when the Nikaido–Isoda function satisfies certain concavity conditions and cannot be made (significantly) positive for a given y, the Nash equilibrium point is (approximately) reached (Contreras et al., 2004). This is used to construct a termination condition for the relaxation algorithm, such that when a given ε is chosen, the Nash equilibrium is obtained when $\max_{y \in \mathbb{R}^m} \Psi\left(x^s, y\right) < \varepsilon$ where s is an iterative step of the relaxation algorithm.

An element $x^* \in X$ is referred to as a Nash normalized equilibrium point if:

$$\max_{y \in X} \Psi\left(x^*, y\right) = 0 \tag{6}$$

Given the concavity conditions, a Nash normalized equilibrium is also a Nash equilibrium point (Aubin, 1980). Finally, the optimum response function is introduced; which is the result of maximizing the Nikaido–Isoda function, where all players try to improve their profit. The optimum response function at the point x is:

$$Z(x) = \arg\max_{y \in X} \Psi(x, y), \quad x, Z(x) \in X \; . \tag{7}$$

This function returns the set of players' actions whereby they all try to unilaterally maximize their respective profit. Thus, by "playing" actions $Z(x)$ rather than x, the players approach equilibrium.

The payoff function of player i is:

$$\phi_i(x_i) = p(P)x_i - c_i(x_i) \tag{8}$$

where:

x_i	Electricity produced by unit i.
$p(P)$	Spot price defined by the market model with a total electricity demand of *P*.
$c_i(x_i)$	Cost of generating x_i MW by unit i.

As the energy price is obtained by inverse demand function, for each period t it is mathematically defined as:

$$p_t(x) = \alpha_t - \rho\left(P_t\right), \tag{9}$$

where:

α_t	Price intercepts.
ρ	Inverse of demand elasticity.

Each player or company i chooses its power generated x_i in order to maximize its profit. In addition, P_t^T is the total thermal power to be produced in period t, given by $D_t - P_t^h$, where D_t is the total demand period, and P_t^H is the electricity generated hydroelectrically in period t.

The power produced by each firm must take into account the operative restrictions of units:

$$P_i^{TMIN} \leq x_i \leq P_i^{TMAX}; \forall i \tag{10}$$

where:

P_i^{TMIN} Minimum limit of thermal-generated electricity by unit i.

P_i^{TMAX} Maximum limit of thermal-generated electricity by unit i.

The cost of producing x_i MW of electricity by thermal unit i is given by:

$$c(x_i) = \frac{\phi_i}{2} x_i^{\,2} + \omega_i x_i \;\; + \xi_i \tag{11}$$

Where ϕ_i, ω_i, ξ_i are cost coefficients associated to operative characteristics of unit i.

3. RESOLUTION STRATEGY

The hydrothermal coordination problem is solved by using dynamic programming in combination with a relaxation algorithm to obtain Nash-Cournot equilibrium points. In this section both algorithms will be explained.

3.1. RELAXATION ALGORITHM

To solve thermal generation considering an oligopolistic market the algorithm presented in (Contreras et al., 2004) is used.

In order to find a Nash equilibrium of a game, having an initial estimate x^0, the relaxation algorithm of the optimum response function, when it is single-valued and the concavity conditions are satisfied, is:

$$x^{s+1} = \left(1 - \alpha_s\right) x^s + \alpha_s Z\left(x^s\right), \quad \text{s = 0,1,2,…} \tag{12}$$

where $0 < \alpha_s \leq 1$. An iterative step is constructed as a weighed average of the improvement point $Z\left(x^s\right)$ and the current point x^s. The optimum response function is calculated after solving an optimization problem, as seen in (4). The average shown in (12) ensures convergence of the algorithm under certain conditions (Krawczyk and Uryasev, 2000), (Uryasev and Rubinstein, 1994). At each stage, the optimum response of a player is chosen, assuming that the rest will play as they did in the previous period. Thus, by taking a sufficient number of iterations, the iterations approach the Nash equilibrium x^* as much as needed.

The theorem that ensures convergence of the relaxation algorithm is presented in full detail in (Contreras et al., 2004). Games in which the strategy space of competing generation and distribution agents is coupled are called coupled constraint games (Rosen, 1965) and possess equilibrium solutions under a rather technical assumption. The assumption is that the game is diagonally strictly concave (DST) (Uryasev, 1990). Therefore, if the relaxation algorithm converges to a certain equilibrium, then, this point is a coupled constraint game solution.

3.2. DYNAMIC PROGRAMMING

Solving the actual problem using Dynamic Programming requires that the path in the time-state graph with highest profit between two given volumes should be found. These two volumes are the starting and the ending volume during the planning horizon.

In each period, the reservoir state is represented by a given value, and the profit associated to a volume from one period to the next one represents the cost of each arc of the path.

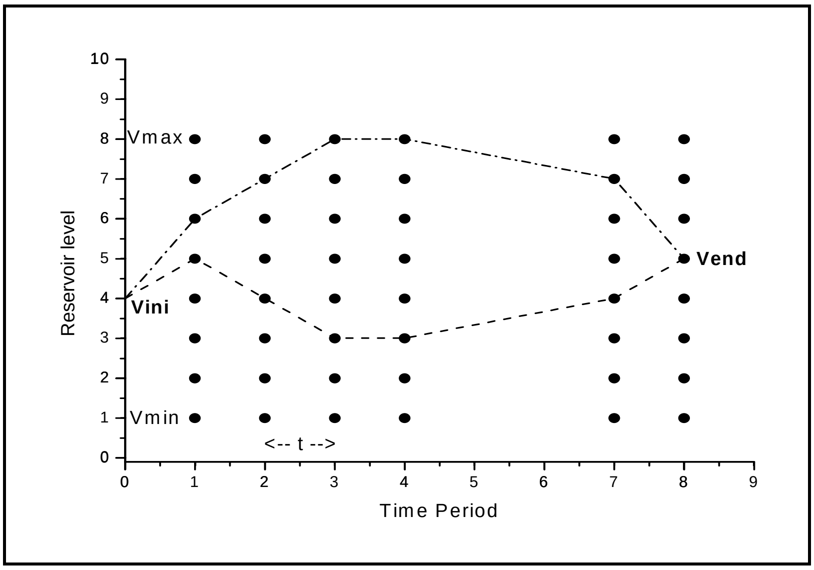

Figure 3: Dynamic Programming Schema.

In Figure 3, Vmin y Vmax are minimum and maximum operative limits of the reservoir respectively, while Vini represents the starting volume and Vend the ending volume on the reservoir during the planning horizon. The difference between these two values must be P^{HGEN}. As it can be observed, there are many possible trajectories, for each volume of the reservoir the decision to be made is on of these:

- generate, a given volume of water in the reservoir flows through the turbine;
- pump, a given volume of water in the lower reservoir is pumped back to the upper reservoir;
- do nothing.

In each period, the volume at the reservoir can take one of a set of possible values, which are represented by dots in Figure 3. Each path has a volume associated to each period and the possible ones must start from Vini and conclude after a fixed planning horizon in Vend. The algorithm must find as result the path which makes more profit to the hydroelectric agent.

The algorithm starts with an initial data load. These values are demand value in each period, starting and ending volume values, hydro and thermal operation limits. In each period, possible volumes are determined taking into account the previous volume value and the data previously mentioned.

Maximum profit Ben(j,k) in a period j for a reservoir state k is calculated as the previously period profit and the profit variation associated to operative decisions that lead to the new state. Mathematically:

$$Ben(j,k) = \underset{x \in PV_j}{Max}[Ben(j-1,x) + \Delta Ben(j-1,x;j,k)],$$

$$Ben(0,k)=0\,, \tag{13}$$

where PV_j is the set of possible levels in the period j-1 from which the level k could be obtained in a period j carring out anyone of the possible operations (generate, pump, nothing) in the plant.

In a given period and state, the Ben matrix represents the maximum profit obtained. P matrix indicates the predecessor state in an optimum path. $\Delta Ben(t-1,x\;;t,v)$ corresponds with the increase or decrease of profit associated to operations performed to vary from a volume x in period t-1 to a volume v in a period t.

Finally, the optimum trajectory can be rebuilt starting by the final volume value and calculating successively its predecessors. The value of $\Delta Ben(t-1,x\;;t,v)$ depends on the amount of energy generated hydraulically and on the price for a MW stated by market. Hence, a Nash-Cournot equilibrium must be find considering that the energy generated by the hydraulic plant is v-x.

4. NUMERICAL RESULTS

In this section a case study from the IEEE 30-bus system is presented. This case does not consider pumped storage plants; therefore, data presented in (Maiorano et al., 2000) must be modified to include this kind of units. This modification consists in changing the thermal characteristics of the plant number 1 to a hydroelectric one, considering cost-coefficients values as zero.

The minimum and maximum generation capacity and costs-function coefficients of the plants are reported in Table I.

Company	Plant	Min. Power	Max. Power	ϕ_i	ω_i	ξ_i
1	1	-60	60	0	0	0
2	2	0	80	0.035	1.75	0
	3	0	50	0.125	1	0
3	4	0	55	0.0166	3.25	0
	5	0	30	0.05	3	0
	6	0	40	0.05	3	0

Table 1: Thermal and Hydroelectric plants characteristics

The algorithms explained in Section 5 is applied to three different periods, one of low demand (off-peak hours), another of high demand (peak hours), and the other one of average demand. Considering the fact that demand is a strictly decreasing function of the price p, the inverse demand function for the three different periods can be modeled as follows:

Period	Demand	Inverse Demand Function
1	Medium	$p=320-2\sum_i\left(PG_i\right)$
2	Low	$p=160-2\sum_i\left(PG_i\right)$
3	High	$p=480-2\sum_i\left(PG_i\right)$

Table 2: Inverse Demand Functions for each period

Where $\sum_i\left(PG_i\right)$ represents the sum of the power generated thermal and hydroelectrically.

The total benefit obtained by the hydroelectric plant generator during the three periods is of $8416.8 and Table 3 shows the prices and hydroelectric generation for each in each period:

Period	Price	Power generated hydroelectrically
1	97.5 $/MW	17 MW
2	58.97 $/MW	-5.6 MW
3	124.33 $/MW	57 MW

Table 3: Results for each period

The power generated column represents power generated or spent in pumping water back to the upper reservoir. Positive values in this column means that the hydroelectric plant has been generating during this period, on the other hand, negative ones means that the unit has been pumping water back to the upper reservoir. The total amount of water which can be used during the planning horizon expressed in MW is 70. To obtain the amount of thermally generated energy which is spent in the pumping operation (Moitre et al., 2005b), the amount of water (traduced to MW) must be multiplied by the efficiency of the pumped storage unit which in this case is of 0.72. For example, in period 2 the electricity consumed by the hydroelectric plant is of 5.6 MW while the amount of water (traduced to MW) pumped back to the upper reservoir is of 4 MW.

5. CONCLUSIONS AND FUTURE WORK

We have presented a simple dynamic programming approach to solve the hydrothermal scheduling problem when pumped storage is considered. We have shown with an also simple example how the algorithm should be applied and the results obtained. This is our first attempt to apply these results to the Argentinean case and we will continue in this research direction both considering more complex methods and example problems. The interest on more complex methods is due to the low efficiency that a dynamic programming approach can have. A next research step will study the combined Nash Equilibrium – Optimization problem as a whole variational inequality trying to find a more efficient algorithm.

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