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INFLUENCE OF GEOMETRIC DISTORTIONS ON WALL PRESSURES IN SILOS DURING GRAVITY DISCHARGE

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RESUMEN

En el presente trabajo se estudian los cambios que sufren las cargas sobre las paredes de silos cilíndricos con imperfecciones geométricas en la pared durante la descarga por acción de la gravedad del material granular. Se utiliza una ecuación constitutiva desarrollada previamente por los autores, en la cual el flujo de los materiales granulares se representa por el flujo de un fluido no newtoniano incompresible. La simulación numérica se realiza por medio de la técnica de elementos finitos. Las paredes de los silos son consideradas rígidas, es decir que carecen de deformación durante el vaciado, y las imperfecciones geométricas presentan simetría axial. Se verifica que las imperfecciones introducen cambios en las cargas, pero su influencia es solamente local, es decir se extiende hasta una distancia tres veces mayor que la imperfección. También se verifica que las imperfecciones internas producen un incremento de carga mayor que el generado por imperfecciones externas.

ABSTRACT

This paper deals with the changes in wall pressures due to geometric distortions during the steady-state gravity discharge of silos. A constitutive equation previously presented by the authors is employed, in which the flow of the granular material is modeled as the flow of an incompressible non-Newtonian fluid. The numerical solution of the problem is achieved by means of finite element. The walls of the silo are assumed to be rigid, and the geometric distortions are axisymmetric. The results show that the geometric distortions induce changes in the loads in the loads on the walls, with a local influence that extends no further than three times the length of the imperfection. Finally, it is found that internal geometric deviations produce larger load changes than outer deviations.

INTRODUCTION

This paper deals with the changes in pressures on the walls of a silo during the discharge. The specific aspect studied is how geometric distortions in the cylindrical shape modify the flow pattern and redistribute pressures in the zone of damage. No previous references to these problems are found in the literature on this topic.

Damage in thin-walled metal structures (such as silos) induces geometric distortions. In the new damaged situation the structure has redistribution of stresses, so that equilibrium is satisfied using a different mechanism of resistance. Such stress redistribution may be of significance to the safety of the structure and assessment of the static behavior should be performed.

The sources of damage, its mechanism, and its consequences may be very different according to the structural system considered. Damage can be associated to geometric deviations in thin-walled metal structural components, and such deviations produce a redistribution of the stress in the structure with respect to the as-designed or perfect situation. It is important to evaluate that stress redistribution, and this is the subject of Ref.[7]. Specific analytical investigations for silos are report in Ref.[5]. For linear elastic structures under increasing load, the stress redistribution is uniquely defined, but for non-linear systems there may be several possible redistribution. Before any remedial action is taken, it is important to understand the equilibrium conditions by means of an appropriate structural analysis.

But damage and imperfections may also modify some functional aspect of a silo. During the discharge of vertical cylindrical silos, a distorted geometry introduces changes in the flow of the bulk stored materials, so that the pressures exerted on the wall are significantly affected. The actual values of changes depend on a number of factors, such as the amplitude and geometry of the distortion, the properties of the bulk material stored, the geometry of the silo, etc; however it is clear that there are overpressures associated to the modification of the flow. For a metal silo structure, one often finds deviations in the meridional directions from the cylindrical shape, caused by various mechanisms. The warning that a change in the geometry due to damage leads to higher pressures during the flow of grains contained in the silo may be found in Eibl, Rombach and Gottlicher[6]

Damage in metal silos is very common in the form of changes in the geometry, with height equal to the height of the plates used in the construction. The origin of such damage is usually associated to wind action, buckling of the empty silo, constructional defects, etc. Several questions arise: Can this damage significantly affect the pressures due to the bulk stored material on the silo walls? Should one be concerned about such changes in pressure, or are they small enough as to be neglected? And if they are significant, what tools do we need to assess their values?

This paper attempts to answer the above questions by means of numerical studies based on a continuo model of the grain stored, and a rigid wall of the silo.

CONSTITUTIVE MODEL

The plastic constitutive model is introduced in this section as a simplification of a previous constitutive formulation to model the frictional and collisional flow of compressible granular materials (see Ref.[1]). In this relation the granular materials are simulated by means of a compressible, non-Newtonian fluid with second order effects and the stress tensor depends on the directional derivatives of the density.

Summary of the general constitutive relations

In a previous paper, the authors introduced a constitutive equation to model the flow of compressible granular materials with friction and collision between particles. A compressible, non-Newtonian fluid with second order effects simulates the granular materials and the stress tensor depends on the directional derivatives of the volume distribution function. The model was fully described in Ref.[1], and only a summary is presented here.

The stress tensor is represented as a function of the strain rate tensor and the gradient of the volume distribution tensor in the form

$$T_{ij} = \left(a_0{}^e + a_0{}^d\right)\delta_{ij} + a_1 D_{ij} + a_2 M_{ij} + a_3 D_{ik} D_{kj} \tag{1}$$

where D_{ij} are the strain rate components; δ_{ij} is the Kronecker delta; M_{ij} are the components of the gradient of the volume distribution tensor; π is the volume distribution; and a_i are coefficients to be written in the following.

The following definitions apply:

$$D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

$$M_{ij} = \frac{1}{2}\left(\frac{\partial \pi}{\partial x_i}\right)\left(\frac{\partial \pi}{\partial x_i}\right) \tag{3}$$

The velocity vector can be written in the form

$$\mathrm{v} = [u_1(\mathrm{x},t), u_2(\mathrm{x},t), u_3(\mathrm{x},t)] \tag{4}$$

Finally, the coefficients $a_0{}^e, a_0{}^d, a_1, a_2$ and a_3 were obtained in Ref.[1] and can be computed from the following expressions:

$$a_0{}^e = -p_c \tag{5}$$

$$a_0^d = -\frac{2(S_{II})_c D_{ii}}{\sqrt{D_{ii}(p_c - p_i)^2 + 4V_{II}(S_{II})_c}} \left[\frac{1}{3} + \frac{(p_c - p_i)^2}{2(S_{II})_c}\right] \tag{6}$$

$$a_1 = \frac{2(S_{II})_c}{\sqrt{D_{ii}(p_c - p_i)^2 + 4V_{II}(S_{II})_c}} \tag{7}$$

$$a_2 = \frac{3\sqrt{2(S_{II})_c}\,(p_c - p_i)}{\sqrt{2(S_{II})_c (M_I)^2 + 9(p_c - p_i)^2 M_{IV}}} \tag{8}$$

$$a_3 = \frac{\varpi}{(\pi_m - \pi)^2} \tag{9}$$

In Eqs. (5-9) π and π_m are the volume distribution function and the highest value that the volume distribution function can reach; ϖ is a parameter that depends on the material; S_{II} is the second invariant of the deviatoric stress tensor; D_{II} is the second invariant of the strain rate tensor; V_{II} is the second invariant of the deviatoric strain rate tensor; and M_I and M_{IV} are functions of the tensor M_{ij} in the form:

$$M_I = M_{ii} \tag{10}$$

$$M_{IV} = (M_{ij})^2 - \frac{2}{3} M_{ij} M_{kk} \delta_{ij} + \left(\frac{M_{kk}}{3}\right)^2 \delta_{ij} \tag{11}$$

The function that represents the second invariant of the deviatoric stress tensor in terms of the critical pressure is

$$(S_{II})_c = \left[\alpha - \delta\, e^{-\tau p_c} + \theta\, p_c\right]^2 \tag{12}$$

where α, δ, τ and θ are parameters of the material and must be obtained from experiments, as explained in Ref.[2]. Eq. (12) represents the critical state in the modified Cap models[11].

The negative (tensile) pressure p_i that the material can have is

$$p_i = -c\frac{\sqrt{3}}{tan(\phi)} \tag{13}$$

where c and ϕ are the cohesion and friction of the material.

Following Gray and co-workers[8] the equation of p_c as a function of the density is

$$p_c = \sigma(\varphi\pi)^z \tag{14}$$

σ and z are parameters of the material; and φ is the density of the individual grains. The constitutive model is completely determined by Eqs. (1-14).

New Assumptions

The complexity of the general model can be reduced by means of two assumptions concerning the behavior of the material during flow. To obtain the new plastic relation two assumptions are introduced: First, that the granular material flows without changes in density during the steady state discharge; and second, that effects of the collisions between particles in the stress tensor are negligible. Each assumption is discussed next:

a) Constant density during steady-state discharge. The problem of interest in our case is the gravity discharge of silos. During the first stage of the flow there is a transient state, with high loads on the walls of the silo; and a second stage is dominated by a steady state flow. The time of duration of the transient response depends on the geometry of the silo and the dimensions of the outlet, as well as on the properties of the bulk stored material, but it may take only a few seconds. The granular solid has deformations during the early stages of flow so that a critical state governs the steady state flow, for this reason the granular solid flows without changes in its density.

b) Collision effects are neglected. In a silo, the influence of collisions between grains on the stress tensor is generally neglected based on experimental evidence, so that authors do not consider collisions in silos as a relevant variable of the problem (see for example Refs.[8,9]). Collisions are not important because the distance between grains during flow inside the silo is small compared with the dimensions of the grains themselves. Moreover, collisions are negligible in cohesive materials.

The importance of the compressibility and the collisional effects in the flow of granular solids was studied in the Ref.[3].

Constitutive Model for the Steady-State Discharge of Silos

The simplifications accepted for steady state flow neglect the influence of second order terms in the constitutive equation, and assume that there is no change in the density of the material. The constitutive equation becomes

$$T_{ij} = -p\delta_{ij} + \left[\frac{\alpha - \delta\, e^{-\tau\, p_c} + \theta\, p_c}{\sqrt{V_{II}}}\right] D_{ij} \tag{33}$$

An apparent viscosity μ_a can be defined in the last equation as

$$\mu_a = \frac{\alpha - \delta\, e^{-\tau\, p_c} + \theta\, p_c}{\sqrt{V_{II}}} \tag{34}$$

The authors presented an alternative derivation of the last equation using non-associated plasticity[2].

FINITE ELEMENT DISCRETIZATION

The finite element formulation for this problem has been described previously (see for example[2,13]) and will not be repeated here. The basic formulation for a viscous flow uses the equilibrium and continuity (or incompressibility) equations. A weak or integral form of the problems is written as the principle of virtual powers plus an integral form of the condition of the incompressible flow. The two conditions are put together into one equation by means of a penalty formulation (including a penalty parameter which takes large values).

The final variables are velocities interpolated within plane or axisymmetric geometry. Nine node Lagrangean elements have been used. The assembled system is non-linear because of the constitutive equation employed. A direct iteration procedure is used to compute the solution, similar to what was done for metal-forming processes a few years ago[12].

NUMERICAL RESULTS

The theme structure employed here to illustrate the influence of geometric distortions is a cylindrical silo with a conical hopper at the lower part, shown in Figure 1. The total height of the hopper and cylinder is H = 25m, the high of the hopper is H_h = 8m, the radius of the cylindrical part is R = 2.5m, and the outlet has a diameter of 1.5m. In this paper is considered a frictional-cohesive material, with $\alpha = 2.4KPa, \tau = 0.25, \theta = 0.35,$ and $\delta = 0.1\text{KPa}$.

The silo walls are assumed to be rigid, for this reason the deformation of the silo structure is not included in the analysis. So that, no attempt is made here to evaluate grain-silo interaction. In similar form that thus used by Ooi and She[10], the geometric imperfections are assumed to be axisymmetric, with the center located at elevation z = 9.5m. Straight lines form the actual shape of the geometric distortion in the meridian. The maximum amplitude of the deviation is denoted as "r", and the extent of the zone of damage in elevation is "h", as shown in Figure 1.

The bulk stored material is discretized using 265 finite elements. Following convergence studies, the mesh was refined in the zone of geometric changes, where pressures changes are highest.

Pressures on the walls of the silo during steady-state discharge are shown in Figure 2, for a perfect geometry and for deviations with amplitudes r = 4, 6, 8, 10 and 12cm (or R/r = 62.5, 41.67, 31.25, 25 and 20.83), and h = 1m (or R/h = 2.5). For example, the maximum pressures obtained for R/r = 31.25 are 2.2 times the values of the pressures in the perfect shell. The relation between pressures changes and r is almost linear, and becomes non-linear for R/r < 30. This is a local effect in the sense that changes in the pressures with respect to the perfect geometry are localized in a zone of 3h.

g

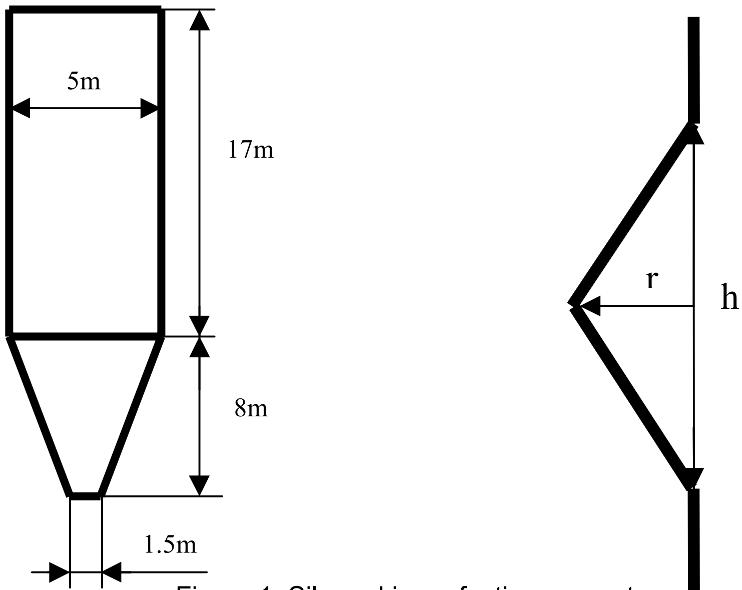

Figure 1. Silo and imperfection geometry.

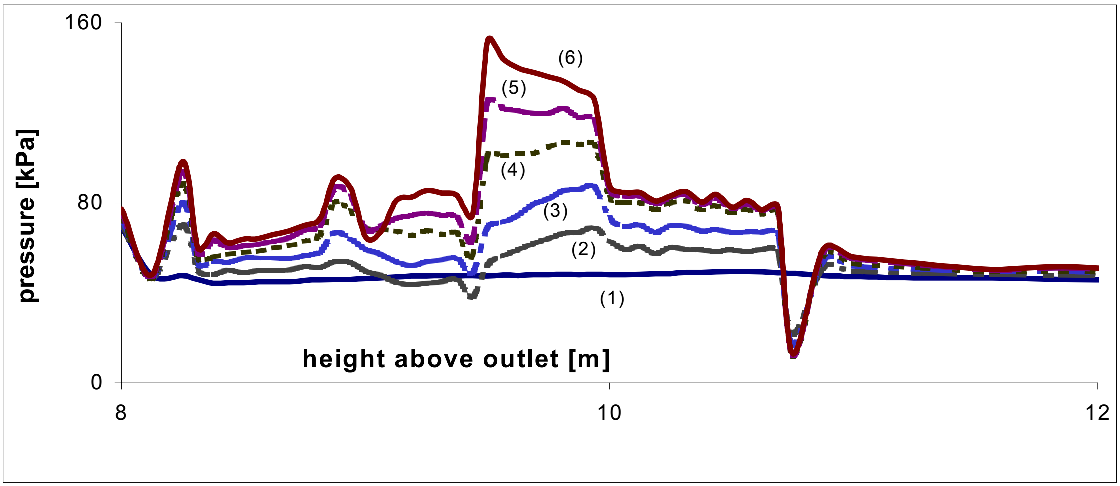

Figure 2. Influence of imperfection amplitude r on pressure on the silo walls, for an inner imperfection. (1) r = 0, (2) r = 4cm, (3) r = 6cm, (4) r = 8cm, (5) r = 10cm, (6) r = 12cm.

For a fixed value of amplitude (R/r = 62.5), the influence of the height of the zone with geometric damage is shown in Figure 3. Shorter imperfections seem to produce a higher change in pressures: for example, with R/h = 5 the maximum pressure changes are 1.27 times the values due to an imperfection with R/h = 2.5. The differences become more important as the amplitude of the imperfection increases.

In all cases the maximum pressures are located near to the top of the geometric distortions, for this class of inner deviations (elevations 9.5m < z < 10m). Outer imperfections, on the other hands, tend to have maximum pressure changes near to the bottom of the imperfection. This is illustrated in Figure 4, where R/r = -62.5, -31.25, and -20.83 are considered, and maximum values are obtained for elevations 9.0m < z < 9.5m.

Finally, the density of the bulk stored material plays a role and has been studied in Figure 5, for R/r = 2.5, and density of 700, 750, and 800kg/m^3. As expected, the changes in pressures increase with the density of the material, and follow an almost linear relation.

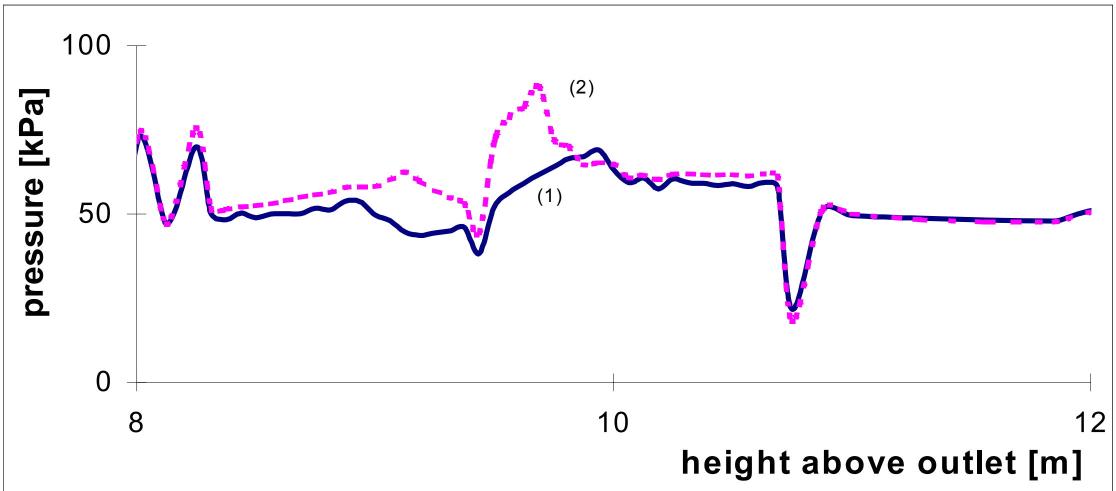

Figure 3. Influence of imperfection extent h on pressure on the silo walls, for an inner imperfection. (1) R/h = 2.5, (2) R/h = 5.

CONCLUSIONS

Imperfections and distortional damage in shells of silos introduce significant changes in the pressures and stresses on the walls computed during the steady-state discharge. Axisymmetric geometric imperfections have a localized effect, affecting a zone not larger than three times the extent of the imperfection, in much the same way as presented by Ooi and She for static actions[10]. There is an almost linear relation between the maximum amplitude of the imperfection and the pressure on the walls of the silo (mainly for small values of r). The extent, on the other hand, is inversely proportional to the pressures, so that short imperfections lead to higher pressures than longer ones.

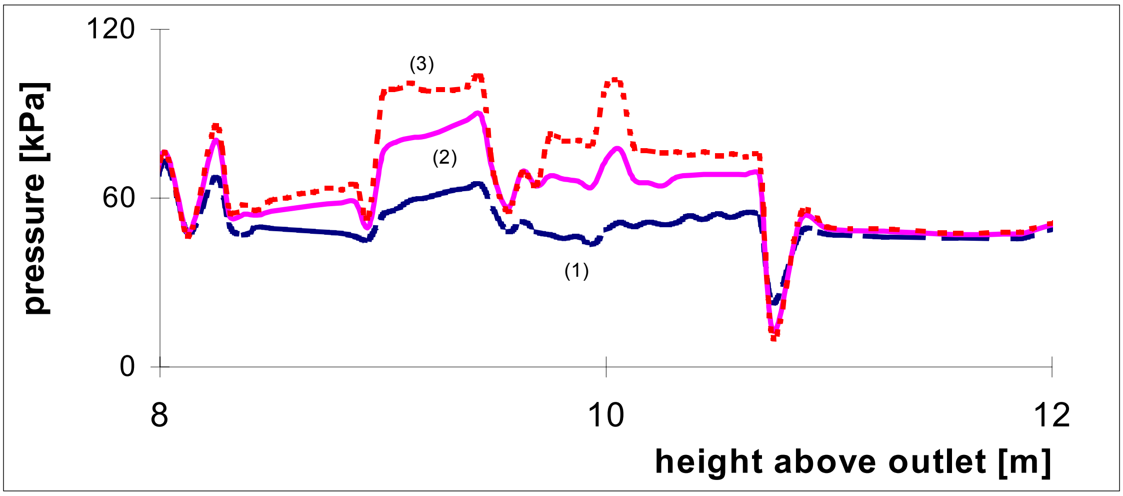

Figure 4. Influence of imperfection amplitude on pressures on the walls of the silo, for an outer imperfection. Data: density of bulk stored material 800kg/m^3; h = 1m. (1) r = -4cm, (2) r = -8cm, (3) r = -12cm

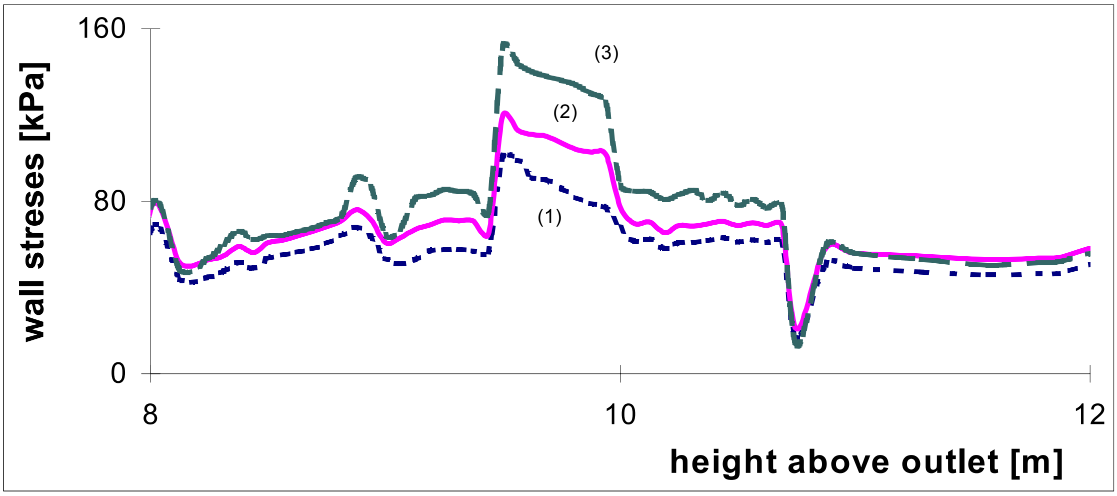

Figure 5. Influence of material density on pressures on the walls of the silo, for an inner imperfection. Data: r = 12cm, h = 1m. (1) 700kg/m^3, (2) 750kg/m^3, (3) 800kg/m^3.

Imperfections that move the wall towards the inside of the silo lead to higher changes in the pressures than outer imperfections, for similar location and imperfection extent and amplitude.

On the part of the imperfection that faces the flow, higher pressures develop in both inner and outer imperfections. A decrease in pressure occurs on the rest of the imperfection surface.

Because high local pressures are generated by imperfections, one should consider their influence on the stresses in the shell itself. This is a form of two types of concentration that may display interaction: a concentration of pressures on the shell and a concentration of stresses in the shell. Further studies are needed to clarify the nature of this interaction, so that one can assess what is the effective increase in stress concentrations due to the local increase in pressures. One may speculate that under the new local pressures on the walls, there will be displacement and some pressure relief.

In the present model the walls are assumed to be rigid, so that the influence of deflections of the walls of the silo are neglected. This is a simplification of the problem, and fluid-structure interaction should be considered in thin-walled metal silos. No such studies have reported up to now.

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