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AN ALGORITHM FOR 3D SEISMIC INVERSION

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ABSTRACT

We present an algorithm to solve an inverse problem originated in seismic exploration. In order to briefly describe the geophysical techique, we can say that while drilling a borehole, the drill-bit generates a signal asumed as a seismic source. The wavefront propagates through the earth, and the direct and reflected waves at the different interfaces are recorded at a line of geophones laid out on the ground. Thus we aim to estimate the wave speeds of the different formations within the earth interior from the measurements at the geophone positions. The forward model is formulated, assuming cylindrical symmetry, by means of the displacements of an elastic solid, and discretized using the Morley mixed finite elements. For the inverse problem, casted as a minimum squares one, we use a quasilinearization algorithm, which allows for a late discretization of the problem. A parallel version of the algorithm is used to solve the included numerical example.

RESUMEN

Presentamos un algoritmo para la resolución de un problema inverso que surge en exploración sísmica. Una breve descripción de la técnica de prospección es la siguiente: Durante la perforación de un pozo petrolero, el trépano genera una señal a la que se la considera como una fuente sísmica. El frente de onda acústico viaja a través del subsuelo, y las ondas directas y las reflejadas en distintas interfaces son grabadas mediante una línea de geófonos dispuesta en la superficie. Así, el objetivo es obtener una estimación de las velocidades de propagación de los distintos estratos en los cuales se asume está formado el subsuelo a partir de los datos obtenidos. El modelo directo, en el cual se asume simetría cilíndrica, es formulado en término de los desplazamientos de un sólido elástico, y es discretizado usando los elementos finitos mixtos definidos por Morley. En las fronteras artificiales se utilizan condiciones de bordes absorbentes, haciéndolas transparentes a ondas que arriban normalmente a ellas. Para el tratamiento del problema inverso, formulado como uno de cuadrados mínimos, utilizamos un algoritmo conocido como cuasilinearización, que permite el planteo en forma continua, discretizando en última instancia. Utilizamos una versión paralela del algoritmo para resolver el ejemplo numérico propuesto.

INTRODUCTION

The aim of this effort is to describe a nonlinear inversion procedure to solve an inverse scattering problem arising from exploration seismology when using the so called seismic while-drilling technology [1]. This technique can be briefly described as follows. During the drilling of a borehole in hydrocarbon exploration, the drill-bit generates a signal that can be regarded as a seismic source. The acoustic wavefront induced by this source propagates through the earth, and the direct and reflected waves at the different interfaces are recorded at a line of geophones laid out on the ground. Thus we aim to estimate the wave speeds of the different formations within the earth interior from the measurements at the geophone positions.
To simplify the problem and reduce the number of parameters to be estimated we assume that our domain of interest Ω is bounded and radially symmetric around the z-axis, located at the center of the borehole, where the drill-bit is positioned at a fixed depth. This drill bit is assumed to behave as a compressional point source of known shape in the time domain. We further assume that the medium is acoustic, ignoring shear waves. At the artificial boundaries of the model we employ absorbing boundary conditions making them transparent to normally arriving waves.
The inverse problem is formulated as an output least-square problem. It is an iterative procedure known as quasilinearization and at each step of the parameter estimation procedure needs to compute the (Fréchet) derivative of the solution with respect to the parameter [2, 3]. This derivative is obtained as the solution of a differential problem identical to the forward problem but with different source and boundary data. The identification procedure is formulated at the continuous level, and then its discrete version is obtained by computing approximations to the solution of the partial differential equations associated with the forward problem and the Fréchet derivatives using an explicit finite element procedure. This way of solving the problem is known to be much more efficient than the classical approach where the the gradients are obtained from the discretized functional as is done for example in [4]. For a thorough description of the technique, see [5]

THE FORWARD AND INVERSE PROBLEMS

The Differential Model

Let

$$\Omega = \{(r,\theta,z):\ 0 \le r < R_B,\ 0 \le \theta < 2\pi,\ 0 < z < z_B\}$$

be a 3D layered and radially symmetric open bounded domain with boundary $\partial\Omega$. Let $\Gamma^T = \partial\Omega \cap \{z = 0\}$ be the part of $\partial\Omega$ associated with the free surface, i.e, the earth–air interface and let $\Gamma = \partial\Omega \setminus \Gamma^T$ denote the bottom and lateral (artificial) boundaries of Ω. Let besides $\rho = \rho(z)$ and $A = A(z)$ denote the mass density and incompressibility modulus of the material, assumed to be bounded above and below by positive constants: $0 < \rho_* \le \rho(z) \le \rho^* < \infty$, $0 < A_{f^*} \le A(z) \le A^* < \infty$.
Let $\vec{u} = \vec{u}(r,z,t) = (u_r(r,z,t),\ 0,\ u_z(r,z,t))$ be the displacement vector in Ω. We will assume that the density ρ is known and we will regard the displacement vector as a function of the incompressibility modulus A, employing the notation $\vec{u}(A)$ to indicate such dependence, and omitting the dependence of $\vec{u}$ on the spatial and temporal variables to avoid cumbersome notation. Recall that the wave speed $c(z)$ is related to the parameter A by the equation $c(z) = (\frac{A(z)}{\rho(z)})^{\frac{1}{2}}$, so that once the parameter A has been estimated it is immediate to obtain the desired wave speed estimate.
We will assume that the propagation of compressional waves in Ω is described by the following forward problem: find $\vec{u}(A)$ such that

$$\rho\,\frac{\partial^2 \vec{u}(A)}{\partial t^2} - \nabla\big(A\nabla\cdot\vec{u}(A)\big) = \vec{f}(r,z,t), \quad (r,\theta,z) \in \Omega,\ t \in I = (0,T), \tag{1}$$

with initial conditions

$$\vec{u}(A)|_{t=0} = \frac{\partial \vec{u}(A)}{\partial t}|_{t=0} = 0, \quad (r,\theta,z) \in \Omega, \tag{2}$$

and boundary conditions

$$-A\nabla \cdot \vec{u}(A) = (\rho A)^{\frac{1}{2}} \frac{\partial \vec{u}(A)\cdot\vec{\nu}}{\partial t}, \quad (r,\theta,z) \in \Gamma, \quad t \in I, \tag{3}$$

$$-A\nabla \cdot \vec{u}(A) = 0, \qquad (r,\theta,z) \in \Gamma^T, \quad t \in I. \tag{4}$$

In (1)–(4) $\vec{f} = (f_r, 0, f_z, t)$ denotes the external source and $\vec{\nu}$ the unit outer normal to $\partial\Omega$. Also, (3) is an absorbing boundary condition which makes the artificial boundary Γ transparent to outward going waves arriving normally to Γ and (4) represents the free–surface condition on Γ^T.

The Inverse Problem.
First we describe the set of admissible parameters $\mathcal{P}$. Assume that Ω consist of the union of N_z-layers Ω_j, where

$$\Omega_j = \{(r,\theta,z),\ 0 \le r < R_B,\ 0 \le \theta < 2\pi,\ z_{j-1} < z < z_j\}, \quad j = 1,\cdots,N_z,$$

and $z_j, j = 0,\cdots,N_z$ is a partition of $(0, z_B)$. Further assume that the parameter $A(z)$ have a constant value on each layer Ω_j. Thus we define $\mathcal{P} = \{A \in L^2(\Omega):\ A_* \le A(z) \le A^*,\ A(z) = \sum_{j=1}^{N_z} A_j \chi_{\Omega_j}\}$, where for any $D \subset \Omega$, χ_D denotes the characteristic function of the set D. Note that $\mathcal{P}$ is a compact convex subset of $L^2(\Omega)$ with the inherited topology and $A \in \mathcal{P}$ has a well defined trace $A|_{\partial\Omega_j}$ on each Ω_j, so that (3)-(4) make sense for elements $A \in \mathcal{P}$. Also note that any element $A \in \mathcal{P}$ can be identified with the vector $\vec{A} = (A_j)_{j=1,\cdots,N_z} \in \mathbf{R}^{N_z}$; we will refer either to A or to $\vec{A}$ indistinctly in the rest of the paper .
Suppose that at time $t = 0$ the medium is excited with a known source function $\vec{f}(r,z,t)$ and that the values $\vec{u}^{obs}(r,z,t) = \left(u_r^{obs}(r,z,t), u_z^{obs}(r,z,t)\right)$ of the displacements induced inside Ω, associated with direct and reflected waves, are recorded at receivers located at the points $r = r_i, z = z^*, i = 1,\cdots,N_r$, inside Ω for all $t \in I$. Then the objective is to infer from the measurements $\vec{u}^{obs}(r_i, z^*, t)$, $1 \le i \le N_r$, the actual value of the parameter A. To formulate the problem in a meaningful fashion, since the solution of the forward differential problem may not have well defined pointwise values, we define the nonlinear averaging map $\Phi : (\mathcal{P}, \|\cdot\|_0) \to L^2(I, \mathbf{R}^{2N_r})$ as follows:

$$\Phi(\vec{u}(A))(r_i, z^*, t) = (\Phi(u_r(A)), \Phi(u_z(A)))(r_i, z^*, t))_{1 \le i \le N_r}, \tag{5}$$

where for any function $g(r,z)$

$$\Phi(g)(r_i, z^*) = \frac{1}{|B(r_i, z^*, d)|} \int_{B(r_i, z^*, d)} g(r,z)\ r dr dz. \tag{6}$$

(Here the radius d of the balls $B(r_i, z^*, d)$ is small enough so that $B(r_i, z^*, d) \cap B(r_j, z^*, d) = \emptyset$ for $i \neq j$). Next, for $\vec{u}^{obs}(\cdot) \in L^2(I; R^{2\times N_r})$ and $A \in \mathcal{P}$, let the cost functional $J(A)$ and its regularized form $J^\beta(A)$ with nonnegative regularization parameter β be defined by the equations

$$J(A) = \frac{1}{2}\sum_{i=1}^{N_r} \int_0^T \left\|\Phi(\vec{u}(A))(r_i, z^*, s) - \vec{u}^{obs}(r_i, z^*, s)\right\|_{\mathbf{R}^2}^2 ds, \tag{7}$$

$$J^\beta(A) = J(A) + \frac{1}{2}\beta \|A\|_0^2, \tag{8}$$

where we denoted by $\|\vec{y}\|_{\mathbf{R}^n}$ the euclidean norm of any vector $\vec{y} \in \mathbf{R}^n$.
Then we formulate our standard least-squares estimation problem as follows:

$$\text{minimize } J(A) \quad \text{over } \mathcal{P}. \tag{9}$$

Also, the regularized least-squares estimation problems is given by

$$\text{minimize } J^{\beta}(A) \quad \text{over } \mathcal{P}. \tag{10}$$

The problem of existence of solutions of the minimization problems (9)–(10), as well as results on the existence, uniqueness and regularity on the solution of (1)–(4) were addressed in [5]. To solve the the minimization problems (9)–(10) we will employ an iterative quasilinearization technique requiring the calculation of the Fréchet derivative $\vec{D}^j_A(\vec{u}) \equiv \vec{D}_A(\vec{u})\chi_{\Omega_j} = ((D^j_A(\vec{u}))_r, (D^j_A(\vec{u}))_z), \quad j = 1, \cdots N_z$, of the solution of the (1)–(4) with respect to the parameter A. The algorithm reads:
For $\vec{A} \in \mathcal{P}$ set

$$\begin{aligned} \mathbf{G}_r(i,j)(t) &= \Phi((\vec{D}^j_A)_r)(r_i, z^*, t)), \\ \mathbf{G}_z(i,j)(t) &= \Phi((\vec{D}^j_A)_z)(r_i, z^*, t), \quad 1 \le i \le N_r, 1 \le j \le N_z. \end{aligned} \tag{11}$$

Let

$$\mathbf{M}^{\beta}(\vec{A}) = \int_0^T \left(\mathbf{G}_r^T(s)\mathbf{G}_r(s) + \mathbf{G}_z^T(s)\mathbf{G}_z(s)\right) ds + \beta\, \Xi, \tag{12}$$

$$\begin{aligned} \left(\mathbf{H}^{\beta}(\vec{A})\right)_j = \int_0^T \Bigg(& \textstyle\sum_{i=1}^{N_r} \mathbf{G}_r(j,i)(s)\left[u_r^{obs}(r_i, z^*, s) - \Phi(u_r(\vec{A}))(r_i, z^*, s)\right] \\ & + \textstyle\sum_{i=1}^{N_r} \mathbf{G}_z(j,i)(s)\left[u_z^{obs}(r_i, z^*, s) - \Phi(u_z(\vec{A}))(r_i, z^*, s)\right]\Bigg)\, ds, \quad 1 \le j \le N_z, \end{aligned} \tag{13}$$

where $\Xi = \operatorname{diag}(|\Omega_1|, \cdots, |\Omega_{N_z}|)$. In (12) $\mathbf{G}_r^T$, $\mathbf{G}_z^T$ denote the trasposes of the matrices $\mathbf{G}_r$, $\mathbf{G}_z$ defined in (11).
Let $\vec{A}^0$ be an initial guess for $\vec{A}$. Then the iterative quasilinearization estimation procedure at the continuos level is defined as follows.

$$\vec{A}^{l+1} = \mathcal{F}^{\beta}(\vec{A}^l) = \vec{A}^l + [\mathbf{M}^{\beta}(\vec{A}^l)]^{-1}\mathbf{H}^{\beta}(\vec{A}^l). \tag{14}$$

THE DISCRETE IDENTIFICATION PROCEDURE

We will employ an explicit finite element procedure to compute the solution of the forward problem and the Fréchet derivatives in order to obtain a discrete version of the identification algorithm (14). Let τ^h, $0 < h < 1$ be a quasiregular partition of Ω into elements $\mathcal{E}_m$ generated by the rotation around the z-axis of rectangles in the (r, z)-variables of diameter bounded by h.
Let

$$\begin{aligned} \mathcal{Q}^0 &= \operatorname{Span}\{(r,0,0), (0,0,1), (0,0,z)\}, && (15) \\ \mathcal{Q} &= \operatorname{Span}\{(r^{-1},0,0), (r,0,0), (0,0,1), (0,0,z)\} && (16) \end{aligned}$$

and set

$$\mathcal{V}^h = \left\{\vec{v} = (v_r, 0, v_z) \in H(div, \Omega) : \vec{v}|_{\mathcal{E}_m} \in \mathcal{Q}^* \; \forall \mathcal{E}_m \in \tau^h\right\}, \tag{17}$$

where $\mathcal{Q}^* = \mathcal{Q}$ for elements $\mathcal{E}_m$ located away from $r = 0$ and $\mathcal{Q}^* = \mathcal{Q}^0$ for the innermost elements $\mathcal{E}_m$ near $r = 0$. The space $\mathcal{V}^h$ is the vector part of the lowest-order mixed finite element space

defined by Morley [6] in an unpublished manuscript and it was employed in [7] to simulate the propagation of waves in a model for full waveform acoustic logging. The property that elements in $\mathcal{V}^h$ be globally in $H(div,\Omega)$ is equivalent to the requirement that their normal components be continuous across interelement boundaries $\Gamma_{pq} = \partial\mathcal{E}_p \cup \partial\mathcal{E}_q$. The natural degrees of freedom for $\vec{v} \in \mathcal{V}^h$ are the values of $\vec{v}\cdot\vec{\nu}$ at the mid points of the edges of the elements $\mathcal{E}_m \in \tau^h$.
Since we want to define an explicit finite element procedure, all integrals involving time derivatives will be computed using a quadrature rule as follows. For a rectangle R of side lengths h_r and h_z and side mid points $a_i = (r_i, z_i), i = 1,2,3,4$,

$$\int_R f(r,z) r dr d\theta dz \approx \frac{2\pi}{4} h_r h_z . \sum_{i=1}^{4} f(r_i, z_i) r_i \tag{18}$$

Similarly, for the boundary integrals we will employ the mid-point quadrature rule.
Let $[f,g]$ and $\langle\langle f,g\rangle\rangle$ denote respectively the inner products (f,g) and $\langle f,g\rangle$ computed approximately using the indicated quadrature rules. Next, let L be a positive integer, $\Delta t = T/L$, and $g^n = g(n\Delta t)$. Set

$$d_t g^n = \frac{g^{n+1} - g^n}{\Delta t}, \qquad \partial g^n = \frac{g^{n+1} - g^{n-1}}{2\Delta t}, \tag{19}$$

$$\partial^2 g^n = \frac{g^{n+1} - 2g^n + g^{n-1}}{(\Delta t)^2}. \tag{20}$$

Then the approximation to the solution $\vec{u}(A)$ of the weak version of (1)–(4) is computed as follows: find $\vec{u}^{h,n}(A) \in \mathcal{V}^h$ such that

$$\begin{gathered}[\rho\, \partial^2 \vec{u}^{h,n}(A), \vec{v}] + \left(A\nabla\cdot\vec{u}^{h,n}(A), \nabla\cdot\vec{v}\right) + \langle\langle (\rho A)^{\frac{1}{2}} \partial\vec{u}^{h,n}(A)\cdot\vec{\nu}, \vec{v}\cdot\nu\rangle\rangle_\Gamma \\ = (\vec{f}^n, \vec{v}), \quad \vec{v}\in\mathcal{V}^h, \quad n = 1,\cdots,L-1, \\ \vec{u}^{h,0}(A) = \vec{u}^{h,1}(A) = 0.\end{gathered} \tag{21}$$

The approximation to the Fréchet derivatives $\vec{D}_A^j(\vec{u})$ are computed in a similar fashion: find $\vec{D}_A^{j,h,n} \in \mathcal{V}^h$ such that

$$\begin{gathered}[\rho\, \partial^2 \vec{D}_A^{j,h,n}, \vec{v}] + \left(A\nabla\cdot\vec{D}_A^{j,h,n}, \nabla\cdot\vec{v}\right) + \langle\langle (\rho A)^{\frac{1}{2}} \partial\vec{D}_A^{j,h,n}\cdot\vec{\nu}, \vec{v}\cdot\nu\rangle\rangle_\Gamma \\ = -\left(\chi_{\Omega_j}\nabla\cdot\vec{u}^{h,n}(A), \nabla\cdot v\right) - \frac{1}{2}\langle\langle \chi_{\Omega_j}\left(\frac{\rho}{A}\right)^{\frac{1}{2}} \partial\vec{u}^{h,n}(A)\cdot\vec{\nu}, \vec{v}\cdot\nu\rangle\rangle_\Gamma, \\ \vec{v}\in\mathcal{V}, \quad n = 1,\cdots,L-1, \\ \vec{D}_A^{j,h,0} = \vec{D}_A^{j,h,1} = 0.\end{gathered} \tag{22}$$

Assume that the stability condition

$$\Delta t \le C_1\, h \tag{23}$$

is satisfied, with C_1 depending on the wave speed. Then the argument employed in [7] can be used here to show that the discrete procedures (21) and (22) are stable and satisfy the following apriori error estimates :

$$\max_{1\le n\le L-1}\left(\left\|d_t(\vec{u}^n - \vec{u}^{h,n}\right\|_0 + \left\|\vec{u}^n - \vec{u}^{h,n}\right\|_{\mathcal{V}}\right) \le C\,(\Delta t^2 + h), \tag{24}$$

$$\begin{gathered}\max_{1\le n\le L-1}\left(\left\|d_t\vec{D}_A^{j,n}(\vec{u}) - d_t\vec{D}_A^{j,h,n}\right\|_0 + \left\|\vec{D}_A^{j,n}(\vec{u}) - \vec{D}_A^{j,h,n}\right\|_{\mathcal{V}}\right) \\ \le C\,(\Delta t^2 + h),\end{gathered} \tag{25}$$

Now we proceed to define the discrete estimation procedure. Set

$$\mathbf{G}_r^{h,n}(i,j) = \Phi((\vec{D}_A^{j,h,n})_r)(r_i, z^*)), \tag{26}$$
$$\mathbf{G}_z^{h,n}(i,j) = \Phi((\vec{D}_A^{j,h,n})_z)(r_i, z^*), \quad 1 \le i \le N_r, 1 \le j \le N_z, \ 1 \le n \le L. \tag{27}$$

Let

$$\mathbf{M}^{\beta,h}(\vec{A}) = \sum_{n=1}^{L} \left((\mathbf{G}_r^{h,n})^T \mathbf{G}_r^{h,n} + (\mathbf{G}_z^{h,n})^T \mathbf{G}_z^{h,n} \right) \Delta t + \beta\, \Xi, \tag{28}$$
$$\left(\mathbf{H}^{\beta,h}(\vec{A}) \right)_j = \sum_{n=1}^{L} \left(\sum_{i=1}^{N_r} \mathbf{G}_r^{h,n}(j,i) \left[u_r^{obs,n}(r_i, z^*) - \Phi(u_r^{h,n}(\vec{A}))(r_i, z^*) \right] \right. \tag{29}$$
$$\left. + \sum_{i=1}^{N_r} \mathbf{G}_z^{h,n}(j,i) \left[u_z^{obs,n}(r_i, z^*, s) - \Phi(u_z^{h,n}(\vec{A}))(r_i, z^*) \right] \right) \Delta t, \quad 1 \le j \le N_z.$$

Let $\vec{A}^0$ be an initial guess for $\vec{A}$. Then the discrete iterative estimation procedure is defined as follows.

$$\vec{A}^{l+1} = \mathcal{F}^{\beta,h}(\vec{A}^l) = \vec{A}^l + [\mathbf{M}^{\beta,h}(\vec{A}^l)]^{-1} \mathbf{H}^{\beta,h}(\vec{A}^l). \tag{30}$$

In the next section we show examples of the implementation of the discrete quasilinearization algorithm (30) to model problems in the context of the seismic while-drilling technology.

NUMERICAL EXAMPLES

For the numerical experiments we used a sequence of domains all having the same horizontal length of 300 m , and increasing depth. This is done in this way because one wants to estimate the velocity up to 500 m below the drill bit. On the top boundary, which coincides with the surface of the Earth, we set 12 equally separated sensors.
For the sake of simplicity, the source was assumed to have a central frequency f_0 of 25 Hz, and a time shape given by $g(t) = -2M\xi(t-t_0)e^{-\xi(t-t_0)^2}$, where M is a scaling factor, $\xi = 8f_0^2$, $t_0 = 1.25/f_0$.
The model is a typical one, consisting of a set of layers of different velocities and thickness. It is assumed that the distribution of velocities is known for the layers above the drill bit.
As the goal is to predict, for example, the existence of a low velocity layer associated with the fluid overpressure region. This leads to optimize the drilling process, diminishing costs and enhacing safety.
Figure 1 shows the behaviour of the algorithm in the vicinity of the above mentioned region when the drill bit is (a) at 1000 m, (b) 1100 m, (c) 1200 m and (d) 1300 m depth. After many numerical examples we decided that the best choice for the algorithm to yield accurate results is to use as initial guess for a fine grid the output of a coarse one, using to get it a very low frequence of 5 Hz. In the example the coarser grid had an inter-node distance of 15 m in both directions, and the time interval was 2 ms. These quantities satisfy the CFL condition, and the same must happen with the ones corresponding to the finer grid. Therefore, as the inter-node distance was assumed to measure 5 m, the time interval was 1 ms.
It can also be observed in Fig. 1 that only 5 iterations on the finer grid are enough to recover at least the low velocity layer of the true model.

CONCLUSIONS

We have presented an algorithm to solve an inverse problem originated in seismic exploration. The forward model was formulated, assuming cylindrical symmetry, by means of the displacements of an elastic solid, and discretized using the Morley mixed finite elements. For the inverse

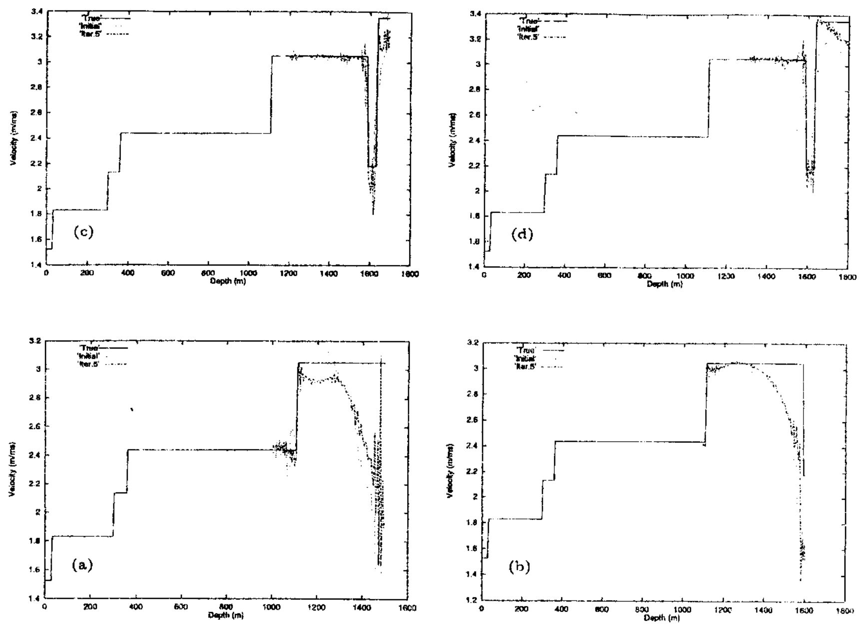

Figure 1: We display the sequence of results of the inversion process, for different depths of the drill bit. In (a) 1000 m, (b) 1100 m, (c) 1200 m and (d) 1300 m

problem, casted as a minimum squares one, we used a quasilinearization algorithm, which allows for a late discretization of the problem. The results yielded by the numerical examples are satisfactory, because it can be seen that the algorithm is able to find the jumps in velocity, in particular, it can determine after few iterations the presence of an overpressure region, main objective in this application.
A parallel version of the algorithm was implemented, but the results were not as good as expected. We continue working on this point.

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