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A 3D PANEL CODE FOR WAVE DRAG CALCULATIONS PART I: GENERAL FORMULATION AND DISCRETIZATION

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SUMMARY

We present a BEM/panel code to compute 3D potential flows about ship-forms with linearized free-surface conditions in order to compute the wave drag as a function of the Froude number. The basic governing equations of potential flow with free surface are the Laplace equation for the velocity potential with appropriated boundary conditions and the free surface condition. The last is based on the Bernoulli equation which relates the surface elevation with the local absolute value of velocity. However this problem is ill-posed in the sense that allows multiple solutions, associated with the existence of a system of trailing gravity waves propagating in both (upstream and downstream) directions. Solutions with upstream propagating trailing waves should be considered non-physical and should be discarded. This is done by means of the addition of an upwind or artificial viscosity term. Details of the upwind technique will be given in another paper [1].
The standard BEM/panel discretization is based in the Green's third theorem and an integral representation of the potential velocity is obtained by means the Morino's formulation. On the surface of the domain, this representation reduces to an integral equation for the source (or monopolar) and the doublet (or dipolar) density layers. In this problem the first is found by application of a linearized boundary condition and the second is the unknown over the surface of the domain. A low order panel method is used for the analytic integrations of the monopolar and dipolar influence coefficients. Then a non-symmetric dense linear system is obtained which is solved by preconditioned Krylov iterative methods, where the coefficient matrix is the sum of the dipolar influence matrix, plus the product of the monopolar influence and the difussive matrices. Both the dipolar and the monopolar influence matrices are evaluated with an exact field integral and they are full populated in general, whereas the difussive matrix is sparse. These properties are taken into account for an iterative solution where the principal CPU time cost is the evaluation of the coefficient matrix.

Key words: 3D BEM, potential flows, wave-drag, ships forms, free surface upwind technique, trailing gravity waves.

INTRODUCTION

The problem of wave making and wave resistance of ships is an old problem in naval engineering and with the advent of both the high speed digital computers and numerical methods, today it is possible to deal with it from the view point of the computational hydrodynamics. The presence of a free surface conduces to a non linear problem and the classic way to deal it, it is to solve a sequence of linear problems and we expect that their solutions converge to the solution of the original problem, where in this case the free surface is a part of the unknowns. At the present we only considered the linearized problem where we assume a ship-like body with constant forward velocity in an infinitely deep and calm uniform sea, as depicted in figure 1. As a first level of description we assume an inviscid 3D flow with a free surface and it is solved with an extended panel method, which it is an extension of the basic Morino's formulation originally proposed for subsonic potential aerodynamic about complex configurations. It is extended by us to include the presence of a free surface with gravity waves for both submerged and surface bodies. Our exposition will process in two steps. In the first step we employ the basic Morino's formulation to solve the so called freezed problem, where the zero position of the free surface is simply a plane and with this assumption we compute the velocity field over such plane. In the second step we employ the extended formulation proposed by us to compute a first correction of the position of the free surface, also called the first linearized surface problem. The previous computed velocity field over the plane is now an input to compute a numerical diffusive matrix inherent to this extended formulation, which results from an upwind surface density technique inspired from the upwind density techniques in transonic flows. In this paper we considered both the basic formulation for the freezed surface problem and the first linearized problem, whereas the upwind surface density technique is discussed in some detail in a companion paper presented at the same conference [1].

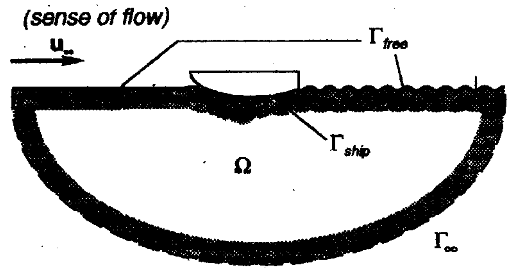

Figure 1: ship-like body with constant forward velocity in a infinitely deep and calm uniform sea.

FORMULATION FOR THE FREEZED SURFACE PROBLEM

In a first level of aerodynamics/hydrodynamics description we assume a 3D flow field inviscid, irrotational, incompressible and stationary, with the flow attached to the surfaces of the body and without propagation waves. With such idealizations the governing differential model is the Laplace differential equation for the total potential Φ^0 of the freezed problem

$$\nabla^2 \Phi^0 = 0 \quad \text{for} \quad x \in \Omega$$

$$\Phi^0(x) = \begin{cases} U_0 x + \phi & x \in (\Omega \cup \Gamma) \\ U_0 x + \phi^* & x \in (\Omega^* \cup \Gamma) \end{cases}$$

where U_0 is the external velocity, ϕ and ϕ^* are the external and internal perturbation potentials respectively, $\mathbf{x} = (x, y, z)$ is the position vector, Ω and Ω^* are the external and internal domains respectively, Γ is the boundary of Ω composed of $\Gamma = \Gamma_B + \Gamma_F + \Gamma_\infty$, where Γ_B is the hull portion of the ship wetted by the flow, Γ_F is the freezed surface assumed by us as known, i.e., a plane of equation $z = 0$, and Γ_∞ is an idealized hemisphere towards infinity.
The boundary conditions for the total potential velocity Φ^0 of the freezed problem are, i) over both the surface of the body Γ_B and the freezed surface the normal velocity V_n is prescribed (Neumann condition),

$$V_n = \frac{\partial \Phi^0}{\partial n} = f_1(x) \quad \text{for} \quad x \in (\Gamma_B + \Gamma_F)$$

ii) and at infinity the total potential velocity of the freezed problem tends to the external (imposed) potential,

$$\Phi^0(x) \to U_0 x \quad \text{for} \quad |x| \to \infty$$

From a mathematical view point it is an exterior Neumann problem [2] for the total potential Φ^0, and it is also equivalent to solve the perturbation potentials ϕ and ϕ^*. The solution in terms of these can be found with the Morino's formulation [3]. It is an integral representation based in the third Green's theorem which is applied for both outer and inner domains for the perturbation potentials ϕ and ϕ^* respectively and combining the resulting expressions we obtain,

$$\alpha\phi = -\frac{1}{4\pi}\int_{\Gamma-\Gamma_\infty} d\Gamma_y \frac{\partial}{\partial n}\frac{1}{r}\mu^0 - \frac{1}{4\pi}\int_{\Gamma-\Gamma_\infty} d\Gamma_y \frac{1}{r}\sigma^0$$

where $\alpha = 1/2$ for $x \in \Gamma - \Gamma_\infty$, $\alpha = 1$ for $x \in \Omega$, $\alpha = 0$ for $x \in \Omega^*$, $r = \|x - y\|_2$ is the Euclidean distance from the source point y to the field point x, $\mu^0 = \phi^* - \phi|_{\Gamma-\Gamma_\infty}$ is the source (or monopolar) surface density, both defined over the discontinuity surface $\Gamma - \Gamma_\infty$ for Φ, whereas at infinity Γ_∞ we assume that $\sigma^0(y) \to 0$ and $\mu^0(y) \to 0$ for $|y| \to \infty$. The super indices over the density layers denote that these are for the freezed problem. The usual specifications for the internal potential are $\phi^* = 0$ or $\phi^0 = U_0 x$. In aeronautical works cited by [4] has been reported that the zero internal perturbation potential option conduces to results of comparable accuracy to those from higher order panel methods for the same density control points. We opted for the first option, i.e., we imposed $\phi^* = 0$ and then the normal and tangential velocities from the external side of the wetted hull of the ship are,

$$V_n = (U_0, \hat{n}) + \left.\frac{\partial\phi}{\partial n}\right|_{\Gamma-\Gamma_\infty} = (U_0, \hat{n}) + \sigma^0$$

$$V_t = (U_0, \hat{t}) + \left.\frac{\partial\phi}{\partial t}\right|_{\Gamma-\Gamma_\infty}$$

where $\hat{n}$ is the normal exterior unit vector, $\hat{t}$ is a tangential unit vector over the exterior side of the body. For the freezed problem all the surfaces are impenetrables so the normal velocity is null $V_n = 0$, and then the monopolar density is found by direct application of the boundary conditions $\sigma^0 = (-U_0, \hat{n})$, whereas for the doublet density layer $\mu^0 = -\phi$ for $x \in \Gamma - \Gamma_\infty$. Then in the basic Morino's formulation for the freezed problem we have,

$$\frac{1}{2}\mu^0 - \frac{1}{4\pi}\int_{\Gamma-\Gamma_\infty} d\Gamma_y \frac{\partial}{\partial n}\frac{1}{r}\mu^0 = -\frac{1}{4\pi}\int_{\Gamma-\Gamma_\infty} d\Gamma_y \frac{1}{r}\sigma^0$$

which can be recognized as a Fredholm integral equation of second kind for μ^0 with a weak singularity and it can be written as,

$$[(1/2)\, I - A(x)]\, \mu^0 = -C(x)\sigma^0$$

$$A(x) = \frac{1}{4\pi}\int_{\Gamma-\Gamma_\infty} d\Gamma_y \frac{\partial}{\partial n}\frac{1}{r} \quad ; \quad C(x) = \frac{1}{4\pi}\int_{\Gamma-\Gamma_\infty} d\Gamma_y \frac{1}{r}$$

where $A(x)$ and $C(x)$ are the dipolar and monopolar influence operators, respectively and I is the identity operator. The solution of its associated discrete version is the basic panel method implemented by us.

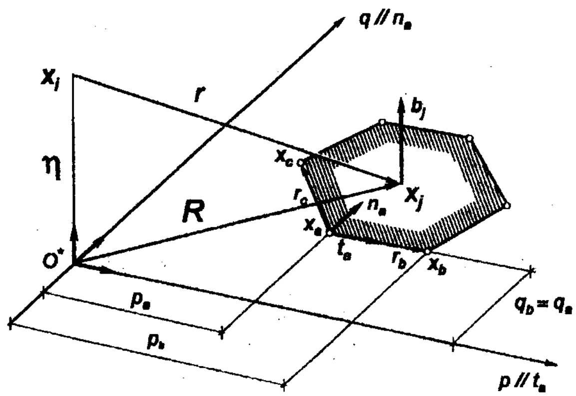

Figure 2: local tern p, q, η according to the side L^k.

FIRST LINEARIZED POTENTIAL PROBLEM

For the second step we will proceed to obtain a first displacement of the zero position of the previous freezed surface. Now the total potential Φ^1 for this first correction can be written as $\Phi^1 = U_0 x + \phi + \psi$ for $x \in \Gamma - \Gamma_\infty$, where ϕ and ψ are the exterior perturbation potentials of the freezed and the first correction problems, respectively, such that the first linearized velocity field is given by $U = U_v + \nabla\psi$, where U_v is the velocity field over the plane of the freezed problem, and $\nabla\psi$ is the first velocity field correction.
For ψ we have the Laplace equation $\Delta\psi = 0$ for $x \in \Omega$ with homogeneous Neumann boundary conditions over the body $\partial\psi/\partial n = 0$ for $x \in \Gamma_B$, whereas over the whole plane we now inject a "transpiration flux" σ such that,

$$\sigma^1 = \partial\psi/\partial n = -(U_v, \nabla\eta)$$

where η is the first incremental correction of the position of the free surface and is found from the application of the Bernoulli equation for unit density $U_0^2/2 = U^2/2 + g\eta$, where g is the gravity acceleration. Then,

$$\eta = \frac{1}{2g}(U_0^2 - U_v^2) - \frac{1}{g}(U_v, \nabla\psi)$$

and the "transpiration flux" is set to,

$$\sigma^1 = \frac{\partial\psi}{\partial n} = \frac{1}{2g}\,(U_v, \nabla U_v^2) + \frac{1}{g}\,(U_v, \nabla)^2\psi \quad ; \quad x \in \Gamma_F$$

Furthermore, because we are interested to obtain the wave drag coefficient C_x as a function of the Froude number $F_n = U_0/\sqrt{gL^*}$, where C_x is a nondimensional wave drag force and L is a characteristic length, we can define $U_v = \alpha_v U_{v1}$, where α_v is a nondimensional velocity factor, U_{v1} is the velocity field over the whole plane of the freezed problem for unit velocity at infinity, then the transpiration flux is null over the body $\sigma^1(x,\psi) = 0$ for $x \in \Gamma_B$, and over the plane it is given by,

$$\sigma^1(x,\psi) = \frac{\alpha_v^3}{2g}\,(U_{v1}, \nabla U_v^2) + \frac{\alpha_v^2}{g}\,(U_{v1}, \nabla)^2\psi \equiv \sigma_v + D\psi \quad ; \quad x \in \Gamma_F$$

where σ_v is a source term and D is the diffusive term. The resulting system of equations for $\mu^1 = -\psi$ is now,

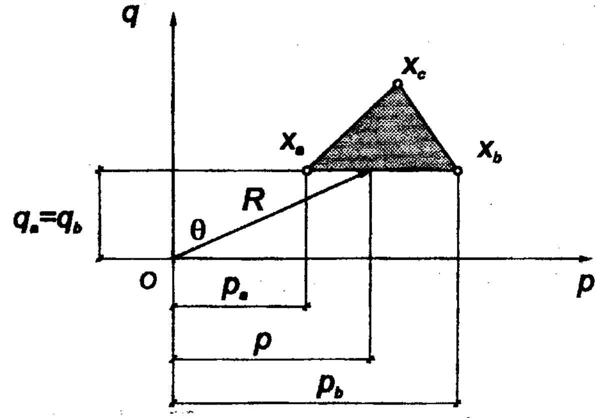

Figure 3: p, q plane according to the k-local tern of the side L^k.

$$[(1/2)\ I - A(x)]\ \mu^1 = -C(x)\ \sigma^1(x, \mu^1)$$

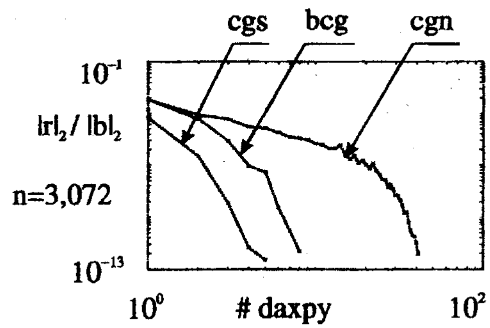

Figure 4: convergence history.

where $x \in \Gamma - \Gamma_\infty$. Thus, in this form we computed u_{v1} only once for the freezed problem and then we computed the solution for a discrete interval of Froude number $\{F_1, F_2, ..., F_i, .., F_n\}$ where for each F_i we assemble and solve the associate discrete system of algebraic equations.

BASIC AND EXTENDED PANEL METHOD

For a numerical solution we opted by a lower panel method where we approximate the assumed regular and finite surface $\bar{\Gamma} = \Gamma_p + \Gamma_b$ by polyhedral surface $\bar{\Gamma}_n$, where we cut the plane a great distance relative to the body and the surface toward infinity Γ_∞ it is not considered. Each face of the polyhedral surface is a flat panel (usually quadrilaterals or triangles). We employ a collocation technique to setup the discrete system of algebraic equations for both the freezed and the first linearized problems where the collocation points are the centroids of the panels. For the first we simply have,

$$(1/2\ I_{nn} - A_{nn})\mu_n^0 = -C_{nn}\sigma_n^0$$

where $A_{nn}, C_{nn} \in R^{n,n}$ are the dipolar and monopolar influences matrices, respectively, $I_{nn} \in R^{n,n}$ is the identity matrix, $\mu_n^0, \sigma_n^0 \in R^{n,1}$ are the dipolar and monopolar column vectors, and n is both the total panels number and the total collocation points present in the polyhedral surface $\bar{\Gamma}_n$. Whereas for the second,

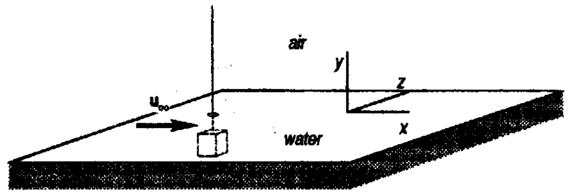

Figure 5: submerged cube.

$$\begin{bmatrix} 1/2\ I_{pp} + A_{pp} - C_{pp}D_{pp} & A_{pb} \\ A_{bp} - C_{bp}D_{pp} & 1/2\ I_{bb} + A_{bb} \end{bmatrix} \begin{bmatrix} \mu_p^1 \\ \mu_b^1 \end{bmatrix} = \begin{bmatrix} -C_{pp}\sigma_v \\ -C_{bp}\sigma_v \end{bmatrix}$$

where $D_{pp} \in R^{pp}$ is the diffusive matrix over the whole plane, A_{pp} and A_{bb} are the dipolar self-influence submatrices over the plane and over the body, respectively, A_{bp} and A_{pb} are the dipolar influence submatrices between them, C_{pp} is the monopolar self-influence matrix over the plane, C_{bp} and C_{pb} are the monopolar influence submatrices between plane and body.

DIPOLAR AND MONOPOLAR MATRICES COMPUTATION

Because each flat panel support both uniform monopolar and dipolar density layers, we can extract these from the discrete integral operators and then for $A_{nn} = [A_{ij}]$ and $C_{nn} = [C_{ij}]$ we have the following definitions

$$A_{ij} = \frac{1}{4\pi}\int_{\Gamma_j} d\Gamma_j\ \frac{(n_j, x_{ij})}{r_{ij}^3} \quad ; \quad C_{ij} = \frac{1}{4\pi}\int_{\Gamma_j} d\Gamma_j\ \frac{1}{r_{ij}}$$

where $r_{ij} = \|x_{ij}\|_2 = \|x_i - x_j\|_2$ is the Euclidean distance from the centroid x_j of the j - manantial panel of the area Γ_j to the collocation point x_i. We can be integrate these in closed form replacing the surface integrals over each flat manantial panel by a linear integral over its closed λ_j contour of m_j edges by means of the Stokes' theorem. Thus for a flat polygonal j-panel situated in an arbitrary position and a field point x_i in the three dimensional space, we consider a local tern according to the side L^k (see figures 2 and 3). The discretized expressions for both the dipolar and monopolar influence coefficients are computed by sums over the m_j edges,

$$A_{ij} = \sum_{k=1}^{m_j} A_j^k(p_a, p_b, q, x_i) \quad ; \quad C_{ij} = \sum_{k=1}^{m_j} C_j^k(p_a, p_b, q, x_i)$$

where $A_j^k(p_a, p_b, q, x_i)$ and $C_j^k(p_a, p_b, q, x_i)$ are the k - edge contribution of the j - mananantial panel for the collocation point x_i according to,

$$C_j^k = M(p_b, q, \eta) - M(p_a, q, \eta) \quad ; \quad A_j^k = D(p_b, q, \eta) - D(p_a, q, \eta)$$

with

$$p_a = (x_{k-1/2} - x_i) \cdot t^k$$

$$p_b = (x_{k+1/2} - x_i) \cdot t^k$$

$$q = (x_{k-1/2} - x_i) \cdot n^k$$

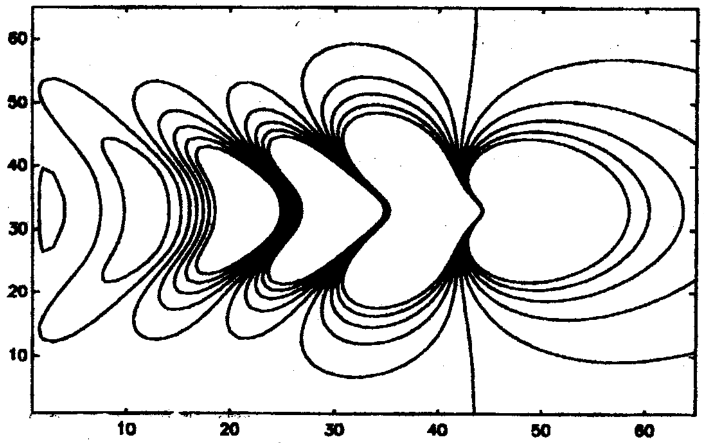

Figure 6: isolines of elevation for a submerged cube.

where p_b and p_a are the abscissas of the side L^k, q the common ordinate according to local dihedral p, q parallel to the side L^k and whose origin is the normal projection of the x_i field point, t^k and n^k are the local unit vectors of the side L^k content in the plan of the j-panel. The functions $M(p,q,\eta)$ and $D(p,q,\eta)$ are the transcendental functions,

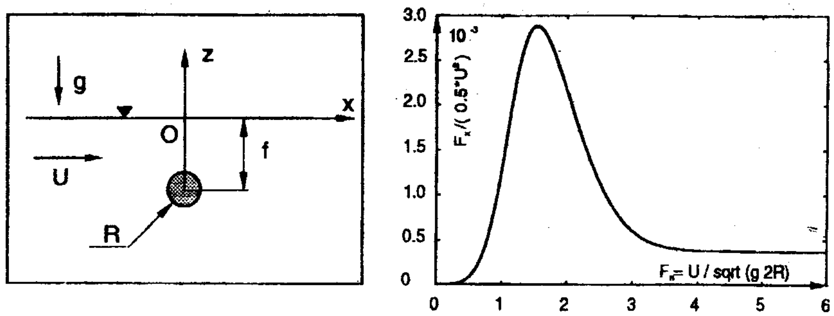

Figure 7: wave drag as a function of the Froude number for a horizontal circular cylinder obtained by the extended panel method.

$$M(p,q,\eta) = 2|\eta| \tan^{-1}\left[\frac{p - |\eta| - \sqrt{p^2+q^2+\eta^2}}{q}\right] - q \ln\left[p + \sqrt{p^2 + (q^2+\eta^2)}\right]$$

$$D(p,q,\eta) = 2 \ \mathrm{sg}(\eta) \tan^{-1}\left[\frac{p - |\eta| - \sqrt{p^2+q^2+\eta^2}}{q}\right]$$

DIRECT AND ITERATIVE SOLVERS

The dipolar and monopolar influence matrices A_n and C_n for both the basic and the extended panel method are in general full populated and non-symmetric, whereas the diffusive matrix D_{pp} of the extended method is sparse. In both cases we solve the algebraic system by direct and iterative methods. For the direct solution we have incorporated the LINPACK direct solver for a full populated matrix system, with LU decomposition and pivoting with backsustitution, whereas for the iterative solution we have implemented the following preconditioned Krylov iterative solvers: Generalized Minimal Residual (GMRES), Conjugate Gradients over Normal Equations (CGN), bi-Conjugate Gradients (bi-CG) and Conjugate Gradients Squared (CGS). As an example we have solved a delta wing of 5 % thickness with 3,072 panels, and the convergence history plot obtained with these iterative solvers it is shown in figure 4.

VECTORIZED ITERATIVE SOLVERS + BEM

For an iterative solution of the algebraic system of equations obtained by a BEM/panel discretization we will take in account that in general the system matrix is full populated. In an iterative solver where the system matrix is full populated, the principal cost are the vector-matrix product operation $y = Ax$ and the vector-matrix transpose product operation $y = A'x$ present in some solvers, where A is the system matrix, A' its transpose, x and y are some input and output vectors. These products must be evaluated a certain number of times in each iteration, for example, in Conjugate Gradients over Normal Equations (CGN) and Bi-Conjugate Gradients (BCG) solvers we have one of each class at each iteration, and for Conjugate Gradients Squared (CGS) we have two of the first class and none of the second one. Because the system matrix is in general dense and non symmetric we discard the possibility to store the full populated matrix in core memory, for example, with $n = 10,000$ panels we need 160 Mbytes of RAM in double precision arithmetic. Furthermore we also discard the disk storage and swapping because its relative lower performance. Then we choose to recompute the system matrix at each matrix-vector product operation and in principle this can be done by row or columns with a double nested loop. Another point to taken in account is that we compute the dipolar and monopolar matrices involving by summation over the side of manantial panel (and eventually geometric symmetries). From heuristic cost considerations we concluded that the lowest operations number is found when we compute the system matrix by columns. Then, in the system matrix A we have a double nested loop with a j-column loop for the all manantial panels and an i-row loop for the collocation points. The column oriented algorithms to be used in vectorial processors can be written as,

i) vector - matrix product operation:

$$y = Ax = A_1x(1) + A_2x(2) + ... + A_ix(i) + ... + A_nx(n)$$

vectorial Fortran semicode

```
y = 0
do j = 1, n
A_j
y = y + A_j x (j)
end do
```

i) vector - matrix transpose product operation:

$$y = A'x = \begin{bmatrix} (A_1, x) \\ (A_2, x) \\ \cdots \\ (A_i, x) \\ \cdots \\ (A_n, x) \end{bmatrix}$$

vectorized Fortran semi code

```
y = 0
do j = 1, n
A_j
do i = 1, n
y (j) = y (j) + A_j (i) x (i)
end do
end do
```

NUMERICAL EXAMPLES

Submerged Cube

Let us consider a cube of unit length submerged at unit depth from its upper face (see figure 5). On the freezed surface we only considered a portion of $1,000 \times 1,000$ length. The cube is off-centered a 20% towards the upwind direction. This is done to capture better the wave pattern. The Froude number based in the cube length is 4.5 whereas based in the plane length is 0.1. The mesh is 64 panels by 64 panels and the isolines of elevation are shown in figure 6. This case can be considered as an approximation of a point pressure perturbation, in which case the characteristic wave pattern should be independent of the details of the submerged body.

Submerged Circular Cylinder

In the following example we considered a submerged circular cylinder. The analytical wave drag force per unit transversal length W_L is,

$$W_L = \frac{4\pi^2 g^3 \rho R^4}{U^4} e^{-2gf/U^2}$$

where R is the cylinder radius, f the depth to its axis, g the gravity acceleration, U the non perturbed velocity. We have computed the numerical drag force obtained by the extended panel method implemented by us for a Froude number F interval and it is shown in figure 7 for a mesh with 300×3 panels over the freezed surface and 128 panels over the cylinder.

Semi-Submerged Sphere

We now consider a semi submerged sphere of unit radius with center over the freezed plane ($z = 0$). In the figure 8 we show the isolines of elevation field for a Froude number of $F = 4.5$ with a mesh with 45×15 panels in the circumferential and radial directions and 96 panels over the wetted submerged semi sphere.

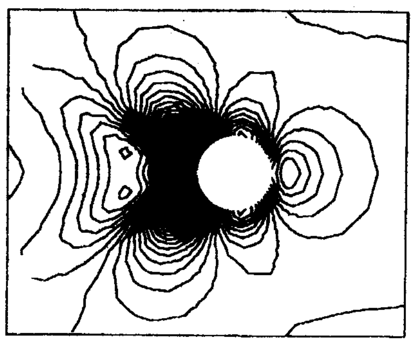

Figure 8: isolines of elevation for a semi submerged sphere at Froude number $F = 4.5$.

Scale Ship Model

Finally we consider a scale model of a ship at Froude number $F = 0.5$ whose isolines of elevation field are shown in figure 9.

CONCLUSIONS

A basic and an extended lower BEM/panel code it was presented. The solution of the wave drag problem in 3D hydrodynamics requires the extension of the basic panel method in order to incorporate the linearized free-surface. This is done by an upwind surface density or transpiration technique. The extended panel method is an interesting tool because we only need to discretize the surfaces of the wetted hull of the ship and a finite portion of the freezed surface.

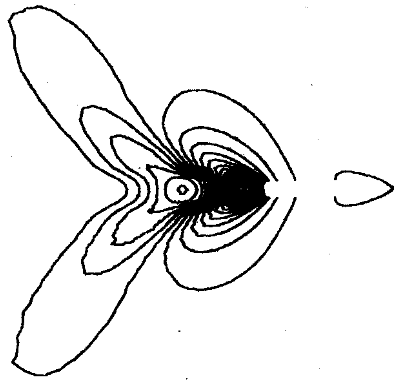

Figure 9: isolines of elevation for a scale ship model at Froude number $F = 0.5$.

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