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# FAILURE PROBABILITY OF ASIMPLE SDOF ELASTIC STRUCTURE SUBYECTED TO AN EVOLUTIONARY MODEL OF GROUND MOTION

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**Abstract.** *This work presents a study on the response of a linear structure to a non-stationary model of earthquake ground motion. This model is built up by simply adding band limited Gaussian distributed processes which are constructed by multiplying a deterministic function of time, called the strength function, and a stationary random process that provides the frequency content. The strength function gives de amplitude evolution in that frequency band.*

*The probabilistic structure of the response is obtained by computing the square mean value of both the displacement response and the velocity response, and the covariance between displacement and velocity from this results the statistics of the response spectrum is derived. Using this result, the influence in the response of many factors such as the earthquake duration and characteristic frequency is studied.*

## 1 INTRODUCTION

The response of a linear system excited by a non stationary random acceleration has been attempted by many authors in the past. Since the early works of Bycroft[1] who used stationary random processes in the evaluation of the response of a SDOF linear system, other works of different authors, like Bogdanoff et al.[2] and Lin[3] also used stationary processes to model the response of linear systems to earthquakes. Later on, the need for a more realistic representation of earthquake response, lead to many researcher to use non-stationary random processes. In this category are the works of Holamn and Hart[4] , Roberts[5], Crempien and Saragoni[6], In all this works, the seismic excitation was represented by a non-stationary but separable model of earthquake, this means that the frequency content of the earthquake was held constant. However, from simple ocular inspection is easy to see that in an acceleration record, the frequency content varies with time. Normally the high frequencies arrive first and lower frequencies arrive towards the end of the acceleration record. This is easy to see for example in the Orion 8244 record of the 1971 San Fernando earthquake which is shown in figure 1, and the record at Pacoima Dam in 1952, which is shown in figure 2.

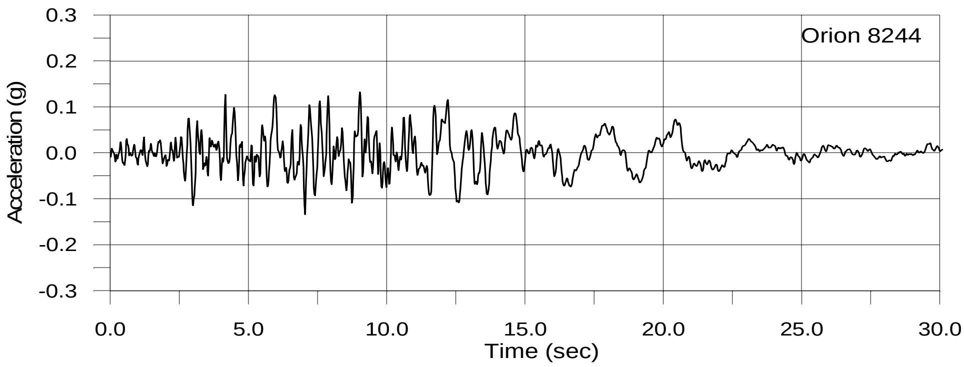


Fig. 1 Acceleration record obtained at Orion 8244, San Fernando Earthquake 1971.

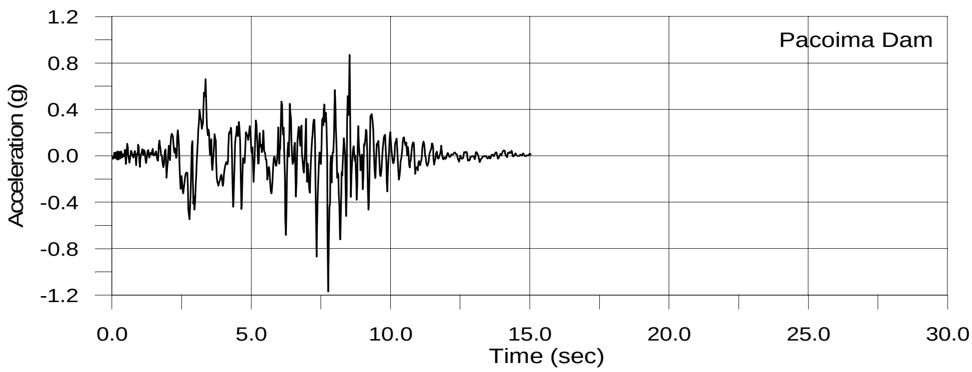


Fig. 2. Acceleration record obtained at Pacoima Dam San Fernando Earthquake 1971.

Based on the works of Priestley[7] and Rice[8], some authors have used more realistic models that take into account frequency evolution in time, but not for seismic structural response but rather for the field of aeronautic engineering. Der Kiureghian and Crempien[9] proposed a simple model to take into account frequency and amplitude evolution. This model is a generalization of Saragoni and Hart[10] model, but in this case it is no separable. In this simple model, the ground acceleration induced by an earthquake is given by:

$$\ddot{u}_g(t) = \sum_{k=1}^{m} \varphi_k(t) s_k(t) \qquad (1)$$

where, $\varphi_k(t)$ is a deterministic function called the strength function that represents the variance of the amplitudes in time in a determined frequency band given by: $D_k = [(-\omega_{k+1}, -\omega_k) \cup (\omega_k, \omega_{k+1})].$, and $s_k$(t) is a stationary random process with zero mean and unitary variance that provides the frequency content in the same frequency band. These frequency bands do not intersect between them.

In this model the strength functions have been chosen as gamma functions (Saragoni and Hart) given by:

$$\varphi_k(t) = \sqrt{\beta_k}\, e^{-\frac{\alpha_k}{2}} t^{\frac{\gamma_k}{2}} \qquad (2)$$

where $\alpha_k$, $\beta_k$, $\gamma_k$ are constants and they are the parameters of the model. This parameters are obtained by equating the expected temporal moments up to order 2 of each frequency component to the temporal moments of the components of a target accelerogram in the same frequency band, this is:

$$\int_0^{\infty} t^i \varphi_k^2(t) dt = \int_0^{t_f} t^i \ddot{u}_{gk}^2(t) dt \qquad i = 0,\ 1 \text{ and } 2 \qquad (3)$$

Crempien and Orosco[11] recommended to use 3 frequency bands of equal Arias[12], Intensity, the Arias intensity is given by:

$$I_{Ak} = \frac{2\pi}{g} \int_0^{t_f} \ddot{u}_{gk}^2(t) dt \qquad (4)$$

Using this method, the parameters of the model were obtained using as target records Pacoima Dam in 1952 and Orion 8244 in 1971. In figures 3 to 4 the actual records and simulated samples are shown, and also the parameters of the model are given tin table 1.

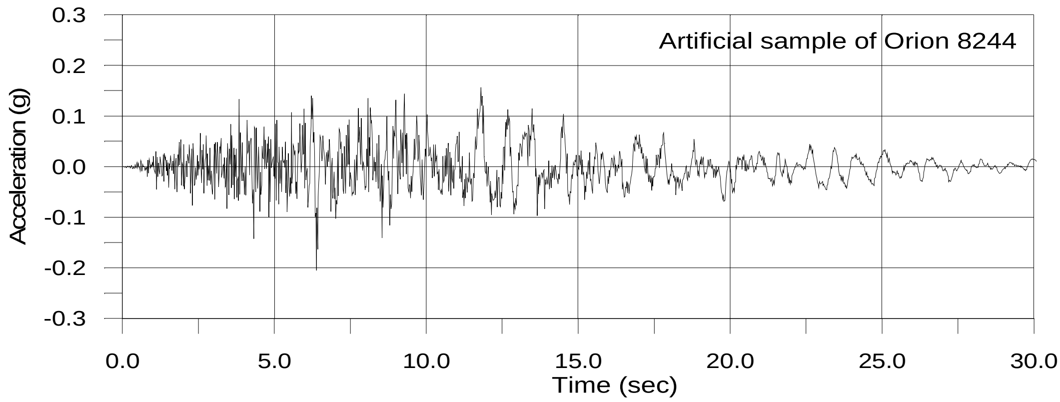


Fig. 3 Artificial acceleration record for the Orion 8244 record

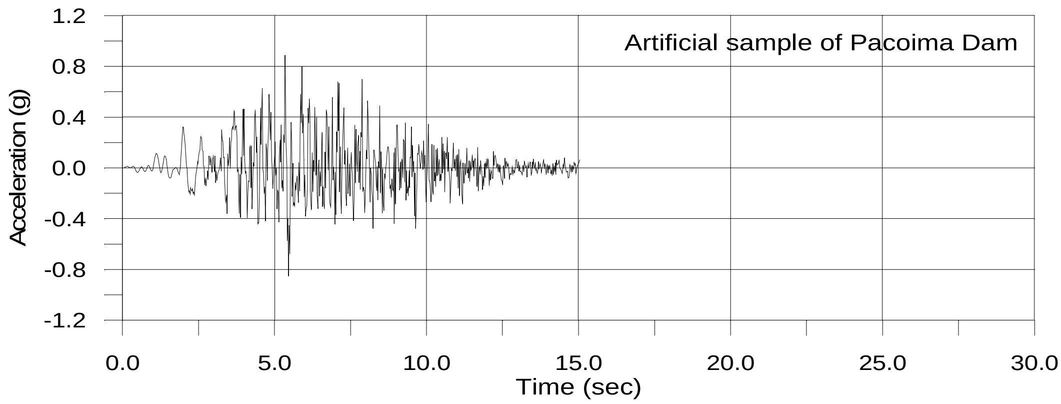


Fig. 4 Artificial sample for the Pacoima Dam accelerogram

## 2 STRUCTURAL RESPONSE

The structural response of a single degree of freedom oscillator to an earthquake model given by equation 1 is given by the solution of the motion equation:

$$\ddot{u}(t) + 2\eta\omega_n \dot{u}(t) + \omega_n^2 u(t) = -\sum_{k=1}^{m} \varphi_k(t) s_k(t) \qquad (5)$$

where $\eta$ is the fraction of critical damping of the structure and $\omega_n$ is the natural frequency of the structure. Because the excitation is Gaussian distributed, and the system linear, the response is a random process that it is also Gaussian distributed. This is important to note because the only thing needed to obtain the probabilistic structure of the response is the mean, which in this case is zero, and the variance that is given by the mean square response which can be computed directly from the Duhamel integral, this is:

$$\sigma^2(t) = E\{u^2(t)\} = E\{\int_0^t\int_0^t h_i(t-\tau_1)h_i(t-\tau_2)\sum_{k=1}^{m}\sum_{l=1}^{m}\varphi_k(\tau_1)\varphi_l(\tau_2)s_k(\tau_1)s_l(\tau_2)d\tau_1 d\tau_2\} \tag{6}$$

Now, without loss of generality, the stationary stochastic processes, $s_k$ and $s_l$ can be selected such that they be orthogonal in the expected value sense, this is:

$$E\{s_k(t)s_l(t)\} = \delta_{kl} \tag{7}$$

where $\delta_{kl}$ is the Kroenecker delta. With this in mind, equation (6) becomes:

$$\sigma^2(t) = \int_0^t\int_0^t h_i(t-\tau_1)h_i(t-\tau_2)\sum_{k=1}^{m}\varphi_k(\tau_1)\varphi_k(\tau_2)R_{ss}(\tau_2,\tau_1)d\tau_1 d\tau_2 \tag{8}$$

where $R_{ss}(\tau_2,\tau_1) = E\{s_k(\tau_1)s_k(\tau_2)\} = R_{ss}(\tau_2-\tau_1)$ which is the autocorrelation function of the stationary random process *$s_k(t)$* and that it depends on the time difference between $\tau_2$ and $\tau_1$. Now, taking into account the Wiener-Kchintchine theorem, the autocorrelation function can be written in terms of the power spectral density function (PSDF)

$$R_{ss_k}(\tau_2,\tau_1) = \int_{D_k} S_k(\omega)e^{i\omega(\tau_2-\tau_1)}d\omega \tag{9}$$

here, $i = \sqrt{-1}$, replacing that last equation in equation (8), and rearranging it one obtains:

$$\sigma^2(t) = \sum_{k=1}^{m}\int_{D_k} S_k(\omega)\|I_k(\omega,t)\|d\omega \tag{10}$$

In this last expression

$$I_k(\omega,t) = \int_0^t h_i(t-\tau)\varphi_k(\tau)e^{-i\omega\tau}d\tau \tag{11}$$

is the evolutionary impulse response function of the system to the random process *k*. To obtain the variance it is necessary to evaluate these expressions. However, the closed form solution for equation 11 first and for equation 8 later is quite complicated to obtain, therefore it is necessary to implement a numerical method to evaluate them.

## 3 NUMERICAL EVALUATION OF THE MEAN QUADRATIC RESPONSE

Equation 11 has a explicit form given by:

$$I_k(\omega,t)=\int_0^t \frac{1}{\omega_A} e^{-\eta\omega_n(t-\tau)} sen\,\omega_a(t-\tau)\varphi_k(\tau)e^{i\omega\tau}d\tau \tag{12}$$

This last expression has a closed form solution when $\varphi_k(t)$ is a step function, this is

$$\varphi(t)=\begin{cases}0 \text{ if } t<0\\ 1 \text{ if } t\geq 0\end{cases} \tag{13}$$

for this kind of strength function, Coughey and Stumpf[13] obtained the following result:

$$\left\|I(\omega,t)\right\|^2=\left\|H(\omega)\right\|^2\left\{1-2e^{-\eta\omega_n t}\left(\eta\frac{\omega_n}{\omega_A}\cos\omega t\sin\omega_A t+\cos\omega t\sin\omega_A t+\frac{\omega}{\omega_A}\sin\omega t\cos\omega_A t\right)\right.$$

$$\left.+e^{-2\eta\omega_n t}\left(\frac{\eta^2\omega_n^2+\omega^2}{\omega_a^2}\sin^2\omega_A t+\cos^2\omega_A t+2\eta\frac{\omega_n}{\omega_A}\sin\omega_A t\cos\omega_A t\right)\right\} \tag{14}$$

In our case, for the strength function given by a Gamma function, the integrals become indefinite Pearson integrals, and their analytical evaluation is much more complex.

Equation 12 can be manipulated algebraically to obtain

$$I_k(\omega,t)=\frac{e^{-\eta\omega_n t}}{\omega_A}\left\{\sin\omega_A t\int_0^t e^{(\eta\omega_n-i\omega)\tau}\cos\omega_A\tau\cdot\varphi_k(\tau)d\tau-\cos\omega_A t\int_0^t e^{(\eta\omega_n+i\omega)\tau}\sin\omega_A\tau\cdot\varphi_k(\tau)d\tau\right\} \tag{15}$$

Now, the strength function is a slow varying function in comparison to the natural period of vibration of the structure, this means that for a small enough interval of time, $\varphi_K(t)$ can be considered constant and the integral can be approximated by:

$$I_1(\omega,t)=\int_0^t e^{(\eta\omega_n-i\omega)\tau}\cos\omega_A\tau\cdot\varphi_k(\tau)d\tau=\sum_{j=1}^{n}\varphi_k(j\Delta t)\int_{(j-1)\Delta t}^{j\Delta t}e^{(n\omega_n-i\omega)\tau}\cos\omega_A\tau d\tau \tag{16}$$

$$I_2(\omega,t)=\int_0^t e^{(\eta\omega_n+i\omega)\tau}\sin\omega_A\tau\cdot\varphi_k(\tau)d\tau=\sum_{j=1}^{n}\varphi_k(j\Delta t)\int_{(j-1)\Delta t}^{j\Delta t}e^{(n\omega_n+i\omega)\tau}\sin\omega_A\tau d\tau \tag{17}$$

Therefore, equation 14 becomes:

$$I_k(\omega,t)=\frac{e^{-\eta\omega_n t}}{\omega_A}\sum_{j=1}^{n}\varphi_K(j\Delta t)\left\{\sin\omega_A t\int_{(j-1)\Delta t}^{j\Delta t}e^{(\eta\omega_n-i\omega)\tau}\cos\omega_A\tau d\tau-\cos\omega_A t\int_{(j-1)\Delta t}^{j\Delta t}e^{(\eta\omega_n+i\omega)\tau}\sin\omega_A\tau\cdot d\tau\right\} \tag{18}$$

This last expression can be further arranged and gives:

$$\begin{aligned}I_K(\omega,t)=\frac{e^{-i\eta\omega_n t}}{\omega_A}\sum_{j=1}^{n}\varphi_K(j\Delta t)\Bigg[&\left\{\sin\omega_A t\left(\frac{e^{(\eta\omega_n-i(\omega-\omega_A))j\Delta t}}{\eta\omega_n-i(\omega-\omega_A}\right)\left[1-e^{-(\eta\omega_n-i(\omega-\omega_A))\Delta t}\right]\right\}\\ &-\left\{\cos\omega_A t\left(\frac{e^{(\eta\omega_n-i(\omega+\omega_A))j\Delta t}}{\eta\omega_n-i(\omega+\omega_A}\right)\left[1-e^{-(\eta\omega_n-i(\omega+\omega_A))\Delta t}\right]\right\}\Bigg]\end{aligned} \tag{19}$$

Equation 19 is a complex equation. The modulus of this expression is plotted for several periods and fractions of critical damping in figure 5 and 6, for a strength function equal to a unitary step function for comparison purposes with the results obtained by Caughey and Stumpf. In these figures is easy to observe that $\|I(\omega,t)\|$ starts as a very wide function of frequency and that as time evolves it gets concentrated at the natural frequency, tending to the frequency response function of the oscillator. For this reason the function $I(\omega,t)$ is often called the evolutionary frequency response function.

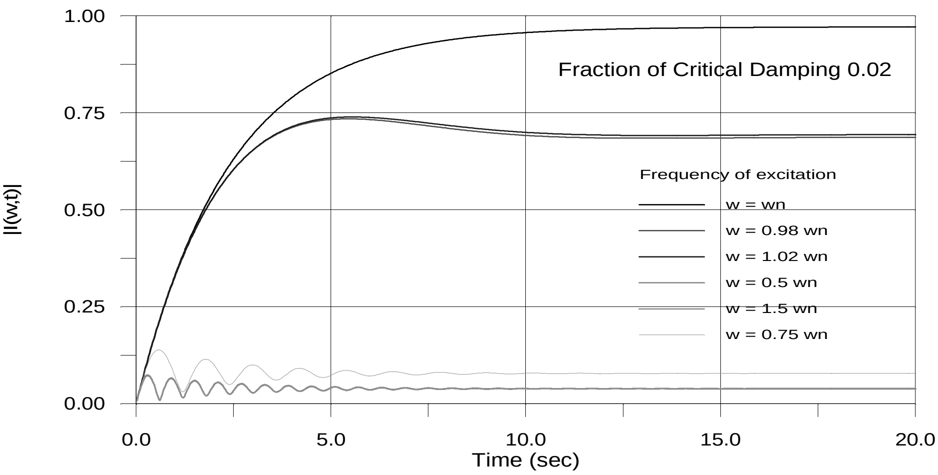


Fig. 5. Evolutionary Impulse Response Function for a step strength function evaluated for a structure with 0.02 of critical damping for different excitation frequencies.

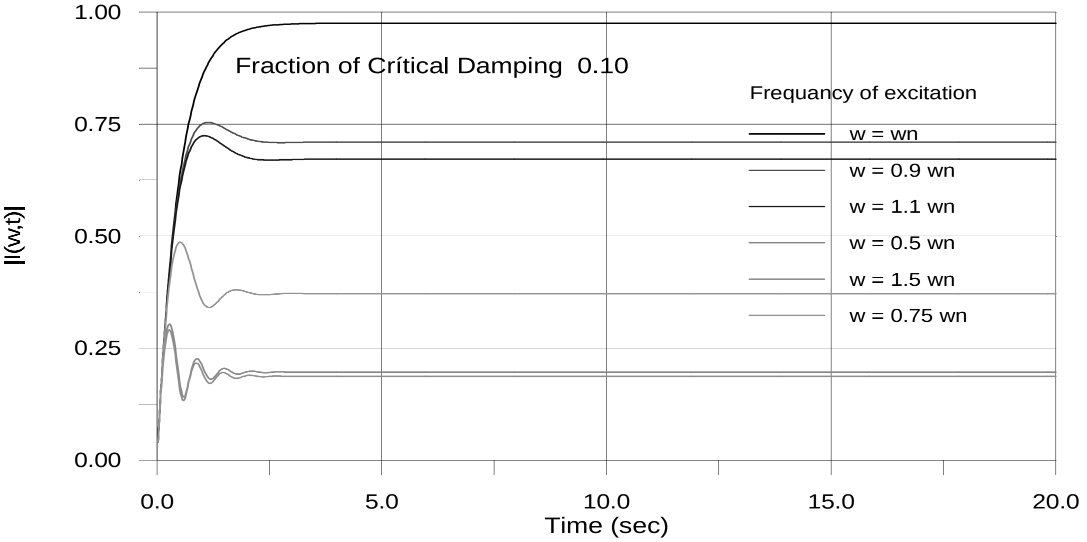


Fig. 6. Evolutionary Impulse Response Function for a step strength function evaluated for a structure with 0.10 of critical damping for different excitation frequencies

From figures 5 and 6 it can be seen that for larger damping the faster that $I(\omega,t)$ gains a permanent value.

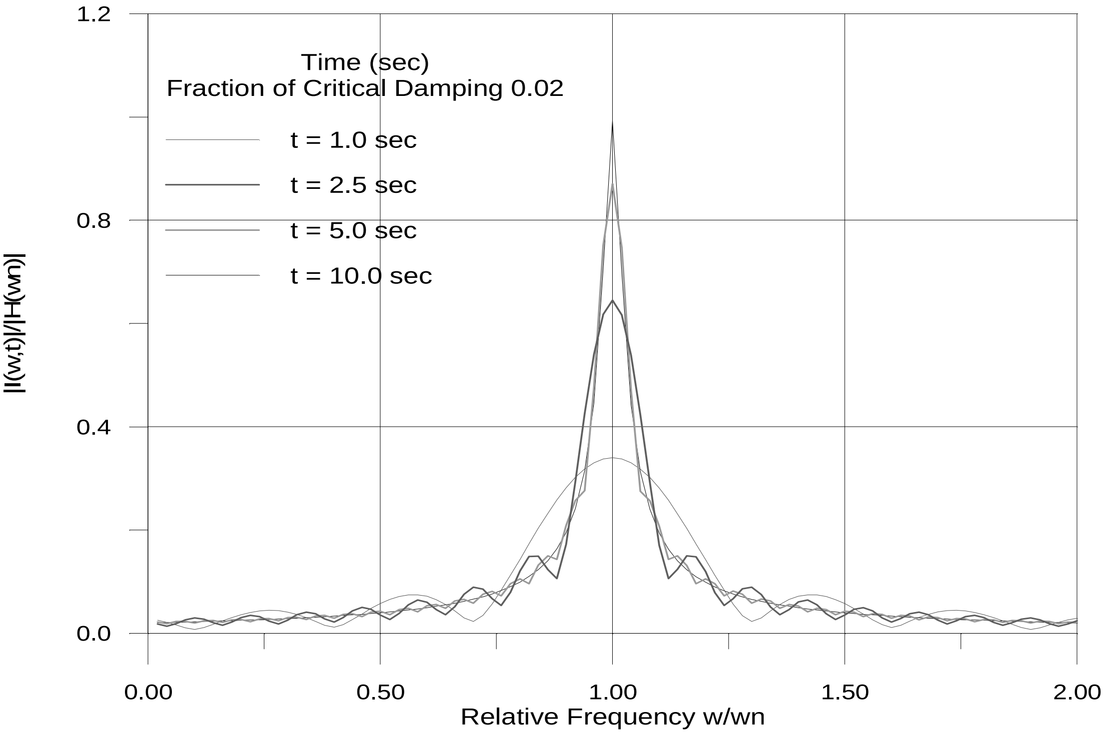


Fig. 7. Function $I(\omega,t)$ evaluated at different times and as a function of the relative frequency

In figure 7 it is possible to observe that the evolutionary frequency response function evolves to $H(\omega)$ as t increases, and that a the beginning the system does not behave has a narrow band system.

In figure 8 and 9 $I(\omega,t)$ is plotted for the Gamma type strength function. In this case it is also easy to observe that $I(\omega,t)$ has a less concentrated behavior than for the step function case. This means that the oscillator tends to respond almost without discrimination to all frequencies at the beginning. Another feature of the modulus of $I(\omega,t)$ is that it is symmetric with respect to the origin. This means that the integral over the frequency domain in equation 10 can be evaluated using just the positive region of the frequency axis.

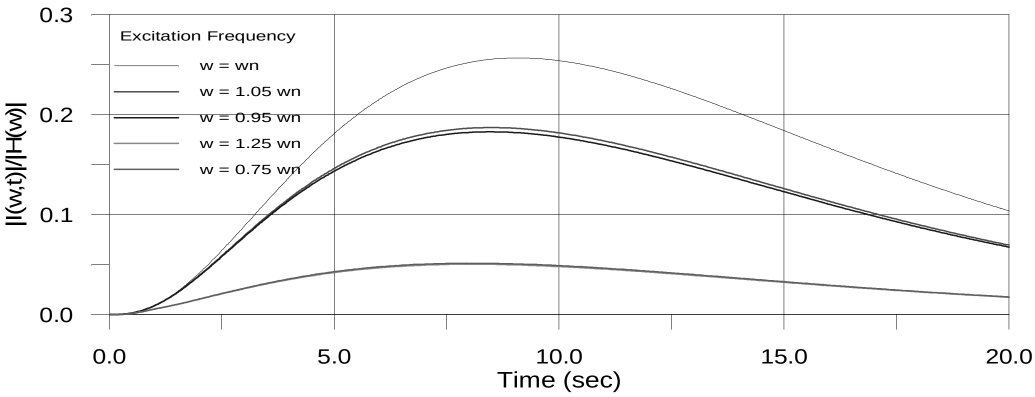


Fig. 8. Evolutionary Frequency Response Function for the parameters of Orion Radial, evaluated at different excitation frequencies.

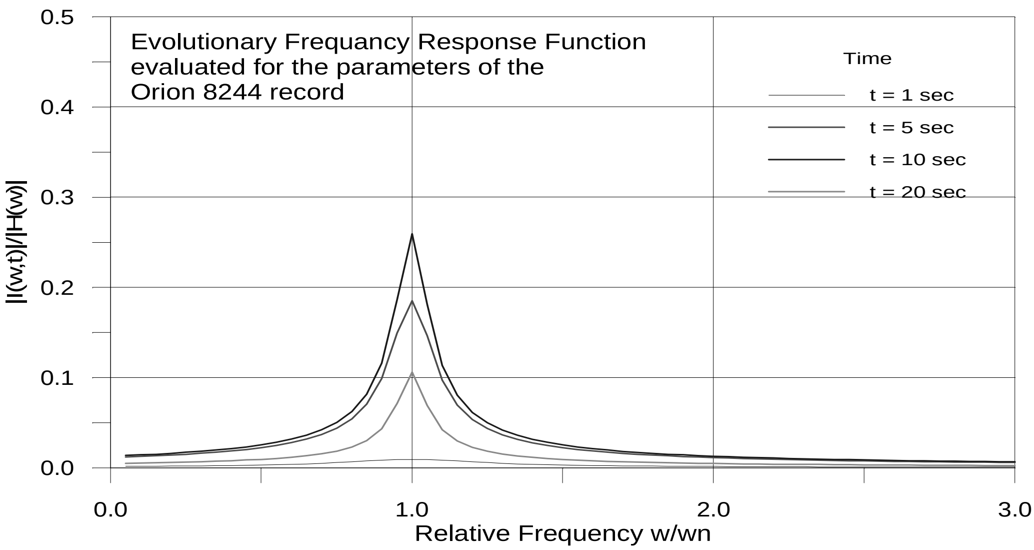


Fig. 9 Evolutionary Frequency Response Function versus relative frequency for the parameters of Orion Radial plotted for different times of evolution.

To obtain the variance $\sigma^2(t)$ a simple approach was selected and a numerical scheme based on Simpson rule is used along the frequency domain. This approach was also followed to compute the covariance function of the velocity and displacement of the structure and also for the mean square value function of the velocity. For this purpose the following derivatives of time for $E\{u^2(t)\}$ wre computed:

$$\frac{dE\{u^2(t)\}}{dt} = 2E\{\dot{u}(t)u(t)\} = 2\sum_{k=1}^{m}\int_{D_K} S_k(\omega)\{\dot{I}_K(\omega,t)\bar{I}_K(\omega,t) + I_K(\omega,t)\dot{\bar{I}}_K(\omega,t)\}d\omega \qquad (20)$$

where $\dot{I}(\omega,t) = \frac{d}{dt}I(\omega,t)$ and can be obtained from the same method of computation for $I(\omega,t)$. The computation of $E\{\dot{u}(t)u(t)\}$ follows the same procedure that the computation of $E\{u^2(t)\}$ in the integration over the frequency domain. Finally, the computation of the expected square value of the velocity response can be obtained from the autocorrelation function of the displacement response, this is:

$$E\{u(t_1)u(t_2)\} = \sum_{k=1}^{m}\int_{D_k} S_K(\omega)I_K(\omega,t_1)\bar{I}_K(\omega,t_2)d\omega \qquad (21)$$

Taking the second partial derivative with respect to $t_1$ and $t_2$ the following relation is obtained:

$$\frac{\partial^2 E\{u(t_1)u(t_2)\}}{\partial t_1 \partial t_2} = E\{\dot{u}(t_1)\dot{u}(t_2)\} = \sum_{k=1}^{m}\int_{D_K} S_K(\omega)\dot{I}_K(\omega,t_1)\dot{\bar{I}}_K(\omega,t_2)d\omega \qquad (22)$$

This last expression for the case $t_1=t_2$ becomes the mean square value of the velocity response of the structure.

$$E\{\dot{u}^2(t)\} = \sum_{k=1}^{m}\int_{D_K} S_K(\omega)\left\|\dot{I}_K(\omega,t)\right\|^2 d\omega \qquad (23)$$

which again can be computed using the same technique used for the mean square displacement. This results are plotted for the case of the model with gamma functions for the strength functions adjusted to the target records of Orion 8244 and Pacoima Dam.

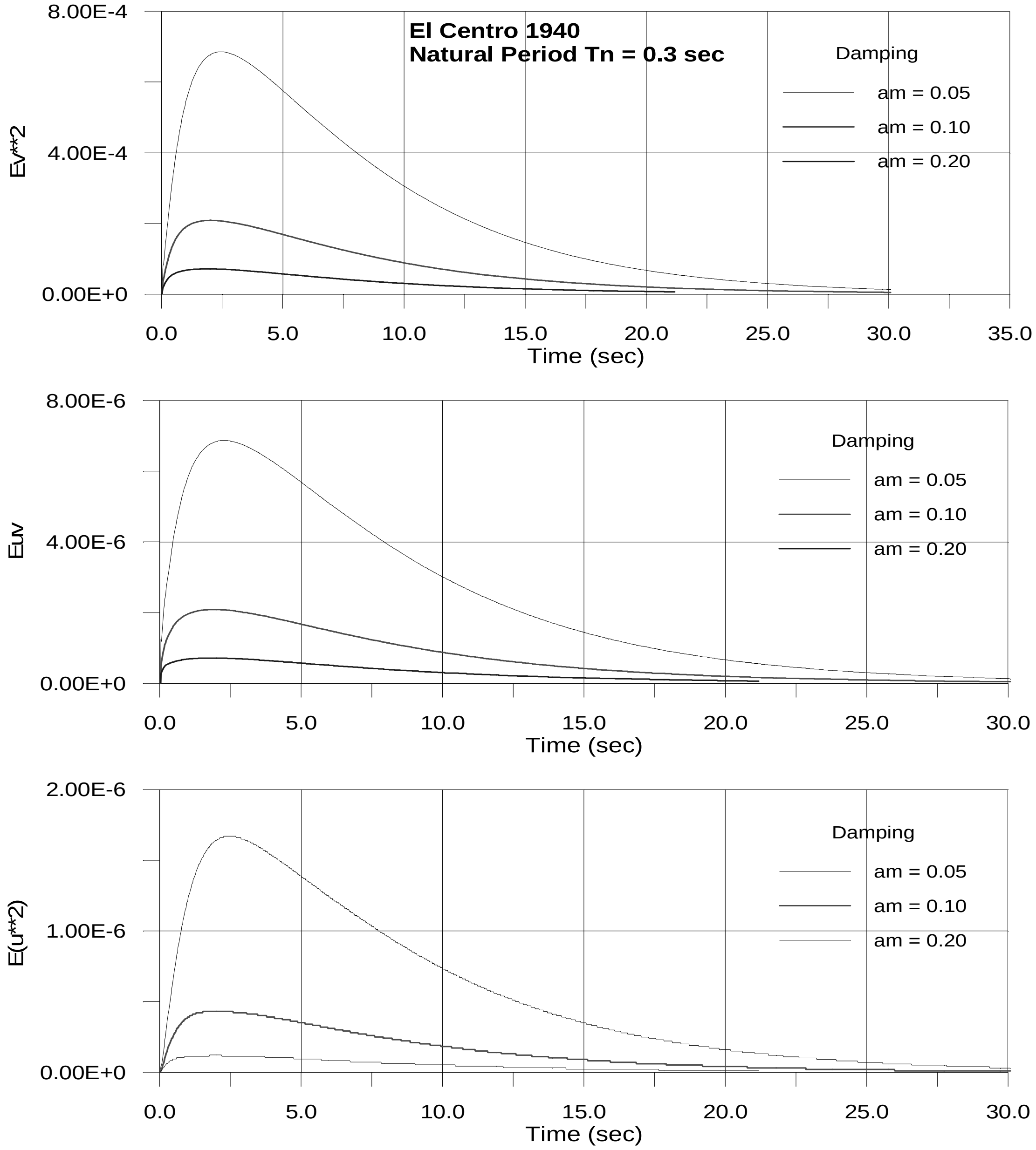


Fig. 11. Square mean value for displacement and velocity and expected value fo the velocity displacement product for El Centro 1940 model

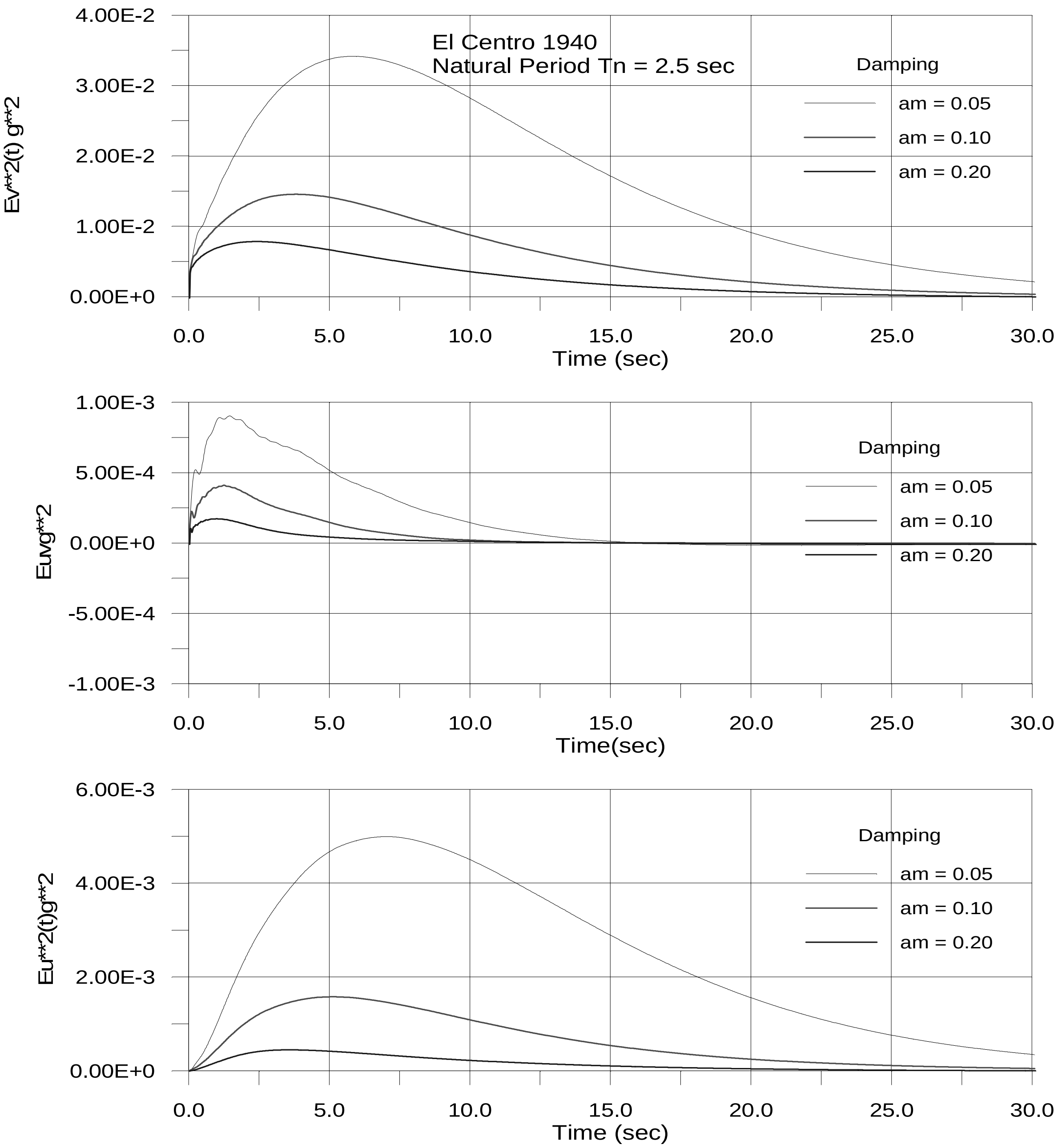


Fig. 12. Square mean value for displacement and velocity and expected value fo the velocity displacement product for El Centro 1940 model. Natural period of vibration Tn = 2.5 sec

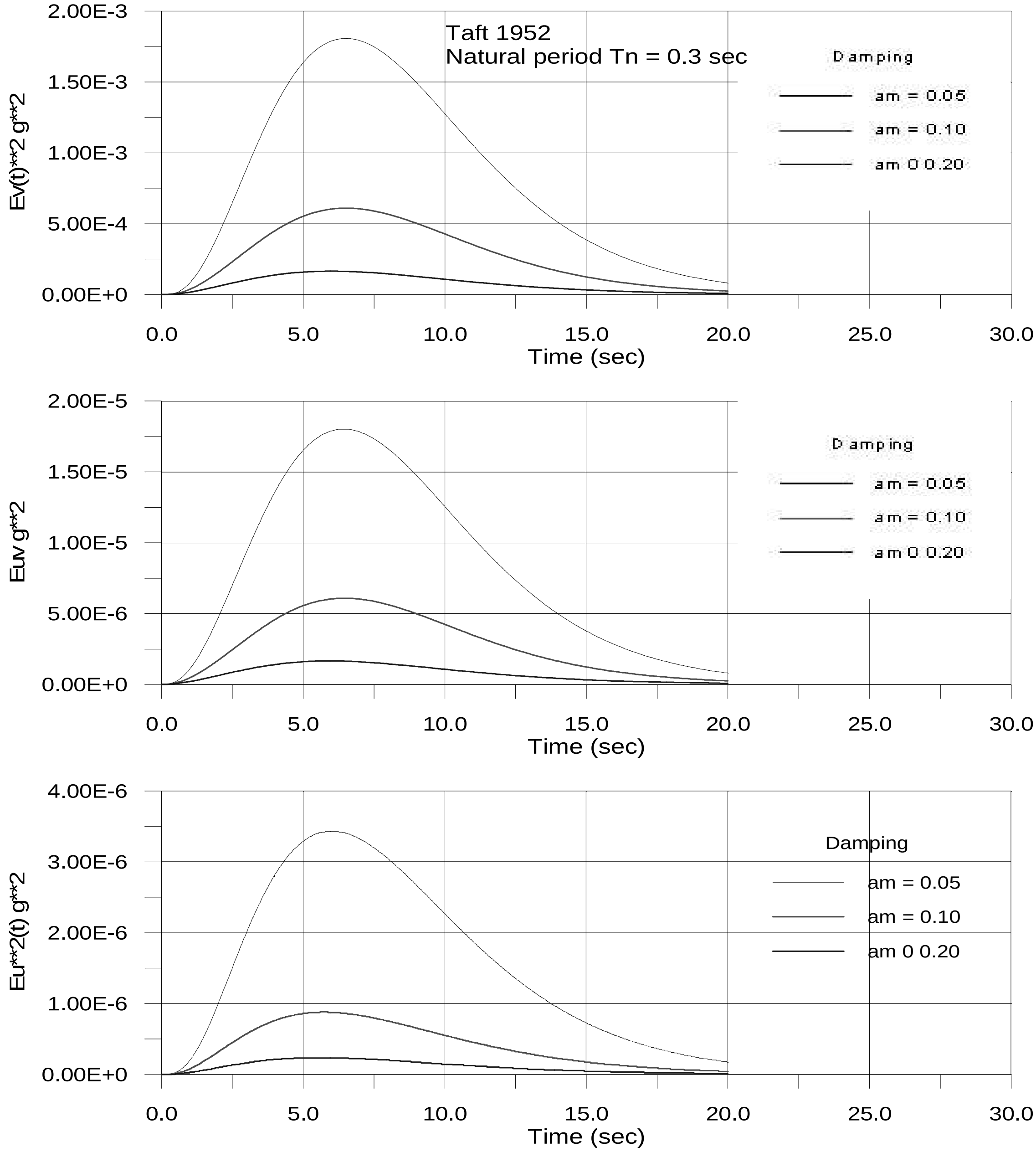


Fig. 13. Square mean value for displacement and velocity and expected value fo the velocity displacement product for Taft 1952 model. Natural period of vibration Tn = 0.3 sec

## 4 PROBABILISTIC STRUCTURE OF AVERAGE RESPONSE SPECTRUM

Once the probabilistic structure of second order is known for the response of the structure it is possible to study the probability structure of the maximum displacement response, this is the displacement spectra.

This studies are related to the study of the crossing of a random displacement process ver a barrier *b.* Rice is the first to study this problem using the principle of the inclusion and exclusion series. This principle gives the exact solution to the problems of computing the probability density function of the first crossing of a barrier *b*. Unfortunately it is no easy to compute the terms of these series. To develop these ideas the main assumption is that the crossings occur independently of each other and that can be modeled as Poisson Processes.

In this same line of thinking Racicot and Moses[14] , proposed a numerical technique to approximately compute the probability of the first crossing for the case of simple linear oscillators. However, Lyon[15] suggested the Poisson Process assumption was not satisfactory because the crossings of a random process over a barrier *b* occur in groups, which implies a dependency between crossings. Using Lyons work, Vanmarke[16] defined an improved method to compute the probability of the first crossing. Also using stationary random processes, Shinozuka[17] first, and Shinozuka and Yang[18] in al latter work and also using stationary random processes computed upper and lower bounds to the probability of the first crossing of a process over a barrier b in a small interval of time. Roberts[19] worked this same idea with non stationary random processes. Yang[20], using these previous studies, compared all the available methods to that date with results obtained from numerical simulation and he concluded that those methods that take into account the grouping of maxima give better results. In all this research the moments of 0 to $2^{nd}$ order of the PSDF play an important role after Vanmarke.

Lutes, Chen and Tzuang[21] gave simple expressions to compute the probability of the first crossing for a special class of problems of a 1DOF system excited by a filtered white noise normally distributed. In these studies the problem of non-stationary excitation is also included for small durations in the same direction are the works by Crandall[22], and Lutes and Chokshi[23].

Der Kiureghian[24] developed a method por the computation of expected response spectrum using theory of stationary random vibration for MDOF linear systems. This method is based in the assumption that the input is wide band Gaussian distributed and the response is stationary. However, this can be used as a good approximation for the response to an excitation also Gaussian but with a duration longer then the natural period of vibration of the structure. Langley[25] assuming the crossings over a barrier follow a Poisson law, finds for non-stationary random processes the probabilistic structure of the response.

## 5 PROBABILITY FUNCTION OF THE RESPONSE SPECTRUM OF A LINEAR STRUCTURE

The expected maximum value of the response of a structure $u_m$ or average response spectrum is defined as the maximum response that is expected that happens only once in a finite interval of time $t_f$. The computation of $u_m$ can be solved using the following approach.

If the excitation is Gaussian distributed, and the structure elastically linear the response process is also Gaussian distributed with zero mean and the second order moment given by the cross correlation matrix. The process of crossings over a barrier *b* can be assumed to be a Poisson random process with non-stationary increments.

The probability that in a small interval of time dt there is a single upward crossing is given in terms of the Gaussian density function of the response variable $u, \dot{u}$ $p_{u,\dot{u}}$ is:

$$\nu^{+}(t)dt = \int_{0}^{\infty} \int_{b-\dot{u}dt}^{b} p_{u\dot{u}}(u(t), \dot{u}(t), t) d\dot{u} du \tag{24}$$

Now, because $dt$ is small, this equation becomes:

$$\nu^{+}(t) = \int_{0}^{\infty} \dot{u} p_{u\dot{u}}(b, \dot{u}(t), t) d\dot{u} \tag{25}$$

where $\nu^{+}$ determines the crossing from the safe region $u(t) \leq b$ to the unsafe region $u(t) > b$ or failure region.
In the same way, for the event f a crossing from the safe region $[-b, 0]$ to the unsafe region $(-\infty, -b)$ we have:

$$\nu^{-} = \int_{-\infty}^{0} \dot{u}(t) p_{u\dot{u}}(b, \dot{u}(t), t) d\dot{u} \tag{26}$$

where the probability density function for the Gaussian jointly distributed variables of displacement $u(t)$ and velocity $\dot{u}(t)$ is given by:

$$p_{u\dot{u}}(u, \dot{u}, t) = \frac{1}{2\pi\sigma_u(t)\sigma_{\dot{u}}(t)(1-\rho_{u\dot{u}}(t))^{1/2}} \exp\left\{-\frac{1}{2}(1-\rho_{u\dot{u}})^{1} \times \left[\left(\frac{u(t)}{\sigma_u(t)}\right)^2 - \frac{2\rho_{u\dot{u}}(t)u(y)\dot{u}(t)}{\sigma_u(t)\sigma_{\dot{u}}(t)} + \left(\frac{\dot{u}(t)}{\sigma_{\dot{u}}(t)}\right)^2\right] - \right\} \tag{27}$$

Combining together equations 28, 29 and 30 for the case of a symmetric barrier crossing problem is:

$$\nu(b,t) = \nu^{+}(t) + \nu^{-}(t) \tag{28}$$

which gives:

$$\begin{aligned}\nu\big((b,t)\big) = {} & W\exp(Gb^2)\int_0^{\infty}\dot{u}(t)\exp(-J\dot{u}^2(t)+Ab\dot{u}(t))d\dot{u} - \\ & W\exp(-Gb^2)\int_{-\infty}^{0}\dot{u}(t)\exp(-J\dot{u}^2(t)+Ab\dot{u}(t))d\dot{u}\end{aligned} \tag{29}$$

where the terms W, J, G, and A are given by:

$$\begin{aligned} W &= \frac{1}{2\pi\sigma_u(t)\sigma_{\dot{u}}(t)(1-\rho_{u\dot{u}}(t)^2)^{\frac{1}{2}}} \qquad & J &= \frac{1}{2(1-\rho_{u\dot{u}}{}^2(t))\sigma_{\dot{u}}^2(t)} \\ G &= \frac{1}{2(1-\rho_{u\dot{u}}{}^2(t))\sigma_u^2(t)} \qquad & A &= \frac{1}{\sigma_u(t)\sigma_{\dot{u}}(t)(1-\rho_{u\dot{u}}^2(t))}\end{aligned} \tag{30}$$

and the correlation function is:

$$\rho_{u\dot{u}}(t) = \frac{E\{u(t)\dot{u}(t)\}}{\sigma_u(t)\sigma_{\dot{u}}(t)} \tag{31}$$

Using tables (Gradshteyn and Ryzhik, 1965) the following result is obtained for the crossing of the displacement response *u(t)* over the barrier *b*:

$$\begin{aligned}\nu(b,t) = W\exp(-Gb^2)\Bigg[ & \frac{1}{2J} + \frac{Ab}{4J}\sqrt{\pi/J}\exp(A^2b^2/4J)(1-\Phi(-\frac{Ab}{2\sqrt{J}})) \\ & + \frac{1}{2J} - \frac{Ab}{4J}\sqrt{\pi/J}\exp(\frac{A^2b^2}{4J})[1-\Phi(\frac{Ab}{2\sqrt{J}})]\Bigg]\end{aligned} \tag{32}$$

where $\Phi(x)$ is the probability integral defined by:

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt \qquad (33)$$

and can be obtained from tables or computer routines. Because of the form of the probability integral the last equation can be reduced to:

$$\nu(b,t) = \frac{W}{2J}\exp(-Gb^2)\left[1+\sqrt{\pi\frac{b^2A^2}{4J}}\exp\left(\frac{b^2A^2}{4J}\right)\left(1-2\Phi\left(\frac{bA}{2\sqrt{J}}\right)\right)\right] \qquad (34)$$

To obtain the total number of crossings of the response process u(t) over the barrier b in a finite interval of time, it is necessary to integrate expression 40 over the time interval T , this is:

$$N(b,T) = \int_0^T \nu(b,t)dt \qquad (35)$$

Under the Poisson process model assumption for the crossings the probability that $\boldsymbol{t_f}$ be the time of the first crossing of the response process above the barrier b is given by:

$$P(T = t_f) = \exp\left\{-\int_0^{t_f} \nu(b,t)dt\right\} \qquad (36)$$

This probability is also the probability that b be the maximum value of the response process in the interval $\left[0,\boldsymbol{t_f}\right]$. If $\boldsymbol{t_f}$ is taken to be the time length of the duration of the earthquake, then $|\boldsymbol{b}|$ is the absolute value of the response with probability **P( b )** of not being exceeded, or in other words **P (b)** is the Probability that **b** be the maximum response in absolute value in the time interval $\left[0,\boldsymbol{t_f}\right]$.

Now, from $P(u \le b)$ it is possible to obtain the probability density function of the maximum just deriving equation (4.18) with respect to b, this is:

$$p_b(b) = \frac{\partial P_b(b)}{\partial b} = \frac{1}{N(0,T)}\int_0^T \frac{\partial \nu(b,t)}{\partial b}dt \qquad (37)$$

In figures 14 and 15 the cumulative probability for the maximum displacement are shown for the case of a structure excited by the family of random process generated by the record of el

Centro 1940. for a natural period of vibration Tn = 0.3 sec and 0.6 sec and for different damping. The larger the damping the probability density function is more concentrated and the cumulative probability function is shifted to the origin.. In figure 15 the same effect can be noticed.

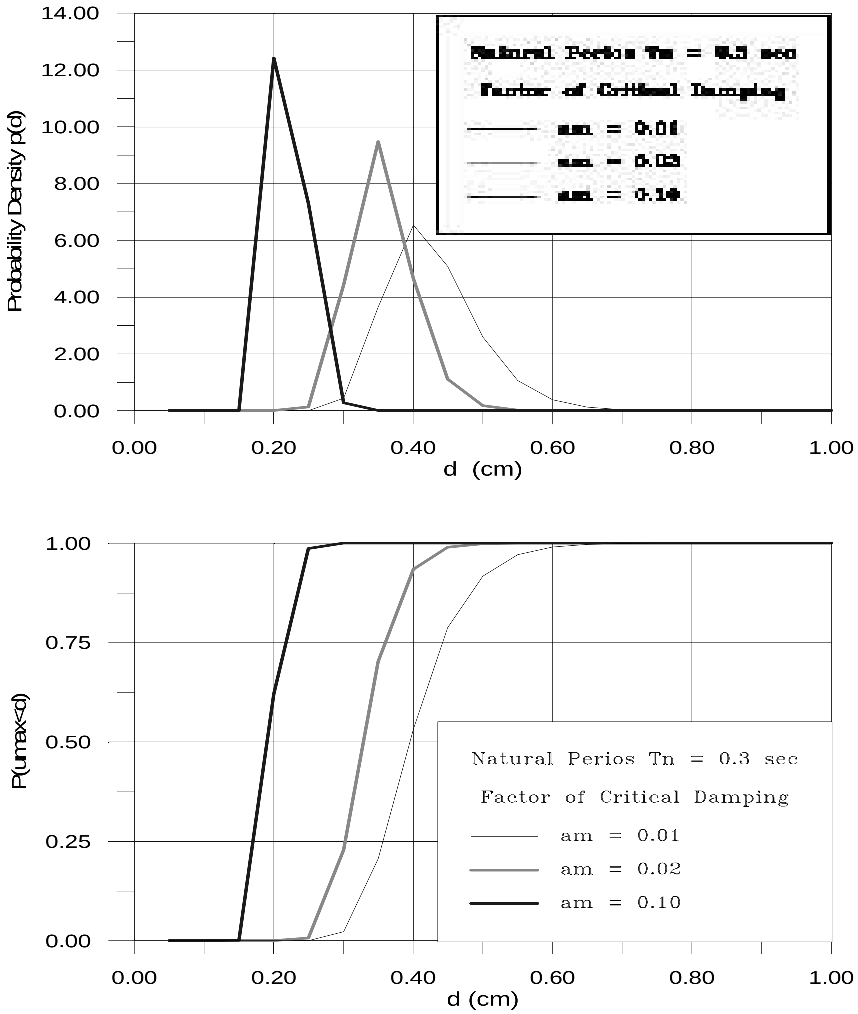


Figure 14. Probability density function and cumulative probability function obtained for a 1DOF structure under the El Centro 1940 record for Tn = 0.3 sec.

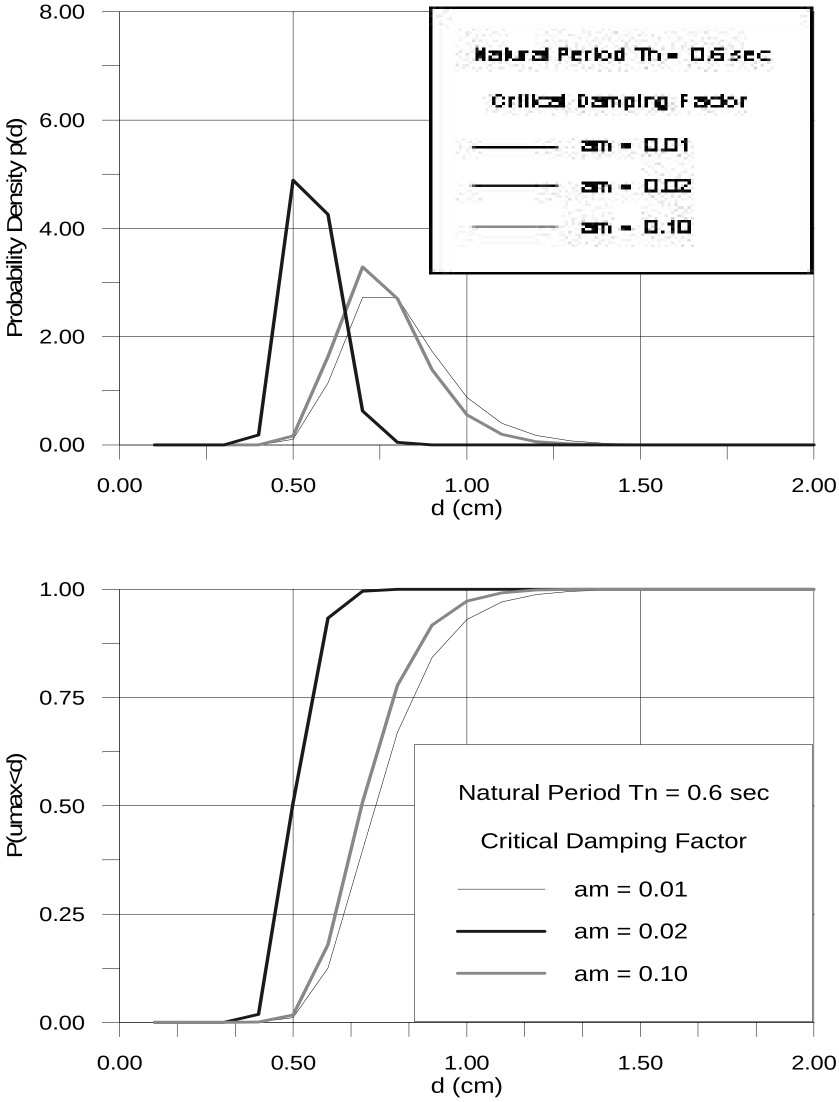


Figure 15. Probability density function and cumulative probability function obtained for a 1DOF structure under the El Centro 1940 record for Tn = 0.6 sec.

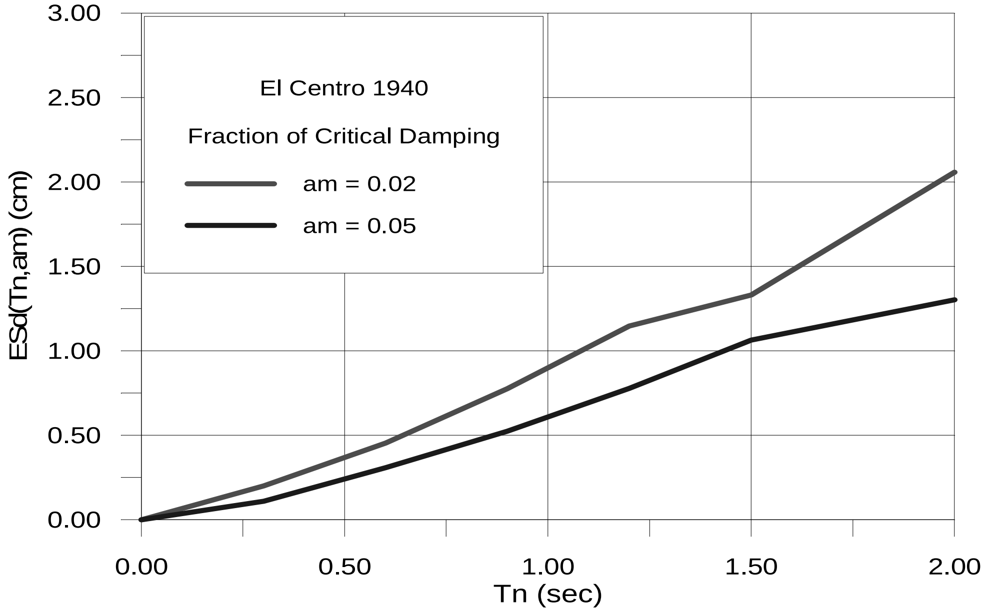


Figure 16. Average Response Spectra for El Centro 1940

## 6 REFERENCES


[ 1 ] Bycroft, G., N., "White Noise Representation of Earthquakes," *Journal of the Engineering Mechanics Division.* ASCE, Vol. 86, No. EM2, pp. 1-16, 1960.

[ 2 ] Bogdanoff, J,. and Goldberg, J., E., and Bernard, M., C., " Response of a Simple Structure to a Random Earthquake Type Disturbance," *Bulletin of the Seismological Society of America*, Vol. 51, pp. 293-310, 1961.

[ 3 ] Lin, Y., K., "Nonstationary Excitation and Response in Linear Systems Treated as Sequences of Random Pulses," *Journal of the Acoustical Society of America*, Vol. 38, No. 3, pp. 453-461, 1965.

[ 4 ] Holman, R.,E., and Hart, G.,C., "Nonstationary Response of Structural Systems," *Journal of the Engineering mechanics Division*, ASCE, Vol. 100, No. EM2, pp. 415-431, 1974.

[ 5 ] Roberts, J.,B., "The Response of Linear Vibratory Systems to Random Impulses", *Journal of Sound and Vibration*, Vol. 2 No. 4, pp.375-390, 1965.

[ 6 ] Crempien J., and Saragoni, G., R., "Influence of the Duration of Earthquake ground Motion in Average Response Spectra," *Proceedings*, Sixth European Conference on Earthquake Engineering, Vol. 2, pp. 143-150, 1978.

[ 7 ] Priestley., "Evolutionary Spectra and Non-Stationary Processes," *Journal Royal Statistical Society B*, Vol. 28, No. 2., pp. 204-230, 1965.

[ 8 ] Rice, S.,O., "Mathematical Analysis of Random Noise," *Bell Systems Technical Journal,* Vol. 23, pp. 282-332, 1944. D

[ 9 ] Der Kiuregian, A. and Crempien, J, "An Evolutionary Model for Earthquake Ground Motion", Structural Safety, Nº 6, 1989: 235 - 246.

[ 10 ] Saragoni, G., R., and Hart, G. "Simulation of Artificial Earthquakes," *International Journal of Earthquake Engineering and Structural Dynamics,*Vol. 2, No. 3, pp.249-267, 1974

[ 11 ] Crempien, J., and Orosco, L., ., "An evolutionary ground motion model for earthquake analysis of structures in zones with little history". Soil Dynamics and Earthquake Engineering, Vol. 20/5-8, pp. 373-379, 2001.

[ 12 ] Arias, A. "A Measure of Earthquake Intensity," in *Seismic Design for Nuclear Power Plants*, R. Hansen (Editor), MIT Press, 1970.

[ 13 ] Caughey, T., K., and Stumpf, H.,J., "Transient Response of Dynamic Systems under Random Excitations*,"* *Journal of Applied Mechanics*, ASME, Vol. 28, No.4, pp. 563-566, 1961.

[ 14 ] Racicot, R., L., and Moses, F., "A First Passage Approximation in Random Vibration," *Journal of Applied Mechanics* Vol. 38, No. 1, pp 143, 1971.

*[ 15 ]* Lyon, R., H., "On the Vibration Satistics of a Randomly Excited Hard Spring Oscillator II," *Journal oj the Acoustical Society of America*, Vol. 33, No. 10, pp. 1395, 1961.

[ 16 ] Vanmarke, E. H., "On the Distribution on the First Passage Time for Normal Stationary Random Processes," *Journal of Applied Mechanics*, March 1975, pp. 215-220

[ 17 ] Shinozuka, M., "Probability of Structural Failure Under Random Loading", *Journal of theEngineering Mechanics Division*, ASCE, Vol. 90, No, EM5, pp. 147-170, 1964.

[ 18 ] Shinozuka M., and Yang, J., N., "On the First Excusion Probability in Stationary Narrow-band Random Vibration," *Journal of Applied Mechanics*, Vol. 38, No. 4, pp. 1017, 1971.

[ 19 ] Roberts, ,., b., "First-Passage Probability for Non Linear Oscillators," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 102, No. EM5, pp. 851-866, 1976.

[ 20 ] Yang , J., N., "First-Excusion Probability in Non Stationary Random Vibrations," *Journal of Sound and Vibration*, Vol. 27, pp. 165-182, 1973.

[ 21 ] Lutes, L., D., Chen, Y., T., and Tzuang, S., H., "First passage Aproximations for Simple Oscillators," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 106, No. EM6, 1980.

[ 22 ] Crandall, S., H., "First-Crossing Probabilities of the Linear Oscillator," *Journal of Sound and Vibration,"* Vol. 12, N.

[ 23 ] Lutes, L., D., and Chokshi, N., C., "Maximum Response Statistic for a Linear Structure," *Proceedings* Fifth World Conference on Earthquake Engineering,, Vol. 11, pp. 2818-2821, Italy, 1972.

[ 24 ] Der Kiureghian, A., "A Response Spectrum Method for Random Vibration Analysis of MDF Systems*," International Journal of Earthquake Engineering and Structural Dynamics*. Vol. 9, pp. 419-435, 1981.

[ 25 ] Langley, R., S., "Comparison of Calculation Methods for the Response of a Linear Oscillator to Non-Stationary Broad-Band Excitation," *Engineering Structures,* Vol. 8., pp. 148-159, 1986.