A REVIEW OF THEORIES ON STRUCTURAL RUPTURE: FRACTURES MECHANICS AND CONTINUUM DAMAGE MECHANICS

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RESUMEN

Son analizadas las hipótesis básicas y los métodos de las teorías de Mecánica de Fractura y de Mecánica del Daño Contínuo, desde el punto de vista de sus aplicaciones estructurales usando técnicas computacionales.

ABSTRACT

The basic hypotheses and methods of Fracture Mechanics and Continuum Damage Mechanics are reviewed, from the point of view of structural applications using computacional techniques.

INTRODUCTION

Although rupture and fracture phenomena are of basic importance in applied solid mechanics and engineering, they are still not well understood and rational theories on the subject are in development.

Historically, the initial work was on brittle materials, pressed up by accidents with the Liberty ships at the end of World War II, the disasters with Comet planes and problems with space rockets.

Nowadays, linear elastic fractures mechanics (LEFM) is a well established technique used in special applications in mechanical, nuclear, aeronautical and civil engineering.

The new ground for research corresponds to the nonlinear (plastic, viscoplastic) range with work in progress along the lines of nonlinear fracture mechanics (NLFM) and continuum damage mechanics (CDM).

FRACTURE MECHANICS

The observation that the theoretical strength of engineering materials (determined on the the base of atomic physics) is one order of magnitude higher than real strength, lead to the hypothesis that real materials contain flaws (dislocations, cracks, etc) that explain such difference. As it is well known, plastic behavior is related to the movement of dislocations. Fracture mechanics, that may be defined as the "mechanics of crack growth" studies a continuum with as initial crack (that may be either real, i.e. observed, or assumed) and uses the tools of classical mechanics (particularly elasticity) without advancing hipotheses on the physics of the fracture process.

Linear Elastic Fracture Mechanics

For the case of static loads, the growth of cracks needs of two conditions to be fulfilled. First, we need a stress high enough at the crack tip to surpass material cohesion. Second, enough energy must flow to the crack tip from the rest of the structure, in order to provide the work needed for the creation of new surfaces, plastic deformation, heat dissipation, etc.

Initially |1| it was thought that the first condition was enough. That lead to the paradox that a cracked body would have stresses approaching infinite values and thus could support no load. This paradox was resolved by Griffith |2| who used a method based on energy balance to analize glass fiber fracture, explaining the effect of thickness on strength.

The method was extended by Orowan and Irwin to included the energy associated with plastic deformation near the newly formed surfaces and more recently by Rice |3| to situations with massive plastification.

Meanwhile, Irwin had introduced the concept of "stress intensity" |5| and had shown that is was equivalent to the Griffith's energy approach (It could be shown that these methods correspond to the energy and static formulations of a stability problem).

The stresses in the neighbourghood of a crack tip opening in mode I (symmetric tension) may be written

$$\begin{bmatrix} \sigma_{xx} \\ \\ \sigma_{yy} \\ \\ \sigma_{xy} \end{bmatrix} = \frac{K_I \cos \frac{\theta}{2}}{\sqrt{2\Pi\, r}} \begin{bmatrix} 1 - \sin \frac{\theta}{2} \ \sin \frac{3\theta}{2} \\ \\ 1 + \sin \frac{\theta}{2} \ \sin \frac{3\theta}{2} \\ \\ \sin \frac{\theta}{2} \quad \frac{3\theta}{2} \end{bmatrix} + 0\,(r^o) \qquad (1)$$

where r, θ are polar coordinates in the x-y plane, with origin at the crack tip. The singular stress distribution in the near field, weighted by the parameter K_I (stress intensity factor) is the same for each distribution of loads or deformations in the outer region of the structure (far field). Different far fields are simply characterized by different values if K. The fact that stresses are infinite for $r \to 0$ should not worry us, because in fact r cannot be zero in a real material and the continnum analysis is not valid at the crack tip.

The corresponding displacements may be written

$$\begin{bmatrix} u_x \\ \\ u_y \end{bmatrix} = \frac{K_I}{2\mu} \sqrt{\frac{r}{2\Pi}} \begin{bmatrix} \cos \frac{\theta}{2} K - 1 + 2 \sin^2 (\frac{\theta}{2}) \\ \\ \sin \frac{\theta}{2} K + 1 - 2 \cos^2 (\frac{\theta}{2}) \end{bmatrix} \qquad (2)$$

Reckoning the energy change dU corresponding to a change da in crack length, we obtain from (1) and (2)

$$\frac{dU}{da} = G_I = \frac{\kappa + 1}{8 \quad \mu} K_1^2 \qquad (3)$$

where μ is the shear modulus and $\kappa = 3 - 4\mu$ for plane strain, $\kappa = (3-\mu)/(1 + \nu)$ for plane stress and ν is Poisson modulus. This most important equation relates Irwin's and Griffith's approaches.

Again, the basic philosophy of the stress intensity method is that the region around the crack tip has a stress distribution which is independent of the loading conditions and geometric shape. Geometry and load conditions influence the singular stress state through parameter K, the stress intensity factor. The critical state of instable crack growth corresponds to $K = K_c$, a material constant to be experimentally determined.

The J-integral

A very important result due to Rice |3 |, shows that the rate of change of potencial energy with length for a crack oriented along the x direction, can be written as the line integral

$$J = -\frac{dp}{da} = \int_{\Gamma} W\, dy - T_i \frac{du_i}{dx}\, ds \qquad (4)$$

where Γ is any path from the lower crack face anticlockwise around the crack tip to the upper face, s is the path length along this contour, W is the strain energy density in the form $dW = \sigma_{ij}\ \varepsilon_{ij}$ and $T_i = \sigma_{ij}\ n_j$ and u_i are forces and displacements along the contour path. Rice showed that this integral is path independent for linear or nonlinear elastic materials. For linear materials, $J \equiv G$ and is an alternative way to determine K through (3), particularly with finite element methods.

Moreover, for a nonlinear elastic relation in the form $e/e_j = A(\sigma/\sigma_y)^N$ stress intensity factors are determined in the form

$\sigma = C\ r^{-(\frac{1}{N}+1)}\ f(\theta)$ where $C = (JE/\sigma_y^2 IA)$. In consequence, for a given nonlinear law N, elastic modulus E, C depends on material constants and the value of J, so that J controls nonlinear fracture problems in the same way as G (or K) controls linear fracture.

As under certain circumstances (radial loading, no unloading) plastic and nonlinear elastic behaviours coincide, J is also valid as a fracture parameter in some situations. The limits of validity, that have to be determined using elastoplastic analysis, are an important research subject in computational (e.g. finite elements) mechanics |1|.

Finite elements applications

Two important consideration in the development of finite element programs for FM are the proper modelling of the crack tip singularity and the interpretation of results in terms of a stress intensity factor K or a crack driving force G.

The use of conventional elements requires a very thin discretization near the crack tip, and is computationaly expensive. Thus, singular elements of different types have been proposed | 6 |.

The simplest way to obtain singular solutions, is to use quarter point elements, which are for example quadratic (8-nodes) isoparametric elements with the midside nodes nearest to the crack tip displaced to the quarter point. Then, we obtain a square root singularity for strains and stresses, as the one in (1).

After modelling the crack tip singularity, the stress intensity factor is obtained from (1) or (2), usually by extrapolation. Using the stiffness method, which gives a better presition in displacements, extrapolation is based on displacements. Alternative methods are the determination of change in strain energy with crack extension which gives G and the use of the J-integral.

CONTINUUM DAMAGE MECHANICS

Continuum damage mechanics (CDM) has been introduced to describe the progressive degradation experienced by the mechanical properties of materials before cracking. It covers the transition between plasticity and fracture mechanics.

The most natural representation of constitutive equations with damage is based on state variables. State variables are used to represent viscoelasticity |7| and plasticity |8| and appear to be particularly convenient to represent large deformations behaviour and the incremental anisotropy due to damage |9|, |10|.

We assume that the changing internal state of a deforming material element can be represented by the pair (S, σ), where $\sigma(t)$ is the current (Cauchy) stress carried by the element and S(t) stands for the n parameters that measure the futurelly relevant aspects of inelastic changes in internal structure that have taken place at time t. Among those state variables we have some used to represent viscous or plastic response and others used to represent damage (providing, in an appropriate statistical sense, the distribution of microcracks). We may assume that failure due to the accumulation of damage will occur when the trajectory of the state point intersects certain parts of the boundary of the state space.

We formulate the constitutive relations in the form (restricting ourselves to the case of small strains)

$$\dot{\varepsilon} = f_1(S,\sigma) + E^{-1}(S)\,\dot{\sigma} \qquad (5)$$
$$\dot{S} = f_2(S,\sigma)$$

The first (5) shows the decomposition of strain ε into an elastic part, with state dependent elastic modulus E (S) and an inelastic part, that depends on the state and the stress level. The second (5) is the law of evolution of the state variables.

Effective stress concept

Many workers, following Kachanov use the concept of effective stress $\bar{\sigma} = M\sigma$ where M is a forth order tensor operator. For isotropic damage and the case of resisting areas independent of the sign of stress we have.

$$\bar{\sigma} = \frac{\sigma}{1-D} \qquad (6)$$

where D is the scalar damage variable. In this case, D takes account of the effective area reduction due to microcracking corrected for micro-stress concentrations and interaction effects. D=0 corresponds to the virgin element and D=1 to the ruptured element.

The so-called principle of strain equivalence |11| states that any constitutive equation for a damaged material is derived from the same potencials as for the virgin material, except that all stress variables are replaced by the effective stress. For example, in elasticity

$$\varepsilon = \frac{\bar{\sigma}}{E} = \frac{\sigma}{E(1-D)} \qquad (7)$$

Now, considering the linear constitutive equation for damage $D = \bar{\sigma}/E_D$, where E_D is a new material parameter, from (7) we obtain.

$$\sigma = E\varepsilon\,(1 - \frac{E}{E_D}\,\varepsilon) \qquad (8)$$

i.e. a nonlinear stress-strain relation with a maximum stress $E_D/4$ and strain-softening behavior. A model of this type may still be coupled with viscous behaviour | 7 |.

Thermodynamics

There is a convenient thermodynamically admissible method of building constitutive relations, that chooses a Helmholtz free energy function ψ (S,σ) and a potential dissipation function Ω (Σ,σ) (where Σ contains the thermodynamic forces conjugated to the state variables S) |11|.

One could, as a simple example, define a dissipation function in the form Ω(Y,σ) where Y is the thermodynamic force corresponding to the damage D, obtaining

$$\dot{\varepsilon}^P = \frac{\partial \Omega}{\partial \sigma} \quad , \quad \dot{D} = \frac{\partial \Omega}{\partial Y} \qquad (9)$$

Examples of this technique, taken from plasticity may be found in refs. 11 and 13 . The final formulation is also similar to that in plasticity, enabling the use of available algorithms. The theory reduces to plasticity when microcracking is obsent or negligible.

Notice however that this convenient formalism is not unique. In fact, Onat |10 | has shown admissible constitutive relations that cannot be put into the potencial form above.

Computational plasticity with damage

For a elastoplastic material with yield function f (S,σ) and plastic potencial function g(S,σ) we may write the relation between rates of stress and strain in the well known form |14|

$$\dot{\sigma} = \left[E - \frac{| E(\frac{\partial g}{\partial \sigma}) | \otimes | E(\frac{\partial f}{\partial \sigma}) |}{A + (\frac{\partial f}{\partial \sigma}) \cdot E(\frac{\partial g}{\partial \sigma})} \right] \dot{\varepsilon} \qquad (10)$$

where A is related to hardening properties. Damage can alter the elastic behaviour, changing E or the plastic behaviour, through changes in f, g and eventually A. Very interesting applications of this formulation to problems in concrete and geomaterials may be found in Refs. 13 and 15 . The introduction of plastic damage through strain-softening leads to some difficulties (lack of objectivity with relation to finite elements mesh size), that should be overcome.

FINAL COMMENTS

A new very good book that covers many fields in FM is ref. 1. For CDMno such general presentation exists but refs. 11 and 12 may be useful. Those texts bring many further references. In our opinion, to study the ideas of Rice, ref. 4 is yet unsurpassed. Many new articles appear in the International Journal of Fracture and Engineering Fracture Mechanics, as well as in Journals on finite element techniques.

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