NONLINEAR TRANSIENT RESPONSE OF RISERS OPERATING IN DEEP WATER SEAS

Nelson F.F. Ebecken
Edison C.P. Lima
Marcio M. Mourelle
COPPE/UFRJ - Civil Engineering Department

RESUMO

Neste trabalho estuda-se a resposta não-linear de um riser instalado em águas profundas. A correta interpretação dos efeitos hidrodinâmicos e a implementação de métodos numéricos eficientes são necessários para este tipo de simulação.

O riser é modelado por elementos de pórtico espacial considerando efeitos axiais flexionais e torsionais.

Alguns casos são apresentados e a representação das ações ambientais e a consideração dos efeitos não-lineares no modelo estrutural são discutidas.

ABSTRACT

In this work the nonlinear transient response of deep-sea riser is focused. For accurate and economic time-domain analysis of deepsea risers, a realistic representation of hydrodynamic forces along the riser and an efficient numerical method are required.

The riser is modelled by three dimensional beam finite elements, which account for coupled axial, bending and torsional deformations.

Some results are presented and the environmental model, hydrodynamic model and structural model are emphasized equally for accurate response prediction.

INTRODUCTION

Marine risers are critical components in offshore drilling and production systems. In essence, a marine riser extends the well from the seafloor to the surface vessel. Tension applied at the top of the riser allows it to resist lateral loads through the beam-column effect. Static lateral loads arise from the normal drag force in a current and from the weight of the riser when its top is offset. Dynamic loads arise from vessel motion and wave action on the riser. Static and dynamic effects are coupled through the nonlinear drag force. In addition, vortex-induced forces may excite the riser in its normal modes of transverse vibration.

For accurate and economic time domain analysis of deep-sea risers, it was used: the well-known advantages of finite element discretizations, modeling the pipe by three-dimensional beam finite elements; an implicit time integration method; coupled axial, bending and torsional deformations; tangent stiffness; nonlinear static equilibrium configurations as initial condition; realistic modeling of the subsurface and subsea hydrodynamic forces; and the slow vessel motion.

2. SIMULATION OF SEA SURFACE ELEVATIONS

A significant aspect of the probabilistic approaches to the dynamic analysis of offshore structures is the proper digital simulation of sea surface elevation records, along with velocity and acceleration records, that are compatible with a specified target power spectrum.

The sea surface is assumed to be a stationary ergodic stochastic process produced by the addition of infinitesimal waves each one with a random phase. The Central Limit Theorem may be invoked to show that these assumptions lead to a Gaussian process [1].

Two basic methods are available for the simulation of a random sea: (a) wave superposition; and (b) linear filters. Most of the currently used algorithms are based on the harmonic wave superposition [2] . The amplitude of each wave component is related to the value of the power spectrum at the frequency corresponding to this component and the phase angles are random numbers uniformly distributed in the interval $(0, 2\Pi)$.

An alternative technique of simulation of surface elevation using a linear filter has been proposed by many authors [2,3,4]. Based on this technique the time series which represents the ocean elevation is obtained by the output of an autoregressive digital filter to white noise input.

The methods based on wave superposition are straightforward and versatile in the sense that they can account for the statistical spacial dependence and cross-correlation of ocean waves. However a large number of wave components must be employed making these methods in several cases more costly than the correspondly linear filter methods.

In this paper was adopted the wave superposition method as can be found in Borgman [2].

A numerical model of a long-crested random sea can be obtained by the summation of a finite number of harmonic waves with amplitudes derived from a target spectral density denoted $S(w)$ in the following manner:

$$\hat{\eta}(x,t) = a\sqrt{2} \sum_{n=1}^{N} \cos(\hat{k}_n x - \hat{w}_n t + \phi_n) \qquad (1)$$

where w is a frequency in rad/s and w_n has a sufficient high value such that $S(w) \cong 0$ for $w > w_n$.

Let $\hat{w}_n = (w_n - w_{n-1})/2$ and the quantity a is given by

$$a = \sqrt{S(\hat{w}_n)\Delta w_n} \qquad (2)$$

in which $\Delta w_n = w_n - w_{n-1}$.

The phase angles θ_n are independent random variables uniformly distributed over the interval $(0, 2\Pi)$, $\hat{k}_n$ is obtained by the solution of $\hat{w}_n = \hat{k}_n g \tanh(\hat{k}_n d)$ where g is the acceleration due to gravity and d is the water depth.

To avoid the periodicity of the time history of sea surface elevations $\hat{\eta}(x,t)$, generated in eq.(1), the set of w_n values is determinated through the cumulative spectrum, defined as

$$S_c(w) = \int_0^w S(\bar{w})\, d\bar{w} \qquad (3)$$

Taking into account (3) it can be observed that the quantity a, given in eq.(2) may be approximated by $a \cong [S_c(w_n) - S_c(w_{n-1})]$ and the periodicity of $\hat{\eta}(x,t)$ can be avoided if the set of w_n values are chosen to make the difference $S_c(w_n) - S_c(w_{n-1})$ constant for all w_n, given by the solution of,

$$S_c(w_n) = \frac{n}{N} S(\infty) \qquad (4)$$

For the Pierson-Moskowitz spectral density [5]

$$S(w) = \frac{A}{w^5} \exp\left(-\frac{B}{w^4}\right) \qquad (5)$$

where

$$B = \frac{5}{4} \left(\frac{2\Pi}{T_o}\right)^{1/4} \qquad (6)$$

$$A = 4B\left(\frac{H_s}{2}\right)^2 \qquad (7)$$

T_o = mean zero cross period

H_s = significant wave height

the equation (4) becomes

$$S_c(\infty) = \frac{A}{4B} \qquad (8)$$

resulting for the solution of eq.(4),

$$w_n = \left(\frac{B}{(\ell n(\frac{N}{n})}\right)^{1/4} \qquad (9)$$

3. SIMULATION OF VELOCITIES AND ACCELERATIONS

The horizontal and vertical components of water velocity and acceleration at some elevation z above the sea bottom may be related to the sea surface elevation through the Airy wave theory as:

$$\hat{v}_x(t) = a\sqrt{2}\sum_{n=1}^{N}\hat{w}_n\frac{\cosh(\hat{k}_n(z+d))}{\text{senh}(\hat{k}_n d)}\cos(\hat{k}_n x-\hat{w}_n t+\phi_n) \qquad (10)$$

$$\hat{v}_z(t) = a\sqrt{2}\sum_{n=1}^{N}\hat{w}_n\frac{\text{senh}(\hat{k}_n(z+d))}{\sinh(\hat{k}_n d)}\sin(\hat{k}_n x-\hat{w}_n t+\phi_n) \qquad (11)$$

$$\hat{a}_x(t) = a\sqrt{2}\sum_{n=1}^{N}\hat{w}_n^2\frac{\cosh(\hat{k}_n(z+d))}{\sin(\hat{k}_n d)}\sin(\hat{k}_n x-\hat{w}_n t+\phi_n) \qquad (12)$$

$$\hat{a}_z(t) = -a\sqrt{2}\sum_{n=1}^{N}\hat{w}_n^2\frac{\sinh(\hat{k}_n(z+d))}{\sin(\hat{k}_n d)}\cos(\hat{k}_n x-\hat{w}_n t+\phi_n) \qquad (13)$$

For an unidirectional spectrum, the cross-spectral density between the horizontal velocity and acceleration is zero. Therefore, the equations (10) to (13) have all the necessary information for the simulation of wave forces. The usual Morison's formula [5] may be applied with the time histories of velocities and accelerations presented in equations (10) to (13).

4. NONLINEAR EFFECTS

The study of the behaviour of marine risers in deep water must take into account basically two important nonlinear effects:

a) the viscous drag forces on the riser that are proportional to the square of the fluid-structure relative velocity;

b) the large displacements induced by the lateral forces and their interaction with the elevated axial forces applied at the riser head.

In the present work, the relative fluid-structure velocity is evaluated at each time step.

The structural nonlinearity is considered by employing a straight finite beam element that has been developed in reference [7]. It considers the equilibrium at the deformed configuration and the strain-displacement relationships are defined for large displacement analysis.

The tangent stiffness matrix is evaluated, at each time step , accounting for the axial forces and the change in the rotational matrix due to the large displacement effects.

As it was stated before, a full nonlinear dynamic analysis in time domain is carried out. The governing equilibrium equation can be written,

$$\underset{\sim}{M}\,{}^{t+\Delta t}\ddot{\underset{\sim}{u}} + \underset{\sim}{C}\,{}^{t+\Delta t}\dot{\underset{\sim}{u}} + {}^{t}\underset{\sim}{K}({}^{t+\Delta t}\underset{\sim}{u}) = {}^{t+\Delta t}\underset{\sim}{R} - {}^{t+\Delta t}\underset{\sim}{F} \qquad (14)$$

Matrix $\underset{\sim}{M}$ is a lumped mass which includes the structural mass, the added mass and any additional mass from internal mud or oil.

The damping matrix $\underset{\sim}{C}$ is also lumped and can be considered as being proportional to the mass matrix $\underset{\sim}{M}$.

These two matrix $\underset{\sim}{M}$ and $\underset{\sim}{C}$ remain constants along the analysis.

The tangent stiffness matrix ${}^{t}\underset{\sim}{K}$ is evaluated at the beginning of each time step and is a function of the geometry and the deformation of the elements.

At the right side of the equilibrium equation, the load vector is divided in two components: the vector $\underset{\sim}{R}$ which includes the nodal equivalent forces due to drag and dead weight, and the concentrated force at the head for the time considered. The vector $\underset{\sim}{F}$ represents the nodal forces exerted by the elements due to their deformation state and is evaluated at each iteration.

Numerical integration of highly nonlinear equation must be done by a step-by-step algorithm. The Newmark method was used for integrating the equilibrium equation and iterations for dynamic equilibrium are carried out for each time step.

In the Newmark method the following assumptions are made:

$${}^{t+\Delta t}\underset{\sim}{u} = {}^{t}\underset{\sim}{u} + \Delta t\ {}^{t}\dot{\underset{\sim}{u}} + \frac{\Delta t^2}{2}\ {}^{t}\ddot{\underset{\sim}{u}} + \alpha\ \Delta t^2({}^{t+\Delta t}\ddot{\underset{\sim}{u}} + {}^{t}\ddot{\underset{\sim}{u}}) \qquad (15)$$

$${}^{t+\Delta t}\dot{\underset{\sim}{u}} = {}^{t}\underset{\sim}{u} + \Delta t\ {}^{t}\ddot{\underset{\sim}{u}} + \delta\ \Delta t({}^{t+\Delta t}\ddot{\underset{\sim}{u}} - {}^{t}\ddot{\underset{\sim}{u}}) \qquad (16)$$

where,

$\delta = 0.25$ and $\alpha = 0.50$.

At each iteration the global stiffness matrix is mantained constant and only the vector $\tilde{F}$ of equation (14), the displacements, velocities and accelerations fields are reestablished.

5. NUMERICAL APPLICATION

The riser has a total length of 1200 m and is filled with mud. 60 cm diameter floating tubes are distributed along the riser to stabilize it. The modelling consists in 14 beam elements with identical mechanical characteristics, shown in Table I.

External diameter of the tube	50 cm
Thickness of the tube wall	2 cm
Modulus of elasticity	2×10^8 kN/m^2
Mass density of the riser	78.48kN/m^3
Mass density of drilling mud	14.72kN/m^3
Mass density of water	9.81kN/m^3
Drag coefficient	0.7
Coefficient of added mass	2.
Effective diameter for current and wave ...	60 cm

TABLE I

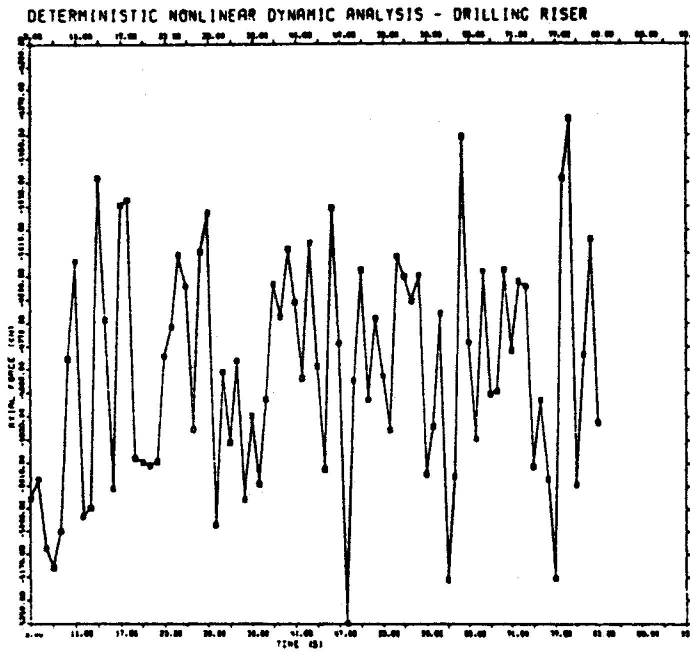
The riser is assumed to be bottom hinged and having a spherical buoy connected at the top. This buoy is 10 m diameter and a 0.125 drag coefficient was adopted. For the deterministic analysis an Airy wave with period of 9 s and an amplitude of 6 m is considered. The profile of the current is linear: 0.5 m/s at the free surface level and 0. at the ball joint.

An integration step of 0.05 s has been adopted. For the random analysis, the Pierson-Moskowitz spectrum is used for a sea state with the following characteristics:

mean zero crossing period = 9 s
significant wave height = 6 m

The wave superposition method was employed for the simulation of this sea state as previously described. The sea surface is assumed to be a stationary ergodic stochastic process produced by the addition of 30 harmonic waves, each one with a random phase, according to equation (1).

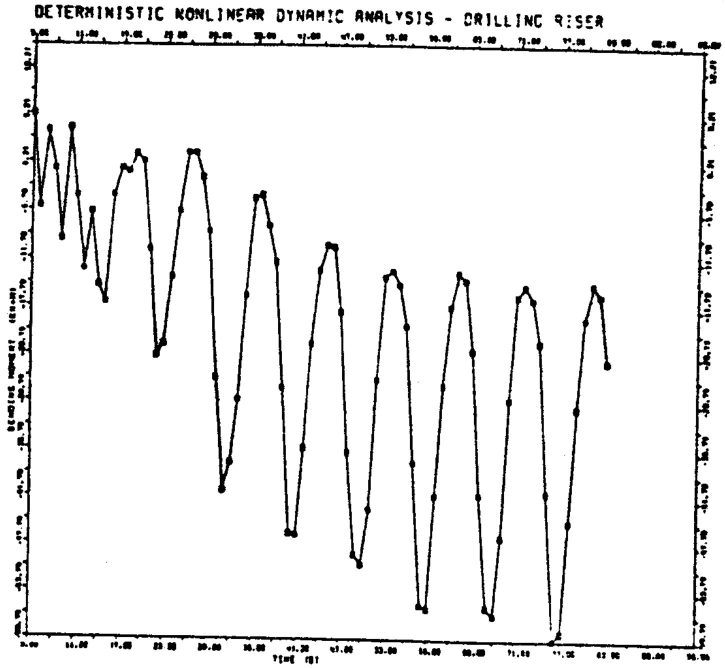
The results related, to the 1080 m level are indicated in figures 1 (axial force) and 2 (bending moment). These values are compared with the expected maximum values obtained in the random analysis considering a 3 hours sea state.



MAXIMA VALUES:

Deterministic	5520 kN
Simulation	6577 kN

FIGURE 1



MAXIMA VALUES:

Deterministic	60 kN.m
Simulation	224 kN.m

FIGURE 2

6. CONCLUSION

A finite element representation of the riser, the use and updating of hydrodynamic and environmental data at each step, and the implicit time integration method provide accurate use of the transient analysis of deep-ocean riser systems.

Equal enphasis has been placed on the environment, hydromechanic and structural models, that are solved together through the fluid-structure interaction. This transient analysis is quite important in providing data for the vessel control system simulation and for the efficiency of the riser operations.

REFERENCES

[1] Clough,R.W. and Penzien, J. - "Dynamic of Structures", McGraw-Hill, 1975.

[2] Borgman, L.E., "Ocean Wave Simulation for Engineering Design", j. of Waterways and Harbors Division, ASCE, Nov,1969.

[3] Samii, K. and Vandiver, J.K. , "A Numerically Efficient Technique for the Simulation of Random Wave Forces on Offshore Structures", OTC-4811, 1984.

[4] Spanos, P.T.D., "ARMA Algorithms for Ocean Wave Modeling", Journal of Energy Resources Technology, Vol. 105, September,1983.

[5] Sarpkaya, T. and Isaacson, M., "Mechanics of Wave Forces on Offshore Structures", Van Nostrand Reinhold Company, 1981.

[6] Safai, V.H., "Nonlinear Dynamic Analysis of Deep Water Risers", Applied Ocean Research, 1983, Vol. 5, nº 4.

[7] Benjamin,A.C., "Nonlinear Analysis of Framed Structures by the Finite Element Method", M.Sc. Thesis, COPPE/UFRJ, 1982.

[8] Chung,J.S.; Whitney,A.K and Loden,W.A.,"Nonlinear Transient Motion of Deep Ocean Mining Pipe, Journal of Energy Resource Technology, vol. 103, March, 1981.

[9] Nordgren, R.P., Dynamic Analysis of Marine Risers with Vortex Excitation, Journal of Energy Technology, vol.104, March, 1982.