TRANSVERSE VIBRATIONS OF CIRCULAR PLATES WITH A SECANT SUPPORT

Patricio A. A. Laura
Roberto H. Gutierrez
Institute of Applied Mechanics
8111 - Puerto Belgrano Naval Base - Buenos Aires, Argentina
Julio C. Utjés
Empresa Nuclear Argentina de Centrales Eléctricas S.A.
Av. L.N.Alem 712 - (1001) Buenos Aires - Argentina

RESUMEN

Se obtuvo una solución aproximada para el problema del título del presente trabajo usando: a) El método de Rayleigh-Schmidt y b) El método de Elementos Finitos, en el caso de placas empotradas y simplemente apoyadas. Se observa un buen acuerdo en los resultados.

Se demuestra que el método de Rayleigh-Schmidt permite la determinación de los autovalores en el caso en que la placa sea constreñida elásticamente contra rotación, los resultados fueron obtenidos en una calculadora programable de bolsillo.

ABSTRACT

An approximate solution for the title problem is obtained using a) The Rayleigh-Schmidt method and b) A finite Element algorithm, in the case of clamped and simply supported plates. Good engineering agreement is shown to exist. It is shown that the Rayleigh-Schmidt method allows for the straightforward determination of eigenvalues in the case where the plate is elastically restrained against rotation, the results been obtained on a pocket programmable calculation.

INTRODUCTION

Continuous, rectangular plates executing small amplitude, transverse vibrations have been treated in many excellent papers [1-2]. On the other hand no information is available in the open literature on vibrating continuous plates of other shapes, e.g. a circular plate with an internal, secant support, see Figure 1.

For the sake of generality it is assumed that the plate edge is elastically restrained against rotation. An analytical solution is obtained by means of the Rayleigh-Schmidt approach [3]. Independently, the cases of clamped and simply supported edges are treated using a finite element algorithm [4] and very good agreement with the analytical solution is shown to exist.

APPROXIMATE ANALYTICAL SOLUTION

A convenient way to formulate an approximate solution of the title problem is in terms of the governing functional

$$J[W] = D\iint_{\bar{P}} \{(W_{\bar{r}\bar{r}} + \frac{W_{\bar{r}}}{\bar{r}} + \frac{W_{\theta\theta}}{\bar{r}^2})^2 +$$

$$+ 2(1-\mu)[(\frac{W_{\bar{r}\theta}}{\bar{r}} - \frac{W_{\theta}}{\bar{r}^2})^2 - W_{\bar{r}\bar{r}}(\frac{W_{\bar{r}}}{\bar{r}} + \frac{W_{\theta\theta}}{\bar{r}^2})]\}\bar{r}d\bar{r}d\theta +$$

$$+ \frac{a}{\phi}\int_0^{2\pi}[W_{\bar{r}}(a,\theta)]^2 d\theta - \rho h\omega^2\iint_{\bar{P}} W^2\bar{r}d\bar{r}d\theta \qquad (1)$$

subject to the boundary conditions:

$$W(a,\theta) = 0 \qquad (2.a)$$

$$W(\bar{r}\cos\theta - \bar{x}_0,\theta) = 0 \qquad (2.b)$$

$$W_{\bar{r}}(a,\theta) = -D\phi[W_{\bar{r}\bar{r}}(a,\theta) + \mu(\frac{W_{\bar{r}}(a,\theta)}{a} + \frac{W_{\theta\theta}(a,\theta)}{a^2})] \qquad (2.c)$$

where $W(\bar{r},\theta)$ is the dynamic displacement amplitude. Expression (2.b) is the edge constitutive relation which defines the flexibility coefficient ϕ.

Introducing the dimensionless variable $r = \frac{\bar{r}}{a}$ in (1) and (2) one obtains:

$$\frac{a^2 J[W]}{D} = \iint_P \{(W_{rr} + \frac{W_r}{r} + \frac{W_{\theta\theta}}{r^2})^2 + 2(1-\mu)[(\frac{W_{r\theta}}{r} - \frac{W_{\theta}}{r^2})^2 -$$

$$- W_{rr}(\frac{W_r}{r} + \frac{W_{\theta\theta}}{r^2})]\}rdrd\theta + \frac{1}{\phi'}\int_0^{2\pi} W_r^2(1,\theta)d\theta - \Omega^2\iint_P W^2 rdrd\theta \qquad (3)$$

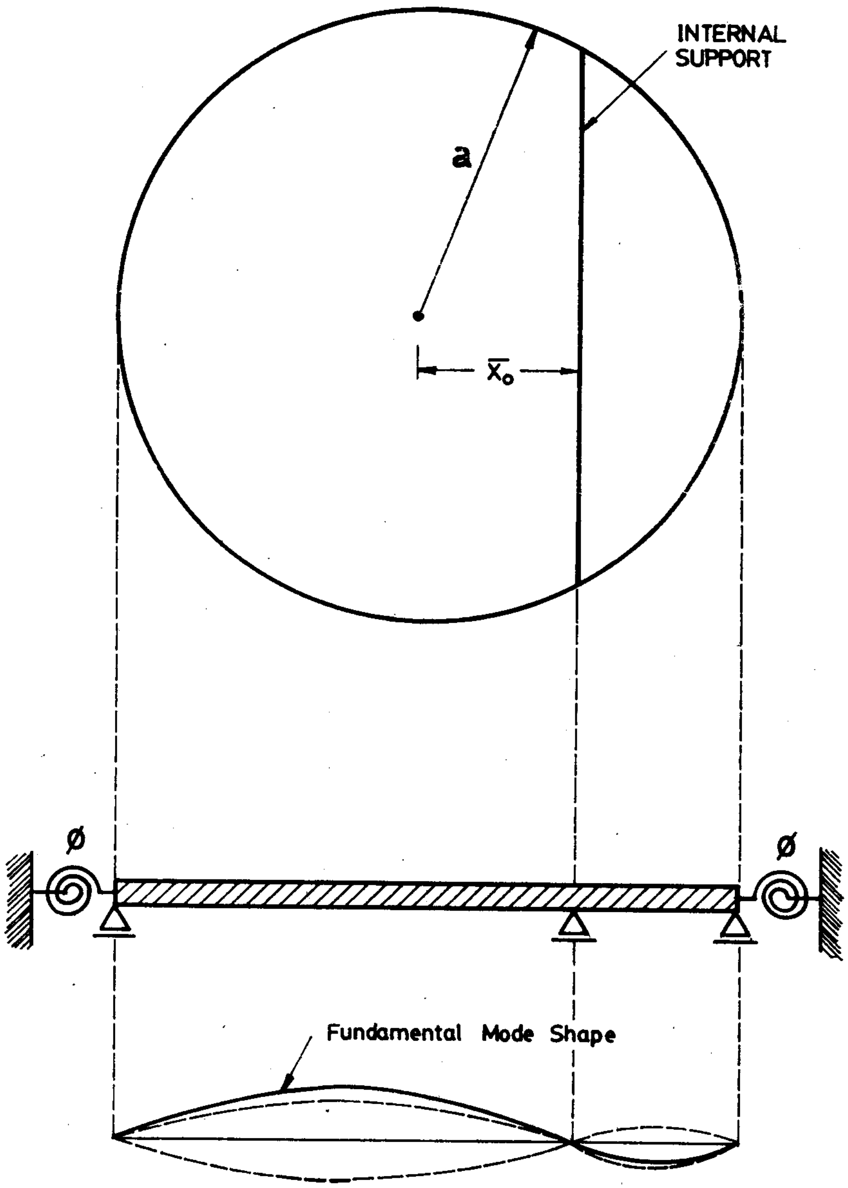

FIGURE 1: Mechanical System Considered in the present investigation.

$$W(1,\theta) = 0 \tag{4.a}$$

$$W\left(r\cos\theta - \frac{\bar{x}_0}{a}, \theta\right) = 0 \tag{4.b}$$

$$W_r(1,\theta) = -\phi'[W_{rr}(1,\theta) + \mu(W_r(1,\theta) + W_{\theta\theta}(1,\theta))] \tag{4.c}$$

where

$$\Omega^2 = \frac{\rho h \omega a^4}{D}$$

$$\phi' = \frac{\phi D}{a}$$

A suitable two-term approximation for the displacement amplitude function is the expression

$$W_a = [C_1(\alpha_1 r^{2\gamma}+\beta_1 r^2+1) + C_2(\alpha_2 r^{2\gamma+2}+\beta_2 r^4+r^2)](r\cos\theta - x_0) \tag{5}$$

where γ is the Rayleigh-Schmidt minimization parameter. The functional relation (5) satisfies identically the essential boundary conditions (4.a, b), but in an approximate sense (4.c), and this is accomplished substituting

$$(\alpha_1 r^{2\gamma}+\beta_1 r^2+1)$$

and

$$(\alpha_2 r^{2\gamma+2}+\beta_2 r^4+r^2)$$

in (4.a) and (4.c) yielding:

$$\alpha_1 = \frac{2[1+\phi'(1+\mu)]}{2\gamma[1+\phi'(2\gamma-1)+\phi'\mu]-2[1+\phi'(1+\mu)]}$$

$$\beta_1 = -\frac{2\gamma[1+\phi'(2\gamma-1)+\phi'\mu]}{2\gamma[1+\phi'(2\gamma-1)+\phi'\mu]-2[1+\phi'(1+\mu)]}$$

$$\alpha_2 = \frac{1+\phi'(5+\mu)}{(\gamma+1)[1+\phi'(2\gamma+1+\mu)]-2[1+\phi'(3+\mu)]}$$

$$\beta_2 = \frac{1+\phi'(1+\mu)-(\gamma+1)[1+\phi'(2\gamma+1+\mu)]}{(\gamma+1)[1+\phi'(2\gamma+1+\mu)]-2[1+\phi'(3+\mu)]}$$

Substituting (5) in the functional (3) and requiring:

$$\frac{\partial J}{\partial C_i} = 0 \tag{6}$$

one obtains, from the non-triviality condition a characteristic equation whose lowest root constitutes the fundamental frequency coefficient of the system Ω_1.

Since this eigenvalue is an upper bound with respect to the exact frequency result, by requiring now that

$$\frac{\partial \Omega_1}{\partial \gamma} = 0 \qquad (7)$$

one is able to obtain the optimum value of Ω_1 corresponding to the functional relation (5).

The procedure is straightforward but rather lengthy to be included here. In general it is more convenient to perform (7) numerically.

NUMERICAL RESULTS

Fundamental eigenvalues of the mechanical system shown in Figure 1 were obtained using the procedure previously described and also by means of SAPIV finite element algorithmic procedure [4] (the finite element net is shown in Figure 2) for the case of clamped and simply supported plates.

Table I shows a comparison of the eigenvalues obtained by using both methodologies. Good engineering agreement is observed for all cases considered.

TABLE I - Comparison of values of $\Omega_1 = \sqrt{\frac{\rho h}{D}}\,\omega_1 a^2$ ($\mu = 0.30$)

* The exact eigenvalue is known since it corresponds to the first antisymmetric mode of a circular plate.

** Eigenvalue determined using $W_a = C_1(\alpha_1 r^{2\gamma} + \beta_1 r^2 + 1)$

	Clamped Plate			Simply Supported Plate		
$\bar{x}_0/a$	Rayleigh Schmidt	Finite Elements	Exact * [5]	Rayleigh Schmidt	Finite Elements	Exact * [5]
0	21.26	21.59	21.26	14.12	13.95	13.94
0.1	20.26	20.23	-	13.56	13.16	-
0.2	18.17	18.07	-	12.30	11.79	-
0.3	16.20	16.15	-	10.96	10.52	-
0.4	14.71	14.58	-	9.82	9.45	-
0.6	12.85	12.32	-	8.24	7.83	-
0.8	11.83	10.94	-	7.27	6.68	-
1.0	10.22**	10.29	10.215	4.98**	4.94	4.977

When $\frac{\bar{x}_0}{a} = 0$ the Rayleigh-Schmidt approach yields a frequency value which is in excellent agreement with the exact result in the case of a clamped plate. On the other hand the difference is of the order of 1% when the plate is simply supported.

Table I illustrates also, in a very clear fashion, the accuracy achieved when a single polynomial is used to determine the fundamental frequency coefficient by means of the Rayleigh-Schmidt procedure.

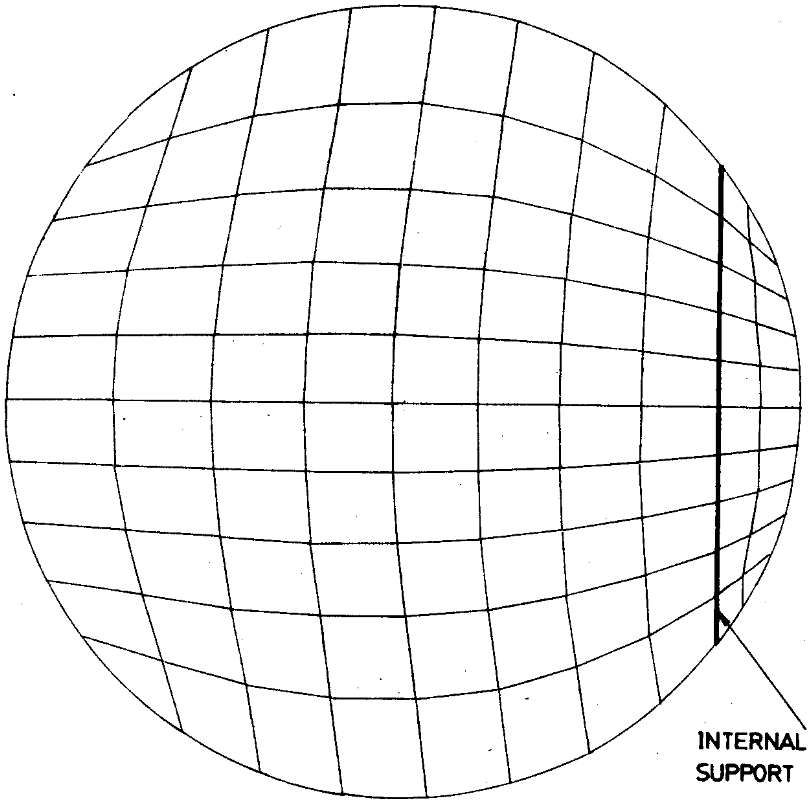

FIGURE 2: Finite Element Net

Table II depicts fundamental frequency coefficients for the case of plates elastically restrained against rotation along the boundary.

When $\frac{\bar{x}_0}{a} = 0$ the results are in good engineering agreement with frequency coefficients already available in the open literature [6].

TABLE II - Values of Ω_1, in the case of Circular Plates elastically restrained against rotation.

ϕ' \ $\frac{\bar{x}_0}{a}$	Present Study 0	Ref.6 0	0.1	0.2	0.3	0.4
0	21.26	21.27	20.26	18.17	16.20	14.71
0.1	18.64	18.55	17.84	16.12	14.42	13.10
1	15.42	14.97	14.81	13.43	11.99	10.77
10	14.34	-	13.77	12.48	11.12	9.96
∞	14.12	13.89	13.56	12.30	10.96	9.82

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