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MODELLING OF THE SQUEEZE FILM AIR DAMPING IN MEMS

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Abstract. We propose a formulation for modeling the squeeze film air damping in micro-plates typical of micro-electromechanical devices for micro switch applications. A special finite element is developed, in which the nonlinear Reynolds equation for compressible film is used to analyze the air pressure field, whereas a standard linear elastic model is used for the displacement field. The formulation is based on a finite element discretization of both the pressure and displacement fields. The coupled equations of motion are established and, for harmonic oscillations, we show that the resulting damping matrix depends on the frequency. The typical dimensions and properties of the MEMS device are in the order of hundred micrometers length and some micrometers (3-8 μm) thick, with a separation from the substrate of also some micrometers (e.g. 3-5 μm). For these dimensions, the influence of damping owing to the surrounding air cannot be neglected, having an important contribution to the quality factor of the device. The influence of plate holes, which are necessary because of the fabrication process, determines also the dynamic behavior of the plate. Examples are presented, with comparisons to results of the bibliography.

1 INTRODUCTION

The damping effect of the air is usually negligible in macroscopic systems. In MEMS (Judy, 2001; Ko, 2007) however, the motion of micro-parts can be affected significantly by the surrounding gas, notably in structures like the one depicted in Figure 1, where a microplate oscillates normally to the substrate. In this case, the gas is forced to squeeze in and out of the space between the surfaces, following the oscillatory motion of the upper plate. The “squeeze film” effect is much more important than the drag force of air acting over the isolated plate, and thus is the subject of intense research at present (Bao and Yang, 2007; Pratap et al. 2007). In order to quickly visualize the physics of the problem, the vibrating microplate may be thought of as an equivalent spring-mass-damper system under harmonic excitation. The transverse deflection $u(x,y,t)$ of a point on the plate is then given by

$$m\frac{\partial^2 u}{\partial t^2} + c_\mathrm{s}\frac{\partial u}{\partial t} + k_\mathrm{s} u = f_\mathrm{drive} + f_\mathrm{fluid}\,, \tag{1}$$

where m is the equivalent mass of the oscillating structure, c_s is the structural damping coefficient of the beam which accounts for internal energy losses, and k_s is the structural spring constant. Also in Eq. (1), f_drive is the external force that excites the beam and f_fluid is the damping force exerted by the gas present in the space between the microplate and the substrate. As it will be shown later, this last term has two components: one is in-phase with the velocity, due to the viscous flow of the gas, and the other is in phase with the displacement, due to the compression of the gas. The viscous effect directly contributes to the damping, while the elastic effect produces a shift in the resonance frequency. Therefore, a proper evaluation of f_fluid is crucial to improve the accuracy of computation of the dynamics of microplates integrated to MEMS. Previous works in the literature considered analytical models (Pandey et al. 2007; Veijola, 2004) and numerical calculations (Pandey and Pratap, 2008; De Pasquale and Veijola, 2008; Nayfeh and Younis, 2004).

The aim of this work is to discuss the modelling and simulation of the squeeze film air damping in MEMS. A special finite element is developed for this purpose. The formulation is based on a finite element discretization of both the pressure and displacement fields. The paper is organized as follows: Section 2 presents the fluid dynamics problem and analytical solutions available. Section 3 describes the numerical formulation of both the squeeze film air damping and the dynamics of a flexible microplate. Section 4 discusses the results of calculations, with comparisons to results of the bibliography.

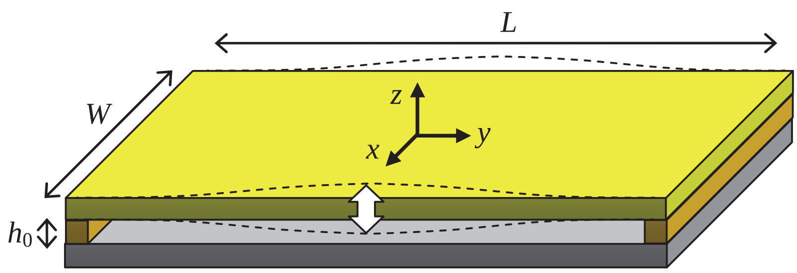

Figure 1: Schematic representation of a vibrating microplate over a substrate.

2 ANALYTICAL MODEL

2.1 Fluid dynamics problem

In the framework of continuum fluid mechanics, viscous forces are determined from the fluid stresses acting on the body surface. Thus the fluid velocity and pressure fields around the vibrating plate should be obtained by solving Navier-Stokes equations and the appropriate boundary conditions. For the particular case of squeeze film flow of an isothermal and compressible fluid, Navier-Stokes equations lead to the nonlinear Reynolds equation, which governs the pressure field associated to the 2D flow (x-y plane) generated by the movement of one plate in the z-direction (Veijola, 2004; Bao and Yang, 2007; Pratap et al. 2007). Given a gas of viscosity μ and density ρ placed in the gap between the surfaces (Figure 1), and assuming that no external forces are present (the body force due to gravity is normally neglected in MEMS) the Reynolds equation is written:

$$\frac{\partial}{\partial x}\left(\frac{Ph^3}{\mu}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{Ph^3}{\mu}\frac{\partial P}{\partial y}\right)=12\frac{\partial(Ph)}{\partial t}. \tag{2}$$

In this expression, $P = p_0 + p$, where p_0 is the ambient pressure and p is the deviatoric pressure generated by the displacement of the upper plate; $h = h_0 + z$, where h_0 is the equilibrium gap width and z is the displacement.

Equation (2) is valid if the system satisfies the following conditions: (*i*) laminar and fully developed flow, (*ii*) pressure does not vary in z-direction, and (*iii*) the fluid does not slip at the walls. The first condition implies low Reynolds number, or equivalently, inertial effects negligible in comparison to viscous effects. For the squeeze film flow related to vibrating plates, this number is defined $\mathrm{Re} = \omega h_0^2\rho/\mu$, where ω is the angular frequency of oscillation of the plate. It is also required a gap width h_0 much lower than the length L and the width W of the plate (Figure 1), so that border effects can be neglected. The second condition imposes that, when the upper plate oscillates out of plane (z-direction), it must describe a small amplitude motion in comparison with h_0. Finally, the third condition means very low Knudsen number; otherwise appropriate corrections should be included. This number is defined $\mathrm{Kn} = \lambda/h_0$, where λ is the mean free path of molecules (inversely proportional to P).

It is worth to add that gas microflows can be treated in the classical framework of continuum fluid mechanics if $\mathrm{Kn} < 0.001$ (Gad-el-Hak, 1999; Berli and Cardona, 2009), i.e. by using Eq. (2) with the no-slip boundary condition. In the range $0.001 < \mathrm{Kn} < 0.1$, Eq. (2) is still valid, but slip boundary conditions must be applied, such as the classical Maxwell slip-velocity equation. When $\mathrm{Kn} > 0.1$, the continuum fluid mechanics breaks down, and there is a transitional flow region towards the free molecule flow, where statistical approaches are required. For example, $\lambda = 65$ nm for ideal gases at room temperature and normal pressure. Therefore, the squeeze film flow in gaps on the order of 2 μm should be corrected for the slip at the walls. The simplest way to do is by replacing μ by the effective viscosity $\mu_{\mathrm{eff}} = \mu/(1+6\mathrm{Kn})$ in Reynolds equation, and therefore in the related results (Bao and Yang, 2007; Pratap et al. 2007).

2.2 Analytical solutions for the linear Reynolds equation

If $p \ll p_0$ and $z \ll h_0$, Eq. (2) may be linearized as follows,

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{12\mu}{h_0^2}\left(\frac{1}{h_0}\frac{\partial z}{\partial t}+\frac{1}{p_0}\frac{\partial p}{\partial t}\right), \tag{3}$$

This expression is the base of several analytical solutions reported in the literature. Here we consider a rectangular plate of length L (y-direction) and width W (x-direction), as shown in Figure 1. The plate undergoes a harmonic vibration of the form $z(y,t)/h_0 = \Phi(y)\,Ae^{i\omega t}$, where A is the normalized amplitude and $\Phi(y)$ is the shape of the deflected plate, which is considered to vary in the y-direction only. For flexible plates clamped at two opposite edges, the exact function $\Phi(y)$ is available, nevertheless it leads to tedious expressions of $p(x,y,t)$ (Zhang et al. 2004). For the purposes of this work, an approximate function for the deflection of the plate in the first resonance mode is used: $\Phi(y) = (2/L)^4 (L/2 - y)^2 (L/2 + y)^2$. In addition, since the flow domain has two edges open and two edges closed, the boundary conditions are written as follows,

$$\partial p/\partial y = 0 \text{ at } y = \pm L/2\,;\;\; p = 0 \text{ at } x = \pm W/2\,. \tag{4}$$

Solving Eq. (3) under these conditions yields (Pandey et al. 2007),

$$p(x,t) = P_{\mathrm{a}} \sum_{n=\mathrm{odd}} \frac{32(-1)^{(n-1)/2}}{15n\pi} \left(\frac{-i\omega A e^{i\omega t}}{i\omega + k_n^2/a^2} \right) \cos\left(\frac{n\pi x}{W} \right), \tag{5}$$

where $k_n^2 = n^2\pi^2 / W^2$ and $a^2 = 12\mu / h_0^2 p_0$. It is worth noting that the gas pressure only varies in the x-direction (transverse to the open edges).

2.3 Damping force over the microplate

The force exerted by the fluid, f_{fluid}, is obtained by integrating the pressure over the whole plate surface. Then the elastic and viscous components of the force are obtained by separating the real and imaginary parts of f_{fluid}. Considering $z(t) = h_0 A\cos(\omega t)$, one has

$$f_{\mathrm{fluid}}(t) = -k_{\mathrm{e}} z(t) + c_{\mathrm{v}}\, dz(t)/dt\,, \tag{6}$$

with the following expressions for the elastic and viscous damping coefficients, respectively,

$$k_{\mathrm{e}}(\sigma) = \frac{512\sigma^2 p_0 LW}{225\pi^6 h_0} \sum_{n=\mathrm{odd}} \frac{1}{n^2(n^4 + \sigma^2/\pi^4)}\,, \tag{7}$$

$$c_{\mathrm{v}}(\sigma) = \frac{512\sigma p_0 LW}{225\pi^4 h_0 \omega} \sum_{n=\mathrm{odd}} \frac{1}{(n^4 + \sigma^2/\pi^4)}\,, \tag{8}$$

In these expressions, $\sigma = 12\mu\omega W^2 / h_0^2 p_0$ is the so-called "squeeze number" (Bao and Yang, 2007; Pratap et al. 2007), which characterize the relative importance of each contribution to the force. At low frequencies the viscous component of the force dominates, then reaches a maximum and finally decreases at high frequencies. In contrast, the elastic component of the force, which is associated to the compressibility of the gas, always grows with ω. The crossover of the force components takes place at $\sigma_{\mathrm{c}} = \pi^2[1+(W/L)^2]$. Basically, if $\sigma << \sigma_{\mathrm{c}}$, the force is purely viscous (damping effect); if $\sigma >> \sigma_{\mathrm{c}}$, the force is purely elastic (spring effect); if $\sigma \approx \sigma_{\mathrm{c}}$, the combination of both effects significantly influence the dynamics of the oscillating plate.

It is useful to analyze the damping coefficients in the absence of compressibility effects. Introducing $\sigma \to 0$ into Eqs. (7) and (8) yields, respectively, $k_{\mathrm{e}} \approx 0$ and $c_{\mathrm{v}} \propto \mu L(W/h_0)^3$.

This result shows that the ratio W/h_0 is a key factor to control the squeeze film damping. On the other hand, having accurate values of h_0 is critical to attain reliable quantitative calculations, mainly because h_0 cannot be directly measured. The fabrication tolerance of MEMS is around 0.15 μm. Therefore, for a gap $h_0 \approx 2$ μm, the uncertainty in the calculation of f_{fluid} may be 25%.

3 NUMERICAL MODEL

This section presents the numerical formulation of the squeeze film air damping of microplates. The non-linear Reynolds equation for compressible film is used to analyze the air pressure field, whereas a standard linear elastic model is used for the displacement field. The formulation is based on a finite element discretization of both the pressure and displacement fields. The coupled equations of motion are established and, for harmonic oscillations, we show that the resulting damping matrix depends on the frequency.

3.1 Dynamic equilibrium of the elastic plate

The weak form of the dynamic equilibrium of the plate is

$$\int_V \delta\varepsilon_{ij}\sigma_{ij}dV + \int_V \delta u_j \rho_s \ddot{u}_j dV - \int_V \delta u_j b_j dV - \int_{S_{tot}} \delta u_j t_j dS = 0\,, \tag{9}$$

where $\mathbf{u}$ is the vector of mechanical displacements; $\mathbf{b}$, the vector of applied volumetric forces and $\mathbf{t}$, the vector of applied surface tractions. In this expression, the first term represents the variation of the energy of deformation; the second, the variation of the kinetic energy. S_{tot} is the total surface of the plate including the superior and inferior faces of the plate.

Considering the particular geometry of the microsystem (Figure 1), the applied traction forces are defined on the superior and inferior faces of the plate, $\mathbf{t}_{\text{sup}} = -\mathbf{n}_{\text{sup}}p_{\text{sup}}$, $\mathbf{t}_{\text{inf}} = -\mathbf{n}_{\text{inf}}p_{\text{inf}}$, and the body applied forces are negligible, $\mathbf{b} = 0$. The vectors $\mathbf{n}_{\text{sup}}$ and $\mathbf{n}_{\text{inf}}$ are the outer normals to the plate surface and we have $\mathbf{u}^{\mathrm{T}}\mathbf{n}_{\text{sup}} = \mathbf{u}^{\mathrm{T}}\mathbf{n}_{\text{inf}} = \mathbf{u}^{\mathrm{T}}\mathbf{n}$. Using the assumption that the pressure on the superior face is constant ($p_{\text{sup}} = p_0$, $p_{\text{inf}} = p_0+p$), the contribution of surface loads can thus be represented by a single surface integral. The variation of the energy of deformation is expressed with respect to the 6x1 strain and stress vectors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$. Moreover, if one only considers the vertical contribution of the displacement to the kinetic energy, the weak form of the dynamic equilibrium takes the final form,

$$\int_V \delta\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\sigma} dV + \int_V \delta\mathbf{u}^{\mathrm{T}}\rho_s \ddot{\mathbf{u}} dV - \int_S \delta\mathbf{u}^{\mathrm{T}}\mathbf{n}\Delta p dS = 0\,. \tag{10}$$

Let us consider the finite element interpolation formulae, $\mathbf{u}(x,y,z,t) = \mathbf{Q}(x,y,z)\mathbf{q}(t)$, $p(x,y,z,t) = \mathbf{R}(x,y,z)\mathbf{p}(t)$, where $\mathbf{q} = [\mathbf{u}_1^{\mathrm{T}} \ldots \mathbf{u}_N^{\mathrm{T}}]^{\mathrm{T}}$ is the nx1 vector of nodal displacements ($n = 3N$), $\mathbf{p} = [p_1 \ldots p_m]^{\mathrm{T}}$ is the nx1 vector of nodal pressures, and $\mathbf{Q}$ (resp. $\mathbf{R}$) is the 3xn (resp. 1xm) vector of interpolation functions. Using the strain-displacement relation $\boldsymbol{\varepsilon} = \mathbf{Bq}$, and the constitutive relation $\boldsymbol{\sigma} = \mathbf{C}\boldsymbol{\varepsilon}$, the weak form becomes,

$$\delta\mathbf{q}^{\mathrm{T}}(\mathbf{M}\ddot{\mathbf{q}} + \mathbf{Kq} - \mathbf{Gp}) = 0\,, \tag{11}$$

with the definitions

$$\mathbf{Q} = \int_V \rho \mathbf{Q}^{\mathrm{T}}\mathbf{Q} dV, \quad \mathbf{K} = \int_V \mathbf{B}^{\mathrm{T}}\mathbf{CB} dV, \quad \mathbf{G} = \int_S \mathbf{Q}^{\mathrm{T}}\mathbf{nR} dS. \tag{12}$$

3.2 Dynamic equilibrium of the compressible film

The squeeze film between the flexible plate and the support is modeled by using the nonlinear Reynolds equation (2), and boundary conditions (4). The weak form of the equilibrium condition is given by,

$$\begin{aligned}&\int_S \delta p\left[\frac{\partial}{\partial x}\left(\frac{ph^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{ph^3}{12\mu}\frac{\partial p}{\partial y}\right)-\frac{\partial(ph)}{\partial t}\right]dS\\ &+\int_0^W \delta p\left(\frac{ph^3}{12\mu}\frac{\partial p}{\partial x}\right)_{y=-L/2} dx-\int_0^W \delta p\left(\frac{ph^3}{12\mu}\frac{\partial p}{\partial x}\right)_{y=L/2} dx=0\end{aligned}. \tag{13}$$

After integration by parts, one obtains

$$\int_S \frac{ph^3}{12\mu}\left(\frac{\partial(\delta p)}{\partial x}\frac{\partial p}{\partial x}+\frac{\partial(\delta p)}{\partial y}\frac{\partial p}{\partial y}\right)dS+\int_S \delta p\left(h\frac{\partial p}{\partial t}+p\frac{\partial h}{\partial t}\right)dS\ . \tag{14}$$

Using $\partial h/\partial t=\mathbf{n}^{\mathrm{T}}\dot{\mathbf{u}}$ and the finite element interpolation formulae, this expression becomes, after normalization,

$$\delta\mathbf{p}^{\mathrm{T}}(\mathbf{E}\dot{\mathbf{p}}+\mathbf{A}\mathbf{p}+\mathbf{H}\dot{\mathbf{q}})=0\,, \tag{15}$$

with the definitions

$$\begin{aligned}&\mathbf{E}(\mathbf{q})=\int_S \frac{h}{p_0}\mathbf{R}^{\mathrm{T}}\mathbf{R}dS, \qquad \mathbf{H}(\mathbf{p})=\int_S \frac{p}{p_0}\mathbf{R}^{\mathrm{T}}\mathbf{n}^{\mathrm{T}}\mathbf{Q}dS,\\ &\mathbf{A}(\mathbf{p},\mathbf{q})=\int_S \frac{ph^3}{12\mu p_0}(\mathbf{R}_x^{\mathrm{T}}\mathbf{R}_x+\mathbf{R}_y^{\mathrm{T}}\mathbf{R}_y)dS.\end{aligned} \tag{16}$$

3.3 Coupled equations

In summary, if a structural damping contribution is considered, the coupled equations of motion are,

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+\mathbf{K}\mathbf{q}-\mathbf{G}\mathbf{p}=0\,, \tag{17}$$

$$\mathbf{E}(\mathbf{q})\dot{\mathbf{p}}+\mathbf{A}(\mathbf{p},\mathbf{q})\mathbf{p}+\mathbf{H}(\mathbf{p})\dot{\mathbf{q}}=0\,, \tag{18}$$

which can be written as

$$\begin{bmatrix}\mathbf{M} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{q}}\\ \ddot{\mathbf{p}}\end{bmatrix}+\begin{bmatrix}\mathbf{C} & 0\\ \mathbf{H} & \mathbf{E}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{q}}\\ \dot{\mathbf{p}}\end{bmatrix}+\begin{bmatrix}\mathbf{K} & -\mathbf{G}\\ 0 & \mathbf{A}\end{bmatrix}\begin{bmatrix}\mathbf{q}\\ \mathbf{p}\end{bmatrix}=0\,. \tag{19}$$

In the particular case $z<<h_0$ and $p<<p_0$, the matrices **E**, **A** and **H** would be constant, the model would be linear, and we would have $\mathbf{H}=\mathbf{G}^{\mathrm{T}}$.

Considering a harmonic behavior of the system, we get

$$\mathbf{p}(s)=-(s\mathbf{E}+\mathbf{A})^{-1}\mathbf{H}s\mathbf{q}(s)\,, \tag{20}$$

with s, the Laplace variable. The pressure variables can then be eliminated from the structural equations

$$(s^2\mathbf{M}+s\mathbf{D}(s)+\mathbf{K})\mathbf{q}(s)=0\,, \tag{21}$$

with the frequency damping matrix (for $\mathbf{C} = \mathbf{0}$)

$$\mathbf{D}(s) = \mathbf{G}(s\mathbf{E} + \mathbf{A})^{-1}\mathbf{H} . \tag{22}$$

Note that for very low frequencies, with $s \to 0$, the squeeze flow contribution is a pure damping resulting:

$$\lim_{s \to 0} \mathbf{D}(s) = \mathbf{G}\mathbf{A}^{-1}\mathbf{H}$$

On the other hand, at the high frequency limit with $s \to \infty$, the squeeze flow contribution is a pure stiffness resulting:

$$\lim_{s \to \infty} \mathbf{D}(s) = \frac{1}{s}\mathbf{G}\mathbf{E}^{-1}\mathbf{H}$$

and we observe that the squeeze film effect induces a stiffening of the micro-structure.

4 RESULTS

We describe below some preliminary results of computations. Further results with applications will be given in the oral presentation.

Consider a rectangular plate with comparable side lengths $W = 2a$ and $L = 2b$ as shown in Fig. 2. The boundary conditions for the squeeze film problem are

$$p(\pm a, y) = 0; \quad p(x, \pm b) = 0$$

Let us suppose the plate is oscillating rigidly in the vertical direction, with very small amplitude of motion. Dimensions of the plate are length $L = 300\,\mu\text{m}$ and width $W = 50$ μm. The plate is separated from the substrate by an air gap of $h_0 = 5\ \mu\text{m}$. The air pressure is $p_0 = 100000\ \text{N/m}^2$ and viscosity is $\mu = 1.8 \times 10^{-5}\,\text{Pa} \cdot \text{s}$, with air assumed incompressible. Only one quarter of the plate is modeled for symmetry reasons, with a mesh of $n_b \times n_a = 150 \times 100$ rectangular bilinear elements.

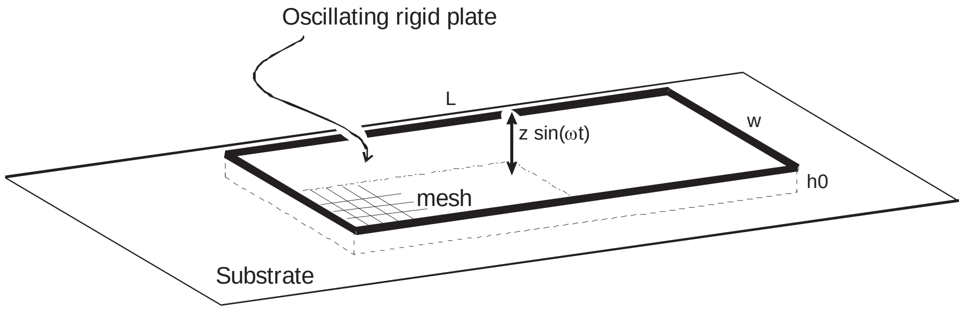

Figure 2: Rigid plate problem

Since the amplitude of motion $z \ll h_0$, and $p \ll p_0$, then the matrices **E**, **A** and **H** are constant, with $\mathbf{H} = \mathbf{G}^{\mathrm{T}}$. The damping coefficient due to vertical motion of the plate can be computed as:

$$d(i\omega) = \mathbf{\Phi}_R^T \mathbf{D}(i\omega) \mathbf{\Phi}_R = \mathbf{\Phi}_R^T \mathbf{H}^T \left(i\omega\mathbf{E} + \mathbf{A}\right)^{-1} \mathbf{H}\mathbf{\Phi}_R$$

where $\mathbf{\Phi}_R$ is the rigid body mode of the plate with components of vertical displacement equal to one.

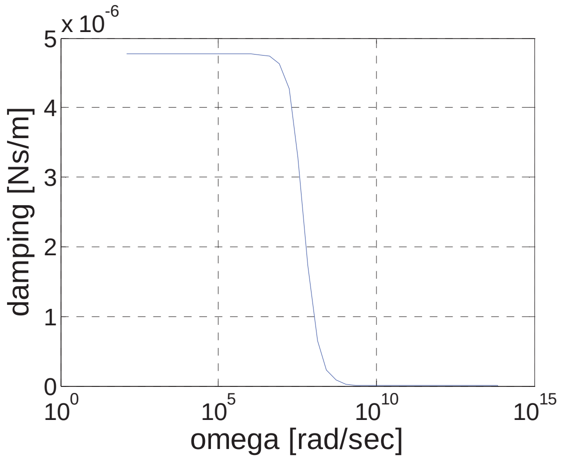

Figure 3: Damping component of the damping coefficient for the rigid plate problem

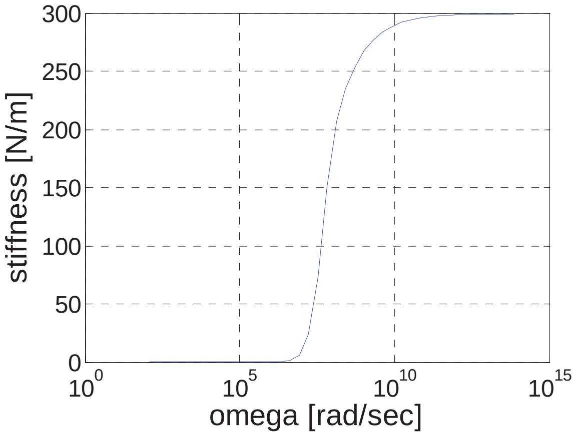

Figure 4: Stiffness component of the damping coefficient for the rigid plate problem

Figures 3 and 4 show the real and imaginary components of the damping coefficient as a function of the frequency of excitation. We may see that for low frequencies of excitation, the damping coefficient is pure damping, while for high frequencies this coefficient is practically a stiffness component.

These values can be compared to limit values obtained from the bibliography (Bao and Yang, 2007). The coefficient of the damping force at $\omega \to 0$ can be written as:

$$c_{\mathrm{v}} = \mu L \left(\frac{W}{h_0} \right)^3 \beta(\eta)$$

where $\eta = W/L$ and $\beta(\eta) = 1 - 0.58\eta$. In this case, $c_{\mathrm{v}} = 4.878 \times 10^{-6}$ Ns/m. On the other hand, the coefficient of the damping force at $\omega \to \infty$ can be written as:

$$k_{\mathrm{e}} = \frac{p_0 WL}{h_0}$$

giving $k_{\mathrm{e}} = 300$ N/m.

5 CONCLUSIONS

A finite element model for the analysis of squeeze film flow in MEMS has been presented. The element has been tested in a simple configuration for which analytical results exist. Comparisons have been made for limit cases of very low and very high frequencies of excitation, displaying a very good agreement.

Future research will be addressed to making further testing, and integrating the element completely in a finite element code specialized for MEMS analysis, to be able to represent other forces that are important in this kind of problems.

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