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FINITE CALCULUS. A PARADIGM FOR DERIVING STABILIZED FINITE ELEMENT METHODS IN COMPUTATIONAL MECHANICS

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Abstract. Finite Calculus (FIC) is a procedure for deriving the governing equations in mechanics assuming that the space-time domain where the balance laws are imposed has a finite size. This is a key difference versus standard infinitesimal calculus which assumes that the dimensions of the balance domain are of infinitesimal size [1].

The consequence of the FIC assumption is that the governing equations, i.e. the momentum equations and their boundary conditions in fluid and solid mechanics, the mass balance equation in fluid mechanics , etc., have additional terms that are a function of the dimensions of the balance domain and the space and time derivatives of the classical infinitesimal equations [1-4].

The so-called *modified governing equations* have many interesting properties .Their analytical solution includes that of the infinitesimal equations and additional solutions that in most cases are negligible versus the “exact” infinitesimal solution. Moreover, the additional terms introduced by the FIC assumption act as stabilization terms in the numerical solution of the modified equations using the FEM or any other numerical method. In addition, the space and time dimensions of the balance domain can be adjusted so as to yield the *exact solution* at the nodes of a finite element mesh [1-7].

The paper presents an overview of recent developments on the FIC approach for solving a variety of problems in computational mechanics with the FEM. In particular the following problems are addressed:

- Convection-diffusion and convection-diffusion-reaction problems
- Incompressible problems in solid and fluid mechanics
- High Reynolds number flows
- Explicit time integration schemes for the transport equation with increased stability and accuracy.

The merits of the FIC-FEM formulation for solving each one of the above problems are discussed. Finally, examples of application showing the potential and advantages of the FIC technique are presented.

References.

[1] Oñate E., Derivation of stabilized equations for numerical solution of advective-diffusive transport and fluid flow problems. *Computer Methods in Applied Mechanics and Engineering*, 151 (1-2):233-265, 1998.

[2] Oñate E., A stabilized finite element method for incompressible viscous flows using a finite increment calculus formulation. *Computer Methods in Applied Mechanics and Engineering,* 182 (3-4):355-370, 2000.

[3] Oñate E., Taylor R.L., Zienkiewicz O. C., and Rojek J., A residual correction method based on finite calculus. *Engineering Computations*, 20 (5-6):629-658, 2003.

[4] Oñate E., Possibilities of finite calculus in computational mechanics. *International Journal for Numerical Methods in Engineering*, 60 (1):255-281, 2004.

[5] Felippa C. A. and Oñate E., Nodally exact Ritz discretizations of 1D diffusion-absorption and Helmholtz equations by variational FIC and modified equation methods. *Computational Mechanics*, 39 (2):91-111, 2007.

[6] Oñate E., Valls A. and García J., Computation of turbulent flows using a finite calculus-finite element formulation. *International Journal for Numerical Methods in Fluids*, 54 (6-8):609-637, 2007.

[7] Oñate E., Nadukandi P., Idelsohn S.R., García, J. and Felippa C., A family of residual-based stabilized finite element methods for Stokes flows. Submitted to *International Journal for Numerical Methods in Fluids*, September 2010.