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TIME DOMAIN MODELING OF BRIDGE DECK FLUTTER AND APPLICATIONS

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Abstract. Time domain modeling of bridge deck flutter and its applications are examined in this article. The frequency dependent aerodynamic self-excited forces acting on a bridge deck are approximated in the Laplace domain by rational functions. The matrix formulation of the rational functions known as "Minimum State" is applied to aerodynamic data obtained experimentally from various types of decks. The precision of the approximations is calculated, and plots are drawn of the approximation functions compared to the available tabular data. The state-space equations of motion describing the aero elastic behavior of a section of a bridge deck are presented. Given the dynamic data of a bridge structure (mass, rotational mass moment of inertia, natural frequencies, stiffness and damping ratios), and assuming that a geometric similarity exists between the profiles of the full-scale bridge deck and the sectional model from which the frequency dependent aerodynamic data was extracted, it is possible to calculate the critical velocity of that particular bridge. This study shows that it is possible to build a catalog of several profiles, characterized by frequency dependent aerodynamic data and the corresponding rational functions. This catalog could form the basis for calculating a bridge's aerodynamic stability without recurring to wind tunnels.

1 INTRODUCTION

Time-domain modeling of bridge deck flutter was examined , e.g., by Wilde et al.(1996), where the frequency dependent aerodynamic unsteady forces on a bridge deck are approximated in the Laplace domain by rational functions. The matrix formulation of the rational approximation suggested by Karpel (1981), called “Minimum State”, is adopted in the present article, and applied to flutter derivatives for various sections reported by Starossek (2009), and Starossek et al. (2009). The precision of the approximations is also presented.

Finally, critical velocity computations of several bridge decks are presented, using time-domain modeling and rational function approximations, considering structural data reported by Thiesemann (2008) and Wilde, Fugino (1998), whose results are compared to the ones obtained by the present authors.

2 EQUATION OF FLUTTER OF A BRIDGE DECK

The critical or flutter velocity is the wind velocity under which the bridge becomes unstable. The flutter problem of airplane wings was solved by Theodorsen (1934), whose approach to define the unsteady aerodynamic forces considered velocity potentials due to the flow around an airfoil profile and the adoption of the Kutta condition. Significant errors are expected if this theory is applied to bridges with bluff girders or profiles which do not conform to Theodorsen hypothesis. Scanlan (1971) introduced the term “flutter derivatives” which expresses the unsteady forces obtained theoretically or experimentally for any type of bridge deck, either in wind tunnels or water channels.

Roger (1977) proposed the modeling method which transforms the aero elastic equations of the motion of an airplane into time domain. The method approximates aerodynamic force coefficients by rational functions of the Laplace variable. The formulation suggested by Karpel (1981), called minimum-state RFA (Rational Function Approximation), is used in this article.

2.1 Equations of motion of the mechanical system bridge deck - wind

The linear dynamic system of a bridge deck subjected to unsteady wind forces is approximated by the following two degrees of freedom equations:

$$m\ddot{h} + c_h\dot{h} + k_h h = -\rho U^2 B\left[K.H_1^*(k).(\frac{\dot{h}}{U}) + K.H_2^*(k).B.(\frac{\dot{\alpha}}{U}) + K^2.H_3^*(k).\alpha + K^2.H_4^*(k).\frac{h}{B}\right] \quad (1)$$

$$I_\alpha\ddot{\alpha} + c_\alpha\dot{\alpha} + k_\alpha\alpha = \rho U^2 B^2\left[K.A_1^*(k).(\frac{\dot{h}}{U}) + K.A_2^*(k).B.(\frac{\dot{\alpha}}{U}) + K^2.A_3^*(k).\alpha + K^2.A_4^*(k).\frac{h}{B}\right] \quad (2)$$

In the above equations, $H_i\ ,i=1{,}4$ and $A_i\ ,i=1{,}4$ are flutter derivatives obtained either experimentally or theoretically. Transforming (1) and (2) into the Laplace domain with zero initial conditions gives:

$$\{(mB)\,s^2 + (c_h B)s + (k_h B)\}\ \mathscr{L}(\frac{h}{B}) = (-\frac{1}{2}\rho U^2 B)[2K^2 H_4^* + 2KH_1^*\frac{sB}{U}]\ \mathscr{L}(\frac{h}{B}) +$$

$$+(-\frac{1}{2}\rho U^2 B)[2K^2 H_3^* + 2KH_2^*.\frac{sB}{U}]\ \mathscr{L}(\alpha)$$

$$\{(I_\alpha)\, s^2 + (c_\alpha) s + (k_\alpha)\}\ \mathscr{L}(\alpha) =$$

$$= (\frac{1}{2}\rho U^2 B^2)[2K^2 A_3^* + 2KA_2^* \frac{sB}{U}]\ \mathscr{L}(\alpha) + (\frac{1}{2}\rho U^2 B^2)[2K^2 A_4^* + 2KA_1^* \frac{sB}{U}]\ \mathscr{L}(\frac{h}{B})$$

Substituting the dimensionless Laplace variable $p = s\,.B/U$ in the above equations results in:

$$\{(mB)\, p^2 (\frac{U^2}{B^2}) + (c_h B) p (\frac{U}{B}) + (k_h B)\}\ \mathscr{L}(\frac{h}{B}) =$$

$$-\frac{1}{2}\rho\, U^2 B\,[2K^2 H_4^* + p.2K.H_1^*]\ \mathscr{L}(\frac{h}{B}) - \frac{1}{2}\rho U^2 B[2K^2 H_3^* + p.2KH_2^*]\ \mathscr{L}(a) \tag{3}$$

$$\{(I_\alpha) p^2 .(\frac{U^2}{B^2}) + (c_\alpha) p.(\frac{U}{B}) + (k_\alpha)\}\ \mathscr{L}(\alpha) =$$

$$(\frac{1}{2}\rho U^2 B^2)[2K^2 A_3^* + 2KA_2^* .p]\ \mathscr{L}(a) + \frac{1}{2}\rho U^2 B^2 [2K^2 A_4^* + p.2K\ A_1^*]\,\}\, \mathscr{L}(\frac{h}{B}) \tag{4}$$

The equations above can be represented in matrix form as follows:

$$[\mathbf{M}.p^2.(\frac{U}{B})^2 + \mathbf{C}.p.(\frac{U}{B}) + \mathbf{K}].\ \mathscr{L}(\mathbf{q}) = [\mathbf{V}_f].[\mathbf{Q}].U^2.\ \mathscr{L}(\mathbf{q}) \tag{5}$$

The matrices that compose equations (5) are:

$$\mathbf{M} = \begin{bmatrix} mB & 0 \\ 0 & I_\alpha \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} c_h B & 0 \\ 0 & c_\alpha \end{bmatrix} = \begin{bmatrix} 2\,\xi_h \omega_h\ m\,B & 0 \\ 0 & 2\,\xi_\alpha \omega_\alpha\ I_\alpha \end{bmatrix}; \tag{6}$$

$$\mathbf{K} = \begin{bmatrix} k_h B & 0 \\ 0 & k_\alpha \end{bmatrix} = \begin{bmatrix} \omega_h{}^2 mB & 0 \\ 0 & \omega_\alpha{}^2 I_\alpha \end{bmatrix}; \tag{7}$$

$$\mathbf{q} = \begin{bmatrix} h/B \\ \alpha \end{bmatrix};\ \mathbf{V}_f = \begin{bmatrix} -0.5\,\rho\, B & 0 \\ 0 & 0.5\,\rho\, B^2 \end{bmatrix}; \tag{8}$$

$$\mathbf{Q} = \begin{bmatrix} 2K^2 H_4^* + p.2KH_1^* & 2K^2 H_3^* + p.2KH_2^* \\ 2K^2 A_4^* + p.2KA_1^* & 2K^2 A_3^* + p.2KA_2^* \end{bmatrix} \tag{9}$$

3 RATIONAL FUNCTION APPROXIMATION (RFA) FOR UNSTEADY AERODYNAMICS

3.1 Karpel's minimum-state RFA

The formulation reported by Karpel (1981), called minimum-state RFA, approximates $\mathbf{Q}$ to $\hat{\mathbf{Q}}(p)$ by the following rational equations:

$$\hat{\mathbf{Q}}(p) = \mathbf{A}_0 + \mathbf{A}_1 p + \mathbf{D}(p\mathrm{I} + \mathbf{R})^{-1}\mathbf{E} \tag{10}$$

For 3 lag terms, the sizes of the matrices in (10) are expressed as:

$$\hat{\mathbf{Q}}(p) = \mathbf{A}_0\,[2x2] + \mathbf{A}_1\,[2x2]p + \mathbf{D}[2x3]\left\{ p\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} + \begin{bmatrix}\lambda_1&0&0\\0&\lambda_2&0\\0&0&\lambda_3\end{bmatrix} \right\}^{-1} .\,\mathbf{E}(3x2) \tag{11}$$

where $\mathbf{R}$ is the diagonal matrix of the lag parameters λ_i, $i = 1, 2, 3$.

The unsteady aerodynamic data $\mathbf{Q}$, obtained from experiments or through Theodorsen functions are determined only for purely imaginary terms of the dimensionless Laplace variable $p = iK$. Thus, the approximation is performed for oscillatory motion only. Substituting $p = \frac{B}{U}s = iK$, where $K = \frac{Bw}{U}$ is the reduced frequency, in (9) and (11), results in

$$\mathbf{Q}(K) = \begin{bmatrix} 2K^2H_4^* + 2K^2H_1^*\,i & 2K^2H_3^* + 2K^2H_2^*\,i \\ 2K^2A_4^* + 2K^2A_1^*\,i & 2K^2A_3^* + 2K^2A_2^*\,i \end{bmatrix} \tag{12}$$

$$\hat{\mathbf{Q}}(K) = \mathbf{A}_0\,[2x2] + \mathbf{A}_1\,[2x2]K\,i + \mathbf{D}[2x3].\left\{ \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}K\,i + \begin{bmatrix}\lambda_1&0&0\\0&\lambda_2&0\\0&0&\lambda_3\end{bmatrix} \right\}^{-1} .\;\mathbf{E}[3x2] \tag{13}$$

3.2 Procedure to approximate $\mathbf{Q}$ to $\hat{\mathbf{Q}}$.

In the minimum state formulation (13), the numerator coefficients for the lag parameters are the product elements of $\hat{\mathbf{Q}}$ and $\mathbf{E}$. Therefore, a two-step iterative linear optimization is employed. First, for the selected initial $\mathbf{R}$ and $\mathbf{D}$, the matrices $\mathbf{A}_0$, $\mathbf{A}_1$, and $\mathbf{E}$ are obtained through a least-squares optimization such that the total approximation error:

$$J = \sqrt{\sum_{i=1}^{2}\sum_{j=1}^{2} w_{ij}\varepsilon_{ij}} \tag{14}$$

is minimized. The weighing factor is denoted by w_{ij}, and the measure of error between the approximating curve and the actual tabular data is:

$$\varepsilon_{ij} = \frac{\sum_n \left\| \hat{Q}_{ij}(p) - Q_{ij}(p) \right\|^2}{M_{ij}} \tag{15}$$

where

$$M_{ij} = \max_{n}\{1, \left\| Q_{ij}\left(iK_n\right)\right\|^2\} \tag{16}$$

In the next step, for the same $\mathbf{R}$ and previously determined $\mathbf{E}$, the matrices $\mathbf{A}_0$, $\mathbf{A}_1$ and new $\mathbf{D}$ are computed. These steps are repeated till the global approximation error (15) converges or reaches the stopping criterion of a maximum number of interactions. The lag parameters λ_l are in the denominator and are found by a non-linear no-gradient optimizer proposed by Nelder and Mead (1965). A Fortran program written by Masukawa (1994) followed the procedure described above to approximate $\mathbf{Q}$ to $\hat{\mathbf{Q}}$. This program was applied to aerodynamic data derived from flutter derivatives obtained from experiments in the water channel of the University of Harburg, as reported by Starossek (2009), in order to calculate the rational approximation functions of eight typical bridge deck profiles, depicted in Figure 1. Plots of the derivatives of these eight typical deck profiles and Theodorsen's for comparison purposes was presented by Thiesemann (2008).

4 RATIONAL FUNCTION APPROXIMATIONS OF EIGHT TYPICAL BRIDGE DECK PROFILES

Eight typical bridge deck profiles studied by Starossek (2009) and Starossek et al. (2009) are shown in Figure 1.

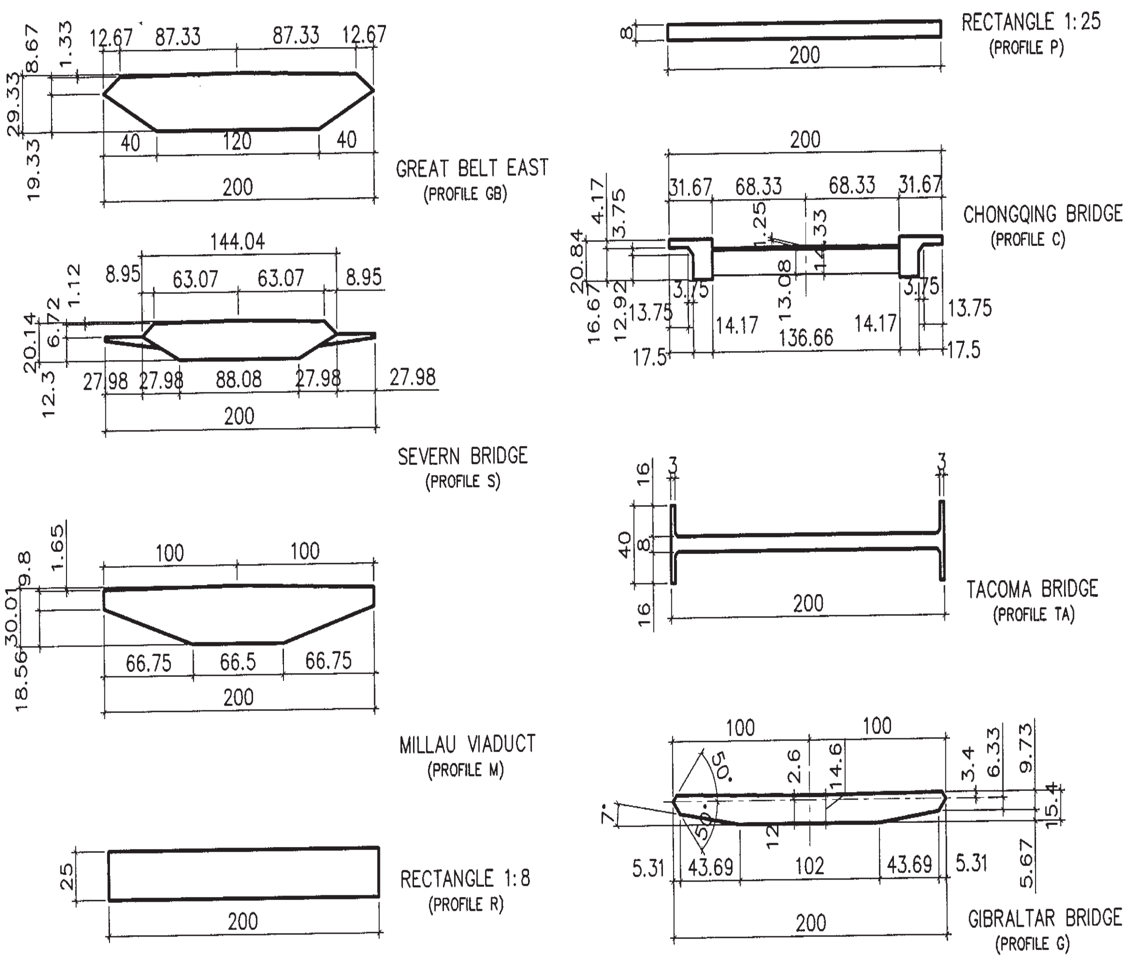

Figure 1- Investigated profiles by Starossek (2009) and Starossek et al. (2009).

4.1.1 Profile GB

GB									
k	H2 *(-1)	H3 *(-1)	A2	A3	k	H1 *(-1)	H4 *(-1)	A1	A4
0.1783	-2.4100	16.2720	-0.9200	4.1750	0.1720	6.1930	2.1250	1.4660	0.4430
0.2676	-0.7750	6.4940	-0.4070	1.7260	0.2580	3.8410	0.9110	0.9170	0.2400
0.3565	-0.1710	3.3680	-0.2100	0.9340	0.3441	2.7430	0.4270	0.6480	0.1600
0.4454	0.0750	2.0240	-0.1270	0.5890	0.4301	1.9780	0.2880	0.5100	0.1060
0.5346	0.2110	1.3520	-0.0850	0.4080	0.5161	1.5880	0.0380	0.4160	0.0750
0.6240	0.2540	0.9640	-0.0610	0.3020	0.6018	1.3100	-0.0260	0.3510	0.0510
0.7129	0.2420	0.7360	-0.0470	0.2340	0.6879	0.9500	-0.3310	0.3100	0.0380
0.8020	0.2440	0.5700	-0.0350	0.1890	0.7736	0.9500	-0.1820	0.2720	0.0290
0.8910	0.2430	0.4550	-0.0290	0.1560	0.8600	0.8620	-0.2280	0.2420	0.0240
0.9805	0.2340	0.3760	-0.0240	0.1320	0.9454	0.7560	-0.2750	0.2230	0.0140
1.0697	0.2230	0.3130	-0.0210	0.1150	1.0314	0.6920	-0.2810	0.2020	0.0110
1.1576	0.2100	0.2640	-0.0170	0.1010	1.1180	0.6490	-0.2890	0.1860	0.0020
1.2472	0.2020	0.2220	-0.0140	0.0910	1.2037	0.5870	-0.3290	0.1750	0.0020
1.3368	0.1960	0.1840	-0.0120	0.0830	1.2897	0.5610	-0.2790	0.1610	-0.0100
1.4254	0.1880	0.1590	-0.0100	0.0760	1.3755	0.4470	-0.2970	0.1600	-0.0170

Table 1 - GB Derivatives

$$A0 = \begin{vmatrix} -0.4636738E+02 & 0.8228331E+01 \\ 0.1035637E+01 & 0.1596801E+01 \end{vmatrix}$$

$$A1 = \begin{vmatrix} 0.8914992E+01 & 0.7375850E+00 \\ 0.8926457E+00 & -0.2251520E+00 \end{vmatrix}$$

$$D = \begin{vmatrix} 0.3428018E+00 & -0.3723400E+03 & 0.3051064E+03 \\ 0.1010667E+00 & 0.4536275E+02 & -0.3639534E+02 \end{vmatrix}$$

$$E = \begin{vmatrix} -0.1936388E+01 & 0.2620034E+02 & 0.3299231E+02 \\ 0.2423165E+01 & -0.7610449E+01 & -0.9443313E+01 \end{vmatrix}$$

$$\lambda 11.\ \lambda 22, \lambda 33 = \begin{vmatrix} 0.4620845E+00 & 0.4960738E+01 & 0.4998248E+01 \end{vmatrix}$$

K	Q11		Q11		Q12		Q12	
	Q11 r (exp)	Q11 r (app)	Q11 i (exp)	Q11 i (app)	Q12 r (exp)	Q12 r (app)	Q12 i (exp)	Q12 i (app)
0.34	0.5032	0.0287	1.4664	1.5041	4.1369	4.1048	-0.6127	-0.5379
0.52	0.4850	0.2169	2.0449	1.9668	3.7189	3.7250	-0.4438	-0.3823
0.69	0.4045	0.2585	2.5982	2.3312	3.4238	3.4738	-0.1738	-0.1480
0.86	0.4261	0.1905	2.9267	2.6835	3.2126	3.2878	0.1190	0.1239
1.03	0.0810	0.0360	3.3840	3.0444	3.0917	3.1521	0.4825	0.3986
1.2	-0.0753	-0.1902	3.7959	3.4213	3.0024	3.0474	0.7911	0.6714
1.38	-1.2530	-0.4982	3.5963	3.8414	2.9921	2.9632	0.9838	0.9429
1.55	-0.8714	-0.8484	4.5483	4.2604	2.9333	2.8977	1.2557	1.1993
1.72	-1.3490	-1.2528	5.1003	4.7034	2.8896	2.8406	1.5432	1.4721
1.89	-1.9664	-1.7087	5.4057	5.1731	2.8920	2.7951	1.7998	1.7469
2.06	-2.3913	-2.2136	5.8889	5.6726	2.8650	2.7606	2.0412	2.0237
2.24	-2.8898	-2.7987	6.4897	6.2372	2.8299	2.7372	2.2511	2.3024
2.41	-3.8133	-3.3960	6.8037	6.8074	2.7624	2.7252	2.5135	2.5667
2.58	-3.7123	-4.0336	7.4645	7.4163	2.6307	2.7232	2.8023	2.8470
2.75	-4.4953	-4.7078	6.7656	8.0665	2.5844	2.7321	3.0558	3.1267
ε_{ij}	ε_{11} =	0.5684926E-01			ε_{12} =	0.7832406E-02		

K	Q21		Q21		Q22		Q22	
	Q21 r (exp)	Q21 r (app)	Q21 i (exp)	Q21 i (app)	Q22 r (exp)	Q22 r (app)	Q22 i (exp)	Q22 i (app)
0.36	0.1049	0.0949	0.3471	0.4283	1.0614	1.1063	-0.2339	-0.2412
0.54	0.1278	0.1649	0.4882	0.5592	0.9884	1.0124	-0.2331	-0.2399
0.71	0.1516	0.1977	0.6138	0.6640	0.9495	0.9614	-0.2135	-0.2162
0.89	0.1568	0.2057	0.7546	0.7681	0.9349	0.9363	-0.2016	-0.1887
1.07	0.1598	0.1969	0.8865	0.8779	0.9330	0.9307	-0.1944	-0.1657
1.25	0.1478	0.1770	1.0171	0.9953	0.9406	0.9378	-0.1900	-0.1493
1.43	0.1439	0.1478	1.1735	1.1285	0.9513	0.9535	-0.1911	-0.1396
1.6	0.1388	0.1153	1.3022	1.2624	0.9726	0.9737	-0.1801	-0.1364
1.78	0.1420	0.0802	1.4319	1.4040	0.9907	0.9990	-0.1842	-0.1388
1.96	0.1001	0.0438	1.5945	1.5526	1.0153	1.0271	-0.1846	-0.1468
2.14	0.0936	0.0074	1.7190	1.7079	1.0526	1.0570	-0.1922	-0.1600
2.32	0.0200	-0.0301	1.8599	1.8788	1.0827	1.0878	-0.1822	-0.1778
2.49	0.0232	-0.0637	2.0284	2.0457	1.1323	1.1172	-0.1742	-0.1986
2.67	-0.1331	-0.0948	2.1422	2.2172	1.1867	1.1483	-0.1716	-0.2244
2.85	-0.2573	-0.1231	2.4217	2.3927	1.2353	1.1789	-0.1625	-0.2538
ε_{ij}	ε_{21} =	0.1366064E-01			ε_{22} =	0.2025828E-01		

$$J = 0.3140073E+00$$

Table 2 - Unsteady Aerodynamic Data - Experimental and approximations - Profile GB

4.1.2 Profile S

S									
k	H2 *(-1)	H3 *(-1)	A2	A3	k	H1 *(-1)	H4 *(-1)	A1	A4
0.1870	0.1820	11.8240	-0.8140	4.1390	0.1867	3.7370	-0.6990	1.4690	0.3240
0.2805	0.7170	4.9160	-0.3970	1.7570	0.2802	2.7460	-0.4930	0.9510	0.2210
0.3741	0.7130	2.7020	-0.2500	0.9660	0.3739	2.0310	-0.4200	0.6850	0.1750
0.4676	0.6820	1.6980	-0.1700	0.6140	0.4673	1.5930	-0.4030	0.5350	0.1260
0.5612	0.6130	1.1890	-0.1310	0.4260	0.5609	1.3190	-0.4550	0.4390	0.1080
0.6545	0.5420	0.8770	-0.1010	0.3170	0.6542	1.1150	-0.4590	0.3700	0.0930
0.7482	0.4930	0.6840	-0.0840	0.2460	0.7478	0.9630	-0.4970	0.3240	0.0750
0.8416	0.4530	0.5430	-0.0710	0.1980	0.8413	0.8530	-0.5370	0.2890	0.0620
0.9347	0.4160	0.4520	-0.0610	0.1650	0.9344	0.7800	-0.5540	0.2590	0.0590
1.0280	0.3820	0.3820	-0.0540	0.1400	1.0280	0.7180	-0.5650	0.2350	0.0510
1.1212	0.3500	0.3320	-0.0470	0.1220	1.1216	0.6760	-0.5730	0.2180	0.0470
1.2148	0.3230	0.2930	-0.0420	0.1080	1.2158	0.6420	-0.5790	0.2010	0.0440
1.3090	0.3020	0.2610	-0.0390	0.0980	1.3085	0.6090	-0.5760	0.1870	0.0410
1.4019	0.2830	0.2360	-0.0370	0.0890	1.4025	0.5890	-0.5830	0.1770	0.0360
1.4953	0.2650	0.2150	-0.0340	0.0830	1.4960	0.5780	-0.5810	0.1690	0.0360

Table 3 - S Derivatives

$$A0 = \begin{vmatrix} -0.6216372E+02 & 0.1647552E+02 \\ 0.2179667E+02 & 0.4522766E+00 \end{vmatrix}$$

$$A1 = \begin{vmatrix} 0.1209033E+02 & -0.3895692E-01 \\ -0.7804527E+00 & -0.4248660E+00 \end{vmatrix}$$

$$D = \begin{vmatrix} 0.2487929E+08 & -0.1675641E+09 & 0.1443836E+10 \\ 0.5315850E+08 & -0.6806189E+09 & 0.2841914E+11 \end{vmatrix}$$

$$E = \begin{vmatrix} -0.4173372E-04 & -0.9753603E-05 & -0.1624806E-06 \\ 0.1550400E-04 & 0.3280318E-05 & 0.5030544E-07 \end{vmatrix}$$

$$\lambda 11.\ \lambda 22,\ \lambda 33 = \begin{vmatrix} 0.4090739E+01 & 0.4479433E+01 & 0.4802952E+01 \end{vmatrix}$$

K	Q11		Q11		Q12		Q12	
	Q11 r (exp)	Q11 r (app)	Q11 i (exp)	Q11 i (app)	Q12 r (exp)	Q12 r (app)	Q12 i (exp)	Q12 i (app)
0.37	-0.1949	-0.0937	1.0422	1.0522	3.3093	3.1601	0.0509	0.4350
0.56	-0.3097	-0.2594	1.7250	1.5854	3.0949	3.1338	0.4514	0.6723
0.75	-0.4696	-0.5013	2.2711	2.1124	3.0257	3.1026	0.7984	0.9250
0.93	-0.7040	-0.8071	2.7828	2.6081	2.9697	3.0715	1.1928	1.1962
1.12	-1.1452	-1.2177	3.3197	3.1321	2.9958	3.0475	1.5445	1.4714
1.31	-1.5717	-1.7249	3.8179	3.6646	3.0054	3.0332	1.8574	1.7809
1.5	-2.2235	-2.3334	4.3083	4.2156	3.0631	3.0357	2.2077	2.1082
1.68	-3.0410	-3.0057	4.8305	4.7650	3.0766	3.0578	2.5667	2.4319
1.87	-3.8699	-3.8163	5.4487	5.3862	3.1593	3.1052	2.9077	2.7843
2.06	-4.7767	-4.7278	6.0703	6.0620	3.2296	3.1798	3.2296	3.1430
2.24	-5.7666	-5.6794	6.8032	6.7635	3.3388	3.2767	3.5198	3.4845
2.43	-6.8467	-6.7696	7.5917	7.5790	3.4594	3.4070	3.8136	3.8421
2.62	-7.8891	-7.9387	8.3411	8.4806	3.5777	3.5651	4.1398	4.1923
2.8	-9.1741	-9.1093	9.2685	9.4206	3.7104	3.7392	4.4493	4.5137
2.99	-10.4023	-10.4007	10.3485	10.5084	3.8457	3.9464	4.7401	4.8387
ε_{ij}	ε_{11} =		0.1452171E-02		ε_{12} =		0.9683110E-02	

K	Q21		Q21		Q22		Q22	
	Q21 r (exp)	Q21 r (app)	Q21 i (exp)	Q21 i (app)	Q22 r (exp)	Q22 r (app)	Q22 i (exp)	Q22 i (app)
0.37	0.0904	0.0962	0.4097	0.4081	1.1584	1.1473	-0.2278	-0.1367
0.56	0.1388	0.1314	0.5974	0.6079	1.1061	1.1322	-0.2499	-0.1992
0.75	0.1957	0.1755	0.7660	0.7975	1.0817	1.1147	-0.2800	-0.2535
0.94	0.2201	0.2214	0.9346	0.9674	1.0739	1.0980	-0.2973	-0.2984
1.12	0.2718	0.2699	1.1049	1.1375	1.0733	1.0861	-0.3301	-0.3325
1.31	0.3184	0.3152	1.2669	1.3010	1.0863	1.0804	-0.3461	-0.3605
1.5	0.3355	0.3548	1.4495	1.4619	1.1016	1.0844	-0.3762	-0.3824
1.68	0.3511	0.3865	1.6366	1.6161	1.1219	1.0988	-0.4023	-0.3999
1.87	0.4121	0.4148	1.8092	1.7849	1.1533	1.1259	-0.4264	-0.4178
2.06	0.4312	0.4400	1.9868	1.9640	1.1836	1.1653	-0.4565	-0.4381
2.24	0.4730	0.4643	2.1939	2.1458	1.2269	1.2133	-0.4727	-0.4620
2.43	0.5203	0.4943	2.3768	2.3523	1.2751	1.2739	-0.4959	-0.4945
2.62	0.5616	0.5333	2.5612	2.5743	1.3434	1.3428	-0.5346	-0.5363
2.8	0.5665	0.5825	2.7853	2.7982	1.3993	1.4138	-0.5817	-0.5855
2.99	0.6445	0.6512	3.0258	3.0474	1.4846	1.4927	-0.6082	-0.6483
ε_{ij}	ε_{21} =		0.1451748E-02		ε_{22} =		0.7396717E-02	

$$J = 0.1413639E+00$$

Table 4 - Unsteady Aerodynamic Data - Experimental and approximations - Profile S

4.1.3 Profile M

M									
k	H2 *(-1)	H3 *(-1)	A2	A3	k	H1 *(-1)	H4 *(-1)	A1	A4
0.1910	-0.2860	15.3370	-0.8300	4.1350	0.1899	5.5610	0.9110	1.5750	0.4150
0.2865	0.4630	6.3610	-0.4120	1.7540	0.2850	3.5630	0.1190	0.9990	0.2240
0.3820	0.6240	3.4620	-0.2570	0.9720	0.3797	2.6120	0.0070	0.7270	0.1810
0.4779	0.6250	2.1790	-0.1790	0.6210	0.4750	2.0270	-0.1800	0.5750	0.1380
0.5732	0.5880	1.5080	-0.1370	0.4350	0.5697	1.6480	-0.2870	0.4710	0.1100
0.6686	0.5330	1.1140	-0.1110	0.3230	0.6649	1.3840	-0.3310	0.3960	0.0870
0.7644	0.4800	0.8570	-0.0920	0.2500	0.7598	1.1980	-0.4060	0.3410	0.0730
0.8595	0.4370	0.6780	-0.0790	0.2010	0.8549	1.0410	-0.4420	0.3050	0.0620
0.9552	0.4000	0.5500	-0.0680	0.1670	0.9503	0.9200	-0.4820	0.2720	0.0500
1.0496	0.3680	0.4550	-0.0600	0.1430	1.0462	0.8350	-0.5080	0.2490	0.0440
1.1453	0.3420	0.3840	-0.0530	0.1240	1.1407	0.7580	-0.5360	0.2290	0.0390
1.2408	0.3200	0.3300	-0.0480	0.1090	1.2359	0.7060	-0.5560	0.2110	0.0360
1.3357	0.3020	0.2900	-0.0440	0.0980	1.3312	0.6590	-0.5760	0.1980	0.0310
1.4319	0.2850	0.2580	-0.0410	0.0890	1.4254	0.6260	-0.5870	0.1860	0.0300

Table 5 - M Derivatives

$$A0 = \begin{vmatrix} -0.7615748E+02 & 0.1425849E+02 \\ 0.7812135E+01 & 0.7378987E+00 \end{vmatrix}$$

$$A1 = \begin{vmatrix} 0.1379135E+02 & 0.5682146E+00 \\ 0.2550696E+00 & -0.2898359E+00 \end{vmatrix}$$

$$D = \begin{vmatrix} 0.1798890E+02 & 0.3674439E+06 & 0.3035072E+06 \\ 0.8673246E+01 & -0.6475102E+05 & -0.6040177E+05 \end{vmatrix}$$

$$E = \begin{vmatrix} -0.3447142E-01 & 0.4754779E-02 & -0.4141133E-02 \\ 0.2613000E-01 & -0.1084968E-02 & 0.1054008E-02 \end{vmatrix}$$

$$\lambda 11.\ \lambda 22,\ \lambda 33 = \begin{vmatrix} 0.5095066E+00 & 0.4946449E+01 & 0.4553581E+01 \end{vmatrix}$$

K	Q11		Q11		Q12		Q12	
	Q11 r (exp)	Q11 r (app)	Q11 i (exp)	Q11 i (app)	Q12 r (exp)	Q12 r (app)	Q12 i (exp)	Q12 i (app)
0.38	0.2629	0.0863	1.6048	1.7234	4.4767	4.4943	-0.0835	0.1070
0.57	0.0773	0.1207	2.3145	2.3155	4.1766	4.2966	0.3040	0.3730
0.76	0.0081	-0.0110	3.0133	2.8465	4.0426	4.1554	0.7286	0.6954
0.95	-0.3249	-0.2923	3.6586	3.3689	3.9810	4.0532	1.1419	1.0572
1.14	-0.7453	-0.7060	4.2797	3.9043	3.9634	3.9867	1.5454	1.4104
1.33	-1.1706	-1.2418	4.8947	4.4636	3.9835	3.9426	1.9059	1.7700
1.52	-1.8748	-1.8942	5.5322	5.0555	4.0058	3.9177	2.2436	2.1353
1.71	-2.5840	-2.6594	6.0859	5.6885	4.0072	3.9110	2.5828	2.5056
1.9	-3.4820	-3.5338	6.6462	6.3717	4.0144	3.9227	2.9196	2.8800
2.09	-4.4478	-4.5129	7.3108	7.1146	4.0104	3.9530	3.2436	3.2568
2.28	-5.5799	-5.5913	7.8910	7.9265	4.0297	4.0024	3.5889	3.6343
2.47	-6.7938	-6.7621	8.6267	8.8162	4.0642	4.0707	3.9411	4.0102
2.66	-8.1656	-8.0170	9.3422	9.7915	4.1392	4.1576	4.3104	4.3824
2.85	-9.5412	-9.3469	10.1752	10.8590	4.2319	4.2626	4.6748	4.7487
ε_{ij}	ε_{11} =	0.9113238E-02			ε_{12} =	0.4745304E-02		

K	Q21		Q21		Q22		Q22	
	Q21 r (exp)	Q21 r (app)	Q21 i (exp)	Q21 i (app)	Q22 r (exp)	Q22 r (app)	Q22 i (exp)	Q22 i (app)
0.38	0.1198	0.1092	0.4545	0.579379	1.2070	1.2587	-0.2423	-0.2493
0.57	0.1455	0.2086	0.6489	0.7454	1.1517	1.1866	-0.2705	-0.2783
0.76	0.2088	0.2657	0.8387	0.8883	1.1350	1.1488	-0.3001	-0.2876
0.96	0.2491	0.2929	1.0378	1.0334	1.1345	1.1367	-0.3270	-0.2967
1.15	0.2857	0.3027	1.2231	1.1897	1.1433	1.1424	-0.3601	-0.3115
1.34	0.3077	0.3045	1.4005	1.3596	1.1550	1.1589	-0.3969	-0.3342
1.53	0.3371	0.3049	1.5747	1.5429	1.1685	1.1820	-0.4300	-0.3652
1.72	0.3625	0.3085	1.7831	1.7383	1.1880	1.2090	-0.4669	-0.4039
1.91	0.3612	0.3188	1.9650	1.9441	1.2189	1.2380	-0.4963	-0.4500
2.1	0.3852	0.3384	2.1801	2.1580	1.2604	1.2673	-0.5288	-0.5025
2.29	0.4060	0.3689	2.3840	2.3778	1.3012	1.2959	-0.5562	-0.5609
2.48	0.4399	0.4114	2.5782	2.6012	1.3424	1.3229	-0.5912	-0.6243
2.67	0.4395	0.4666	2.8069	2.8257	1.3988	1.3477	-0.6280	-0.6919
2.86	0.4876	0.5343	3.0233	3.0494	1.4598	1.3698	-0.6725	-0.7630
ε_{ij}	ε_{21} =	0.6215548E-02			ε_{22} =	0.1864355E-01		

J = 0.1967680E+00

Table 6 - Unsteady Aerodynamic Data - Experimental and approximations - Profile M

4.1.4 Profile P

P									
k	H2 *(-1)	H3 *(-1)	A2	A3	k	H1 *(-1)	H4 *(-1)	A1	A4
0.1884	-2.6090	18.9870	-1.1440	4.5960	0.1881	6.5240	1.4840	1.7350	0.5690
0.2826	-0.4970	7.6440	-0.5700	1.9790	0.2821	3.9970	0.6220	1.0760	0.3420
0.3766	0.1310	4.0810	-0.3610	1.1050	0.3760	2.8860	0.1920	0.7800	0.2500
0.4709	0.3180	2.5390	-0.2600	0.7130	0.4702	2.2560	-0.0090	0.6170	0.1900
0.5651	0.3690	1.7430	-0.2040	0.5040	0.5641	1.8710	-0.0970	0.5110	0.1770
0.6593	0.3790	1.2830	-0.1710	0.3770	0.6582	1.5910	-0.2160	0.4400	0.1500
0.7536	0.3640	0.9910	-0.1450	0.2980	0.7523	1.3940	-0.2710	0.3830	0.1400
0.8475	0.3380	0.7950	-0.1310	0.2420	0.8466	1.2400	-0.3220	0.3410	0.1260
0.9417	0.3160	0.6550	-0.1190	0.2020	0.9409	1.1300	-0.3500	0.3080	0.1170

Table 7 - P Derivatives

$$A0 = \begin{vmatrix} -0.3878034E+02 & 0.1179900E+02 \\ 0.1736960E+02 & -0.3271855E-01 \end{vmatrix}$$

$$A1 = \begin{vmatrix} 0.1031018E+02 & 0.3890370E+00 \\ -0.9762490E+00 & -0.4880674E+00 \end{vmatrix}$$

$$D = \begin{vmatrix} 0.1445158E+02 & -0.2722159E+02 & -0.1114228E+02 \\ 0.2344997E+01 & 0.2788701E+02 & 0.1023353E+02 \end{vmatrix}$$

$$E = \begin{vmatrix} -0.2788161E-01 & 0.3257732E+02 & -0.1003661E+03 \\ 0.6916698E-01 & -0.1007044E+02 & 0.2898910E+02 \end{vmatrix}$$

$$\lambda 11.\ \lambda 22,\ \lambda 33 = \begin{vmatrix} 0.3904822E+00 & 0.4824437E+01 & 0.4999693E+01 \end{vmatrix}$$

K	Q11		Q11		Q12		Q12	
	Q11 r (exp)	Q11 r (app)	Q11 i (exp)	Q11 i (app)	Q12 r (exp)	Q12 r (app)	Q12 i (exp)	Q12 i (app)
0.38	0.4201	0.3979	1.8470	1.9200	5.3922	5.3514	-0.7409	-0.6976
0.56	0.3961	0.4140	2.5453	2.5680	4.8853	4.8802	-0.3176	-0.3218
0.75	0.2171	0.2741	3.2635	3.2418	4.6304	4.6428	0.1486	0.0963
0.94	-0.0159	0.0137	3.9907	3.9444	4.5034	4.5201	0.5640	0.5243
1.13	-0.2469	-0.3416	4.7633	4.6872	4.4534	4.4736	0.9428	0.9245
1.32	-0.7486	-0.7769	5.5141	5.4748	4.4616	4.4743	1.3180	1.2984
1.5	-1.2270	-1.2539	6.3115	6.2653	4.5019	4.5070	1.6536	1.6500
1.69	-1.8461	-1.8185	7.1093	7.1494	4.5678	4.5599	1.9421	1.9654
1.88	-2.4787	-2.4388	8.0027	8.0872	4.6471	4.6339	2.2419	2.2819
ε_{ij}	ε_{11} =	0.6332079E-03			ε_{12} =	0.4066785E-03		

K	Q21		Q21		Q22		Q22	
	Q21 r (exp)	Q21 r (app)	Q21 i (exp)	Q21 i (app)	Q22 r (exp)	Q22 r (app)	Q22 i (exp)	Q22 i (app)
0.38	0.1611	0.1787	0.4912	0.496049	1.3052	1.3238	-0.3249	-0.3202
0.57	0.2178	0.2330	0.6852	0.6891	1.2648	1.2650	-0.3643	-0.3669
0.75	0.2827	0.2877	0.8820	0.8915	1.2538	1.2487	-0.4096	-0.4054
0.94	0.3361	0.3477	1.0914	1.0982	1.2646	1.2567	-0.4612	-0.4538
1.13	0.4506	0.4185	1.3009	1.3100	1.2877	1.2803	-0.5212	-0.5134
1.32	0.5199	0.5038	1.5250	1.5259	1.3110	1.3135	-0.5946	-0.5844
1.51	0.6339	0.6004	1.7341	1.7329	1.3538	1.3521	-0.6587	-0.6664
1.69	0.7224	0.7211	1.9551	1.9520	1.3905	1.3913	-0.7527	-0.7535
1.88	0.8286	0.8621	2.1813	2.1694	1.4331	1.4332	-0.8443	-0.8547
ε_{ij}	ε_{21} =	0.8538965E-03			ε_{22} =	0.3365859E-03		

J = 0.4722678E-01

Table 8 - Unsteady Aerodynamic Data - Experimental and approximations - Profile P

4.1.5 Profile R

R									
k	H2 *(-1)	H3 *(-1)	A2	A3	k	H1 *(-1)	H4 *(-1)	A1	A4
0.1868	-5.5250	22.8360	0.1430	3.6810	0.1866	8.6300	3.8360	1.1550	-0.1580
0.2802	-1.9840	8.7850	-0.0820	1.8180	0.2798	4.8570	2.0770	0.8400	-0.1680
0.3735	-0.6880	4.4190	-0.1040	1.0960	0.3731	3.4130	1.2180	0.6680	-0.1710
0.4669	-0.1330	2.6410	-0.1200	0.7400	0.4664	2.4500	0.6590	0.5960	-0.1050
0.5603	0.1350	1.7430	-0.1210	0.5380	0.5597	1.8770	0.2490	0.5320	-0.0580
0.6534	0.2010	1.2400	-0.1200	0.4100	0.6530	1.4890	0.0550	0.4860	-0.0500
0.7471	0.2670	0.9140	-0.1130	0.3250	0.7462	1.2200	-0.0800	0.4420	-0.0230
0.8407	0.2910	0.7060	-0.1100	0.2630	0.8398	1.0070	-0.2030	0.4150	-0.0030
0.9336	0.3000	0.5670	-0.1050	0.2160	0.9328	0.8670	-0.3450	0.3820	0.0250
1.0270	0.3110	0.4650	-0.1000	0.1790	1.0263	0.8160	-0.4670	0.3450	0.0490
1.1208	0.3140	0.4030	-0.0960	0.1490	1.1200	0.7400	-0.5480	0.3130	0.0660
1.2134	0.3060	0.3610	-0.0900	0.1250	1.2139	0.7130	-0.6320	0.2800	0.0820

Table 9 - R derivatives

$$A0 = \begin{vmatrix} -0.7015957E+02 & 0.1227222E+02 \\ 0.1193326E+02 & 0.1235425E+00 \end{vmatrix}$$

$$A1 = \begin{vmatrix} 0.1548062E+02 & 0.5953427E+00 \\ -0.5559490E+00 & -0.3316040E+00 \end{vmatrix}$$

$$D = \begin{vmatrix} 0.1297802E+02 & -0.1105907E+02 & -0.1443114E+02 \\ -0.8607228E+00 & 0.2337023E+01 & 0.2664362E+01 \end{vmatrix}$$

$$E = \begin{vmatrix} -0.3307328E-01 & 0.9982782E+01 & -0.3243901E+02 \\ 0.9181806E-01 & -0.2605759E+01 & 0.5358793E+01 \end{vmatrix}$$

$$\lambda 11.\ \lambda 22,\ \lambda 33 = \begin{vmatrix} 0.3378882E+00 & 0.2416326E+01 & 0.3989396E+01 \end{vmatrix}$$

K	Q11		Q11		Q12		Q12	
	Q11 r (exp)	Q11 r (app)	Q11 i (exp)	Q11 i (app)	Q12 r (exp)	Q12 r (app)	Q12 i (exp)	Q12 i (app)
0.37	1.0683	0.9634	2.4034	2.4059	6.3717	6.3093	-1.5416	-1.5376
0.56	1.3013	1.2175	3.0430	3.1266	5.5168	5.5214	-1.2459	-1.1811
0.75	1.3565	1.2900	3.8010	3.7140	4.9319	5.0223	-0.7679	-0.7308
0.93	1.1468	1.1936	4.2634	4.1758	4.6067	4.6852	-0.2320	-0.2901
1.12	0.6240	0.8962	4.7040	4.5851	4.3775	4.4148	0.3390	0.1859
1.31	0.1876	0.3731	5.0794	4.9485	4.2353	4.2112	0.6865	0.6759
1.49	-0.3564	-0.3451	5.4348	5.2907	4.0814	4.0736	1.1923	1.1520
1.68	-1.1453	-1.3405	5.6812	5.6954	3.9916	3.9841	1.6453	1.6623
1.87	-2.4014	-2.5690	6.0348	6.1896	3.9537	3.9497	2.0919	2.1735
2.05	-3.9353	-3.9288	6.8763	6.7759	3.9236	3.9648	2.6242	2.6515
2.24	-5.4993	-5.5435	7.4260	7.5490	4.0499	4.0264	3.1555	3.1428
2.43	-7.4504	-7.3106	8.4053	8.5021	4.2524	4.1296	3.6045	3.6149
ε_{ij}	ε_{11} =		0.2806254E-02		ε_{12} =		0.1856888E-02	

K	Q21		Q21		Q22		Q22	
	Q21 r (exp)	Q21 r (app)	Q21 i (exp)	Q21 i (app)	Q22 r (exp)	Q22 r (app)	Q22 i (exp)	Q22 i (app)
0.37	-0.0440	-0.0745	0.3217	0.2999529	1.0271	1.1031	0.0399	0.0417
0.56	-0.1053	-0.1274	0.5263	0.5102	1.1417	1.1791	-0.0515	-0.0206
0.75	-0.1904	-0.1710	0.7439	0.7519	1.2232	1.2421	-0.1161	-0.0975
0.93	-0.1827	-0.1956	1.0371	1.0094	1.2908	1.2957	-0.2093	-0.1798
1.12	-0.1454	-0.1940	1.3332	1.3081	1.3512	1.3470	-0.3039	-0.2765
1.31	-0.1706	-0.1556	1.6579	1.6275	1.4004	1.3918	-0.4099	-0.3827
1.49	-0.1025	-0.0800	1.9690	1.9409	1.4512	1.4270	-0.5046	-0.4908
1.68	-0.0169	0.0439	2.3413	2.2735	1.4870	1.4553	-0.6219	-0.6108
1.87	0.1740	0.2124	2.6589	2.5982	1.5062	1.4742	-0.7322	-0.7347
2.05	0.4129	0.4106	2.9072	2.8909	1.5104	1.4832	-0.8438	-0.8539
2.24	0.6623	0.6561	3.1410	3.1772	1.4974	1.4840	-0.9648	-0.9796
2.43	0.9667	0.9333	3.3008	3.4352	1.4724	1.4765	-1.0602	-1.1038
ε_{ij}	ε_{21} =		0.3653706E-02		ε_{22} =		0.5290542E-02	

J = 0.1166507E+00

Table 10 - Unsteady Aerodynamic Data - Experimental and approximations - Profile R

4.1.6 Profile C

C									
k	H2 *(-1)	H3 *(-1)	A2	A3	k	H1 *(-1)	H4 *(-1)	A1	A4
0.1872	1.4070	19.5320	0.6280	2.0210	0.1863	8.0320	0.7750	0.7500	0.0010
0.2806	-0.0650	8.8000	0.1790	1.0850	0.2794	5.2110	0.8450	0.5550	-0.1630
0.3741	-0.2840	4.8040	0.0520	0.6790	0.3725	3.8840	0.1690	0.4490	-0.0810
0.4676	-0.2730	2.9640	-0.0110	0.4620	0.4657	2.8800	0.3170	0.3980	-0.0250
0.5608	-0.2240	1.9480	-0.0210	0.3380	0.5587	2.2570	0.2840	0.3300	-0.0090
0.6544	-0.1640	1.3420	-0.0260	0.2610	0.6522	1.9090	0.0690	0.2860	0.0180
0.7476	-0.1400	0.9310	-0.0250	0.2130	0.7452	1.5570	0.1300	0.2470	-0.0040
0.8407	-0.0820	0.6300	-0.0220	0.1830	0.8380	1.3090	0.1230	0.2280	-0.0050
0.9339	-0.0050	0.4310	-0.0260	0.1630	0.9314	1.0960	0.0680	0.2030	0.0020
1.0270	0.0560	0.3060	-0.0280	0.1470	1.0247	0.9280	0.0330	0.1900	-0.0130
1.1204	0.1030	0.2300	-0.0310	0.1330	1.1184	0.7280	-0.0110	0.1880	-0.0180
1.2144	0.1550	0.1740	-0.0390	0.1240	1.2116	0.6080	-0.0900	0.1820	-0.0210
1.3085	0.2150	0.1540	-0.0460	0.1130	1.3046	0.4780	-0.2280	0.1880	-0.0190
1.4025	0.2480	0.1640	-0.0510	0.0990	1.3981	0.3710	-0.3420	0.1940	-0.0060
1.4960	0.2500	0.1910	-0.0540	0.0840	1.4917	0.2950	-0.4860	0.2110	0.0060

Table 11 - C Derivatives

$$A0 = \left| \begin{matrix} -0.2056546E+03 & 0.6813178E+02 \\ 0.1800625E+02 & -0.4204705E+01 \end{matrix} \right|$$

$$A1 = \left| \begin{matrix} 0.2392765E+02 & -0.4138189E+01 \\ -0.5919932E+00 & 0.2390796E-01 \end{matrix} \right|$$

$$D = \left| \begin{matrix} -0.5259069E+03 & 0.1695228E+01 & -0.8942568E+01 \\ -0.1223348E+02 & -0.1792081E+00 & 0.9454919E+00 \end{matrix} \right|$$

$$E = \left| \begin{matrix} -0.3665337E-03 & 0.7386826E+06 & 0.1398486E+06 \\ 0.9839661E-03 & -0.3118553E+06 & -0.5905491E+05 \end{matrix} \right|$$

$$\lambda 11.\ \lambda 22,\ \lambda 33 = \left| \begin{matrix} 0.2610189E-01 & 0.4995768E+01 & 0.4993358E+01 \end{matrix} \right|$$

K	Q11		Q11		Q12		Q12	
	Q11 r (exp)	Q11 r (app)	Q11 i (exp)	Q11 i (app)	Q12 r (exp)	Q12 r (app)	Q12 i (exp)	Q12 i (app)
0.37	0.2151	-0.0605	2.2291	2.0154	5.4732	6.0680	0.3943	0.6896
0.56	0.5276	0.1425	3.2538	3.3721	5.5420	5.8552	-0.0409	-0.0706
0.74	0.1876	0.3919	4.3112	4.4493	5.3783	5.5192	-0.3179	-0.5132
0.93	0.5500	0.6671	4.9968	5.3888	5.1839	5.1071	-0.4775	-0.7608
1.12	0.7092	0.9064	5.6362	6.1235	4.9011	4.6745	-0.5636	-0.8456
1.3	0.2348	1.0539	6.4959	6.6304	4.5970	4.2010	-0.5618	-0.7886
1.49	0.5775	1.0779	6.9164	6.9735	4.1632	3.7375	-0.6260	-0.5888
1.68	0.6910	0.9204	7.3536	7.1365	3.5619	3.3316	-0.4636	-0.2762
1.86	0.4719	0.5659	7.6062	7.1490	3.0072	2.9613	-0.0349	0.1727
2.05	0.2772	-0.0579	7.7946	7.0446	2.5820	2.6842	0.4725	0.6973
2.24	-0.1101	-0.9638	7.2848	6.8579	2.3097	2.4859	1.0344	1.3393
2.42	-1.0569	-2.0978	7.1398	6.6435	2.0528	2.3974	1.8286	2.0533
2.61	-3.1046	-3.5942	6.5089	6.4191	2.1092	2.4275	2.9447	2.8194
2.8	-5.3482	-5.3979	5.8017	6.2386	2.5807	2.5699	3.9025	3.5750
2.98	-8.6519	-7.3838	5.2516	6.1447	3.4197	2.8418	4.4760	4.3856
ε_{ij}	ε_{11} =	0.7625500E-01			ε_{12} =	0.7123245E-01		

K	Q21		Q21		Q22		Q22	
	Q21 r (exp)	Q21 r (app)	Q21 i (exp)	Q21 i (app)	Q22 r (exp)	Q22 r (app)	Q22 i (exp)	Q22 i (app)
0.37	0.0003	0.2822	0.2081	0.1402	0.5663	0.6523	0.1760	0.0789
0.56	-0.1018	0.2300	0.3465	0.2382	0.6833	0.6937	0.1127	0.0831
0.75	-0.0899	0.1663	0.4984	0.3458	0.7602	0.7481	0.0582	0.0828
0.94	-0.0434	0.0883	0.6905	0.4806	0.8080	0.8135	-0.0192	0.0718
1.12	-0.0225	0.0048	0.8241	0.6404	0.8504	0.8830	-0.0528	0.0489
1.31	0.0613	-0.0736	0.9732	0.8168	0.8941	0.9615	-0.0891	0.0100
1.5	-0.0178	-0.1499	1.0972	1.0295	0.9525	1.0420	-0.1118	-0.0450
1.68	-0.0281	-0.2135	1.2808	1.2686	1.0347	1.1172	-0.1244	-0.1119
1.87	0.0139	-0.2573	1.4088	1.5173	1.1373	1.1928	-0.1814	-0.1974
2.05	-0.1092	-0.2816	1.5959	1.8001	1.2404	1.2582	-0.2363	-0.2915
2.24	-0.1801	-0.2796	1.8812	2.0999	1.3356	1.3189	-0.3113	-0.4031
2.43	-0.2466	-0.2511	2.1373	2.3956	1.4629	1.3690	-0.4601	-0.5254
2.62	-0.2587	-0.1911	2.5600	2.7154	1.5477	1.4074	-0.6300	-0.6564
2.8	-0.0938	-0.0996	3.0338	3.0385	1.5579	1.4320	-0.8025	-0.7866
2.99	0.1068	0.0160	3.7563	3.3435	1.5039	1.4453	-0.9668	-0.9283
ε_{ij}	ε_{21} =	0.6929462E-01			ε_{22} =	0.4441194E-01		

$$J = 0.5110714E+00$$

Table 12 - Unsteady Aerodynamic Data - Experimental and approximations- Profile C

4.1.7 Profile TC

Tacoma									
k	H2 *(-1)	H3 *(-1)	A2	A3	k	H1 *(-1)	H4 *(-1)	A1	A4
0.1890	-2.0180	23.7260	1.9830	-1.2590	0.1897	12.4580	3.4800	1.5310	-1.1750
0.2836	0.9950	11.0460	1.0820	-0.2320	0.2845	6.0030	-0.9210	0.8970	0.2340
0.3780	-0.4730	6.7710	0.6530	0.1610	0.3790	4.1100	2.0070	0.5820	0.3340
0.4721	-1.4050	4.0530	0.3530	0.2660	0.4738	3.6850	0.0920	-0.3380	-0.4280
0.5661	-1.2590	2.2990	0.2360	0.2720	0.5687	2.6280	1.1150	-0.4070	0.2900
0.6597	-1.0180	1.2600	0.1410	0.2660	0.6633	2.2900	0.7940	0.1070	-0.3450
0.7532	-0.6830	0.5460	0.1110	0.2760	0.7583	1.6830	1.0230	0.1280	-0.3110
0.8454	0.3470	-0.2030	0.0120	0.2770	0.8525	1.1850	0.9520	0.1750	-0.2730
0.9389	0.2930	0.1450	-0.0600	0.2160	0.9480	0.8840	0.7180	0.1720	-0.2550
1.0338	0.1390	0.2580	-0.0690	0.1970	1.0430	0.1500	0.5240	0.2420	-0.2960
1.1333	0.4180	0.3700	-0.1510	0.1620	1.1395	-0.3110	-0.0350	0.3680	-0.2640
1.2272	0.3620	0.4740	-0.1530	0.0960	1.2344	-0.2650	-0.4020	0.4020	-0.1420
1.3222	0.2300	0.5140	-0.1260	0.0540	1.3301	-0.2110	-1.5380	0.4940	0.0650
1.4164	0.1160	0.5100	-0.0970	0.0310	1.4254	0.8450	-1.7510	0.3360	0.2360

Table 13 - TC derivatives

$$A0 = \begin{vmatrix} -0.1545739E+02 & 0.4288549E+00 \\ -0.2694613E+01 & 0.3400656E+01 \end{vmatrix}$$

$$A1 = \begin{vmatrix} 0.6283213E+01 & 0.2084439E+01 \\ 0.2112671E+01 & -0.1167155E+01 \end{vmatrix}$$

$$D = \begin{vmatrix} 0.1444638E+03 & 0.9907199E+04 & -0.3690009E+04 \\ 0.7118608E+00 & 0.3574415E+04 & -0.2289577E+03 \end{vmatrix}$$

$$E = \begin{vmatrix} -0.3285084E-01 & 0.1425661E-02 & -0.1041667E-01 \\ 0.1176005E+00 & -0.3036590E-02 & -0.7812500E-02 \end{vmatrix}$$

$$\lambda 11.\ \lambda 22,\ \lambda 33 = \begin{vmatrix} 0.2505000E+01 & 0.2505000E+01 & 0.2505000E+01 \end{vmatrix}$$

K	Q11		Q11		Q12		Q12	
	Q11 r (exp)	Q11 r (app)	Q11 i (exp)	Q11 i (app)	Q12 r (exp)	Q12 r (app)	Q12 i (exp)	Q12 i (app)
0.38	1.0016	3.2015	3.5856	-0.4429	6.7835	6.5683	-0.5770	-0.1392
0.57	-0.5963	2.6912	3.8867	-0.5482	7.1068	6.4004	0.6402	-0.1707
0.76	2.3058	2.0220	4.7220	-0.5279	7.7418	6.1801	-0.5408	-0.1607
0.95	0.1652	1.2307	6.6171	-0.3598	7.2277	5.9343	-2.5055	-0.1065
1.14	2.8851	0.3559	6.8000	-0.0336	5.8931	5.6476	-3.2272	0.0013
1.33	2.7950	-0.5667	8.0612	0.4506	4.3871	5.3446	-3.5445	0.1611
1.52	4.7058	-1.5059	7.7418	1.0849	2.4780	5.0356	-3.0998	0.3706
1.71	5.5354	-2.4367	6.8902	1.8559	-1.1607	4.7450	1.9841	0.6108
1.9	5.1619	-3.3401	6.3553	2.7474	1.0226	4.4466	2.0664	0.9035
2.09	4.5605	-4.2032	1.3055	3.7422	2.2057	4.1610	1.1883	1.2307
2.28	-0.3636	-5.0177	-3.2306	4.8237	3.8019	3.8776	4.2952	1.6065
2.47	-4.9005	-5.7790	-3.2304	5.9763	5.7107	3.6389	4.3613	1.9673
2.66	-21.7664	-6.4856	-2.9862	7.1864	7.1889	3.4045	3.2168	2.3669
2.85	-28.4612	-7.1379	13.7348	8.4419	8.1853	3.1880	1.8618	2.7818
ε_{ij}	ε_{11} =		0.1584989E+01		ε_{12} =		0.2368647E+01	

K	Q21		Q21		Q22		Q22	
	Q21 r (exp)	Q21 r (app)	Q21 i (exp)	Q21 i (app)	Q22 r (exp)	Q22 r (app)	Q22 i (exp)	Q22 i (app)
0.38	-0.3382	0.2155	0.4406	0.3613662	-0.3600	-0.1042	0.5670	0.0881
0.57	0.1515	0.1359	0.5808	0.5602	-0.1493	-0.0083	0.6961	0.1104
0.76	0.3837	0.0315	0.6687	0.7785	0.1841	0.1174	0.7466	0.1091
0.94	-0.7686	-0.0919	-0.6069	1.0200	0.4744	0.2578	0.6295	0.0822
1.13	0.7504	-0.2283	-1.0531	1.2861	0.6972	0.4214	0.6049	0.0250
1.32	-1.2145	-0.3722	0.3767	1.5768	0.9262	0.5944	0.4909	-0.0619
1.51	-1.4306	-0.5187	0.5888	1.8910	1.2526	0.7708	0.5038	-0.1771
1.69	-1.5874	-0.6639	1.0175	2.2264	1.5839	0.9367	0.0686	-0.3102
1.88	-1.8333	-0.8048	1.2366	2.5807	1.5233	1.1071	-0.4231	-0.4729
2.07	-2.5762	-0.9394	2.1062	2.9510	1.6842	1.2701	-0.5899	-0.6554
2.27	-2.7423	-1.0664	3.8226	3.3349	1.6646	1.4319	-1.5516	-0.8654
2.45	-1.7310	-1.1851	4.9005	3.7299	1.1566	1.5681	-1.8433	-1.0672
2.64	0.9199	-1.2954	6.9913	4.1339	0.7553	1.7019	-1.7623	-1.2910
2.83	3.8360	-1.3971	5.4614	4.5449	0.4975	1.8255	-1.5568	-1.5236
ε_{ij}	ε_{21} =		0.1392574E+01		ε_{22} =		0.1551113E+01	

J = 0.2626275E+01

Table 14 - Unsteady Aerodynamic Data - Experimental and approximations Profile TC

4.1.8 Profile G

G									
k	H2 *(-1)	H3 *(-1)	A2	A3	k	H1 *(-1)	H4 *(-1)	A1	A4
0.1882	-0.8200	9.8600	-0.7960	1.2410	0.1879	3.7690	0.6510	0.6300	0.2580
0.2825	-0.3590	4.2110	-0.5340	0.5170	0.2818	2.4040	0.3730	0.3950	0.2120
0.3767	-0.1480	2.3190	-0.3860	0.2640	0.3756	1.7340	0.2820	0.2630	0.1920
0.4707	-0.0610	1.4860	-0.3040	0.1420	0.4697	1.3700	0.1860	0.1850	0.1720
0.5651	-0.0230	1.0360	-0.2440	0.0770	0.5637	1.1440	0.1370	0.1310	0.1560
0.6590	-0.0130	0.7800	-0.2020	0.0390	0.6582	0.9870	0.1070	0.0950	0.1400
0.7534	-0.0120	0.6190	-0.1690	0.0140	0.7534	0.8760	0.0830	0.0650	0.1280
0.8475	-0.0160	0.5040	-0.1410	-0.0010	0.8466	0.7930	0.0770	0.0440	0.1150
0.9414	-0.0250	0.4220	-0.1190	-0.0100	0.9403	0.7270	0.0790	0.0260	0.1040
1.0355	-0.0350	0.3570	-0.1000	-0.0150	1.0344	0.6710	0.0830	0.0120	0.0910
1.1293	-0.0450	0.3020	-0.0840	-0.0170	1.1284	0.6210	0.0900	0.0020	0.0790
1.2234	-0.0500	0.2570	-0.0700	-0.0170	1.2224	0.5720	0.1030	-0.0060	0.0680
1.3172	-0.0580	0.2160	-0.0590	-0.0160	1.3178	0.5290	0.1140	-0.0120	0.0560
1.4120	-0.0580	0.1830	-0.0500	-0.0130	1.4107	0.4860	0.1250	-0.0140	0.0460

Table 15 - G derivatives

$$A0 = \begin{vmatrix} 0.1428823E+02 & -0.1354282E+01 \\ -0.9541573E+00 & 0.2429820E+00 \end{vmatrix}$$

$$A1 = \begin{vmatrix} 0.4700185E+00 & 0.2530285E+00 \\ 0.8508760E-01 & -0.2557940E+00 \end{vmatrix}$$

$$D = \begin{vmatrix} 0.9727750E+11 & -0.7808196E+07 & 0.9139604E+07 \\ -0.1695936E+11 & 0.1306211E+07 & -0.1565547E+07 \end{vmatrix}$$

$$E = \begin{vmatrix} -0.7014327E-08 & 0.1456413E-03 & 0.1915604E-03 \\ 0.3613225E-08 & -0.8407563E-04 & -0.1075921E-03 \end{vmatrix}$$

$$\lambda11.\ \lambda22,\ \lambda33 = \begin{vmatrix} 0.2515063E+01 & 0.2012862E+01 & 0.2130107E+01 \end{vmatrix}$$

K	Q11		Q11		Q12		Q12	
	Q11 r (exp)	Q11 r (app)	Q11 i (exp)	Q11 i (app)	Q12 r (exp)	Q12 r (app)	Q12 i (exp)	Q12 i (app)
0.38	0.1838	0.0945	1.0641	1.1367	2.7945	2.7926	-0.2324	-0.1852
0.56	0.2369	0.2150	1.5270	1.5917	2.6893	2.7050	-0.2293	-0.2038
0.75	0.3183	0.3210	1.9571	2.0177	2.6324	2.6427	-0.1680	-0.1745
0.94	0.3282	0.3800	2.4176	2.4227	2.6341	2.6220	-0.1081	-0.1167
1.13	0.3483	0.3933	2.9083	2.8474	2.6470	2.6530	-0.0588	-0.0611
1.32	0.3708	0.3842	3.4208	3.3214	2.7102	2.7253	-0.0452	-0.0334
1.51	0.3769	0.3852	3.9776	3.8555	2.8107	2.8193	-0.0545	-0.0480
1.69	0.4415	0.4231	4.5465	4.4111	2.8958	2.9095	-0.0919	-0.1036
1.88	0.5588	0.5228	5.1425	5.0329	2.9922	2.9892	-0.1773	-0.2011
2.07	0.7105	0.6941	5.7441	5.6685	3.0622	3.0417	-0.3002	-0.3280
2.26	0.9168	0.9380	6.3262	6.2958	3.0809	3.0612	-0.4591	-0.4725
2.44	1.2313	1.2303	6.8378	6.8657	3.0770	3.0467	-0.5986	-0.6234
2.64	1.5837	1.6138	7.3491	7.4566	2.9982	3.0037	-0.8051	-0.7639
2.82	1.9900	2.0004	7.7372	7.9424	2.9186	2.9314	-0.9250	-0.9029
ε_{ij}	ε_{11} =	0.2347324E-02			ε_{12} =	0.1077755E-02		

K	Q21		Q21		Q22		Q22	
	Q21 r (exp)	Q21 r (app)	Q21 i (exp)	Q21 i (app)	Q22 r (exp)	Q22 r (app)	Q22 i (exp)	Q22 i (app)
0.38	0.0728	0.1004	0.1779	0.158595	0.3517	0.3777	-0.2256	-0.2242
0.57	0.1347	0.1438	0.2509	0.2290	0.3302	0.3520	-0.3410	-0.3369
0.75	0.2167	0.2105	0.2968	0.2926	0.2997	0.3143	-0.4382	-0.4402
0.94	0.3035	0.2959	0.3265	0.3371	0.2517	0.2607	-0.5389	-0.5409
1.13	0.3966	0.3932	0.3330	0.3563	0.1967	0.1955	-0.6234	-0.6280
1.32	0.4852	0.4933	0.3293	0.3472	0.1355	0.1238	-0.7019	-0.6985
1.51	0.5812	0.5870	0.2951	0.3111	0.0636	0.0510	-0.7674	-0.7517
1.69	0.6593	0.6631	0.2523	0.2559	-0.0057	-0.0146	-0.8101	-0.7875
1.88	0.7356	0.7255	0.1839	0.1816	-0.0709	-0.0765	-0.8438	-0.8124
2.07	0.7790	0.7675	0.1027	0.0972	-0.1287	-0.1290	-0.8577	-0.8275
2.26	0.8048	0.7890	0.0204	0.0087	-0.1734	-0.1711	-0.8569	-0.8363
2.45	0.8129	0.7921	-0.0717	-0.0743	-0.2035	-0.2030	-0.8381	-0.8416
2.63	0.7780	0.7783	-0.1667	-0.1619	-0.2221	-0.2244	-0.8190	-0.8457
2.82	0.7323	0.7531	-0.2229	-0.2341	-0.2073	-0.2389	-0.7974	-0.8506
ε_{ij}	ε_{21} =	0.4850071E-02			ε_{22} =	0.9551460E-02		

J = 0.1335163E+00

Table 16 - Unsteady Aerodynamic Data - Experimental and approximations - Profile G

5 PLOTS OF THE UNSTEADY AERODYNAMIC DATA AND APPROXIMATION CURVES

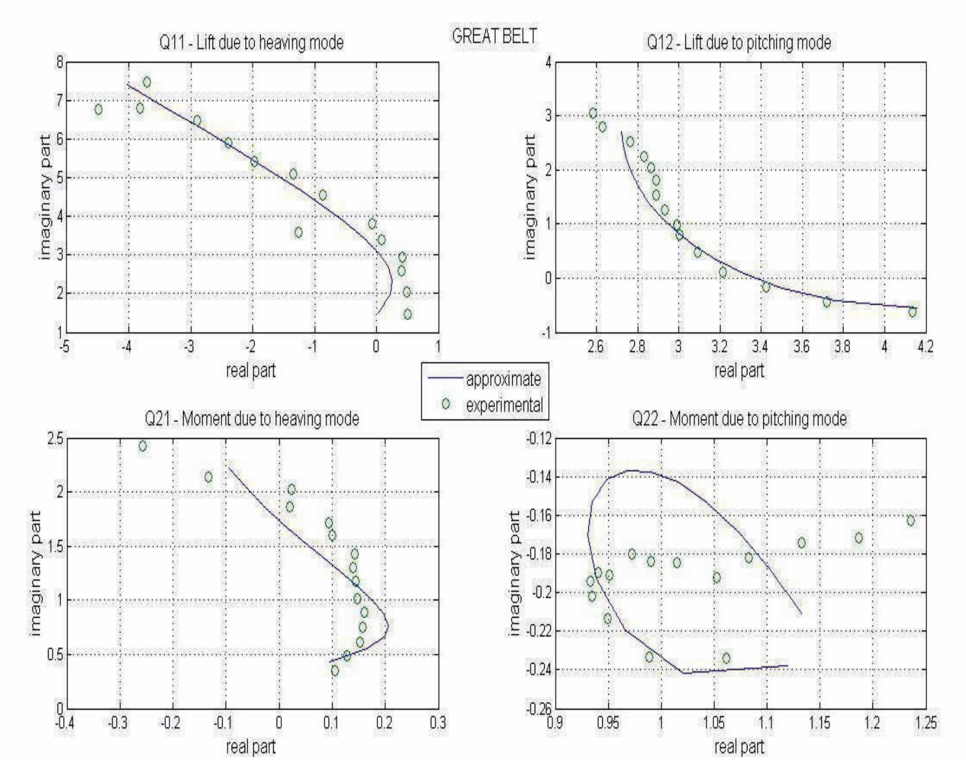

Figure 2 - Plots of GB unsteady aerodynamic data

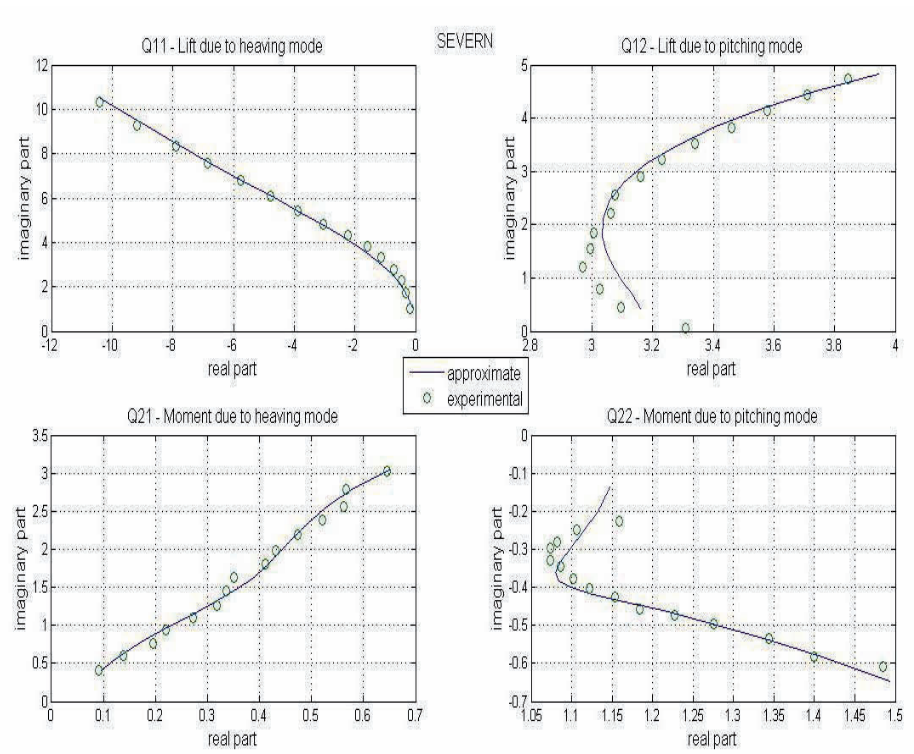

Figure 3 - Plots of S unsteady aerodynamic data

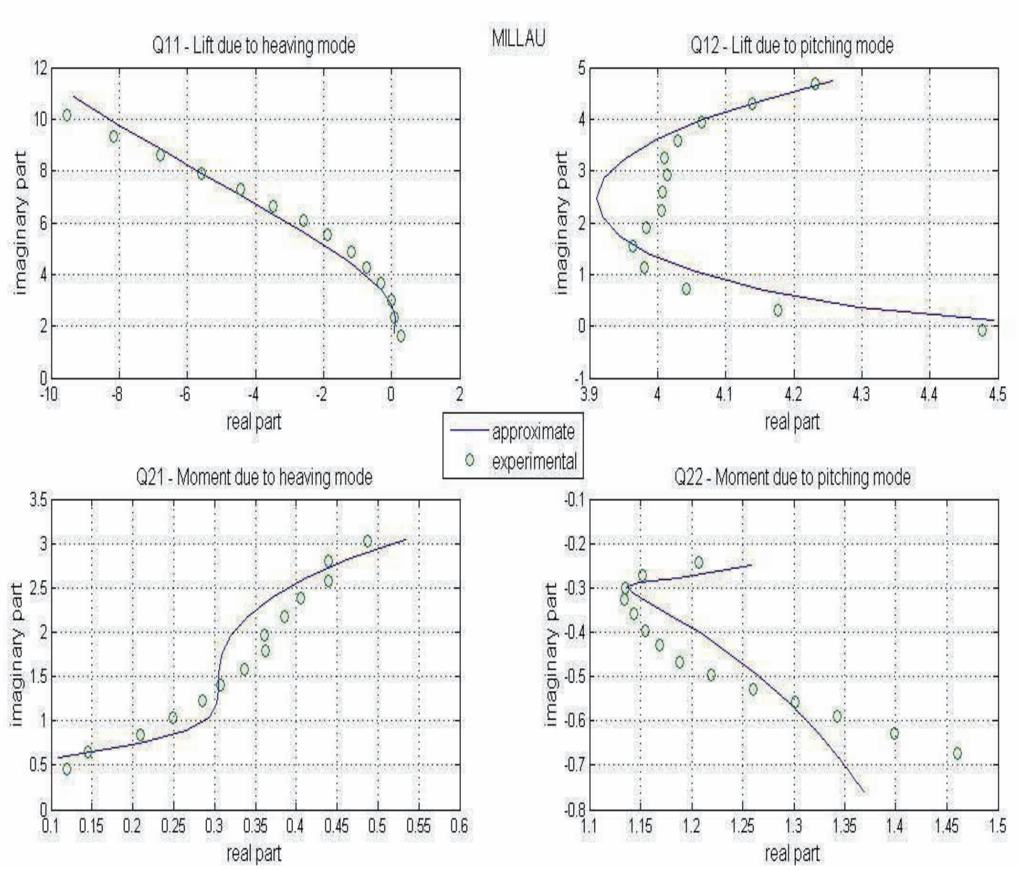

Figure 4 - Plots of M unsteady aerodynamic data

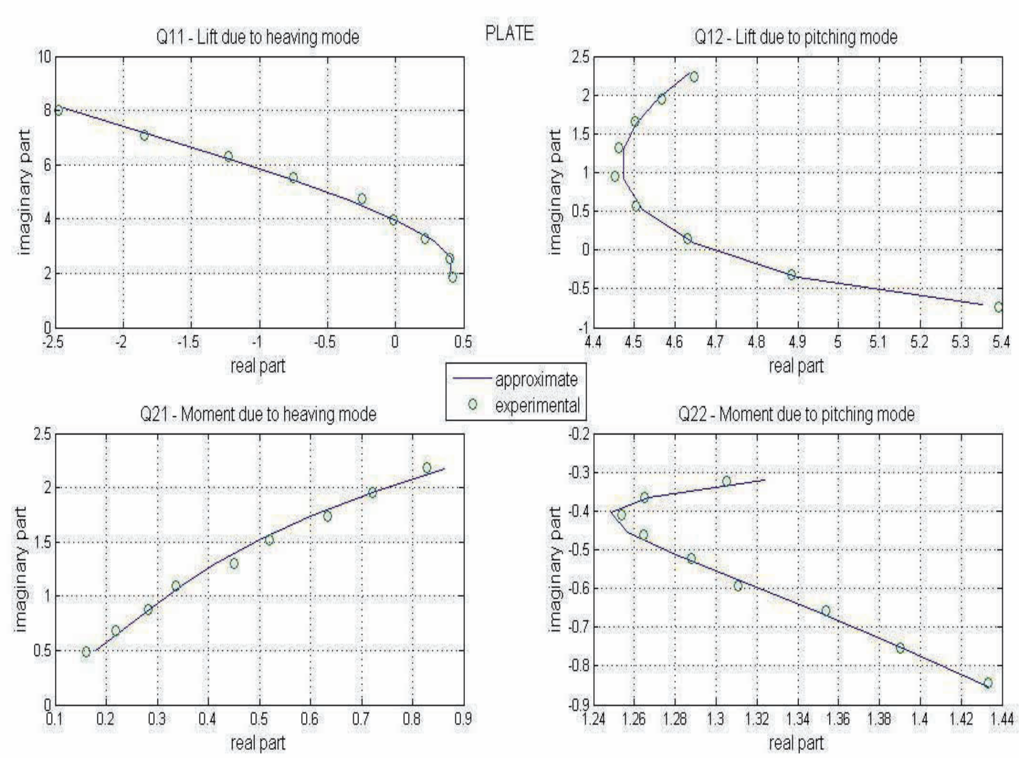

Figure 5 - Plots of P unsteady aerodynamic data

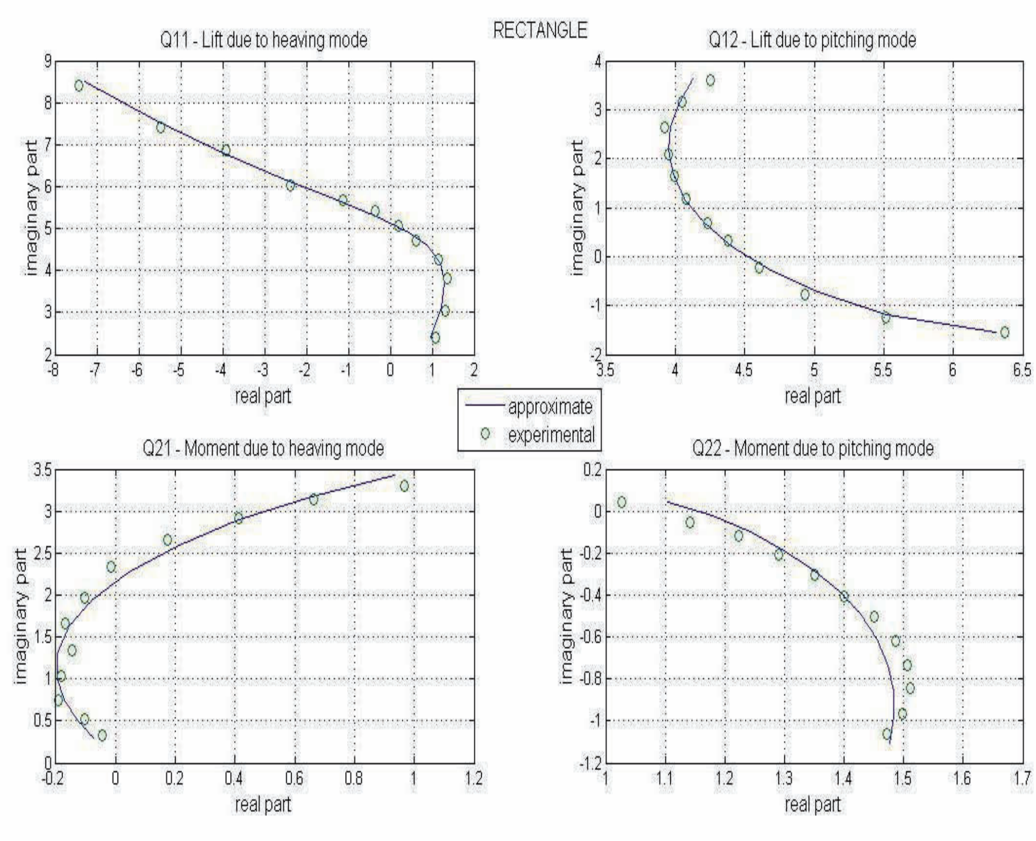

Figure 6 - Plots of R unsteady aerodynamic data

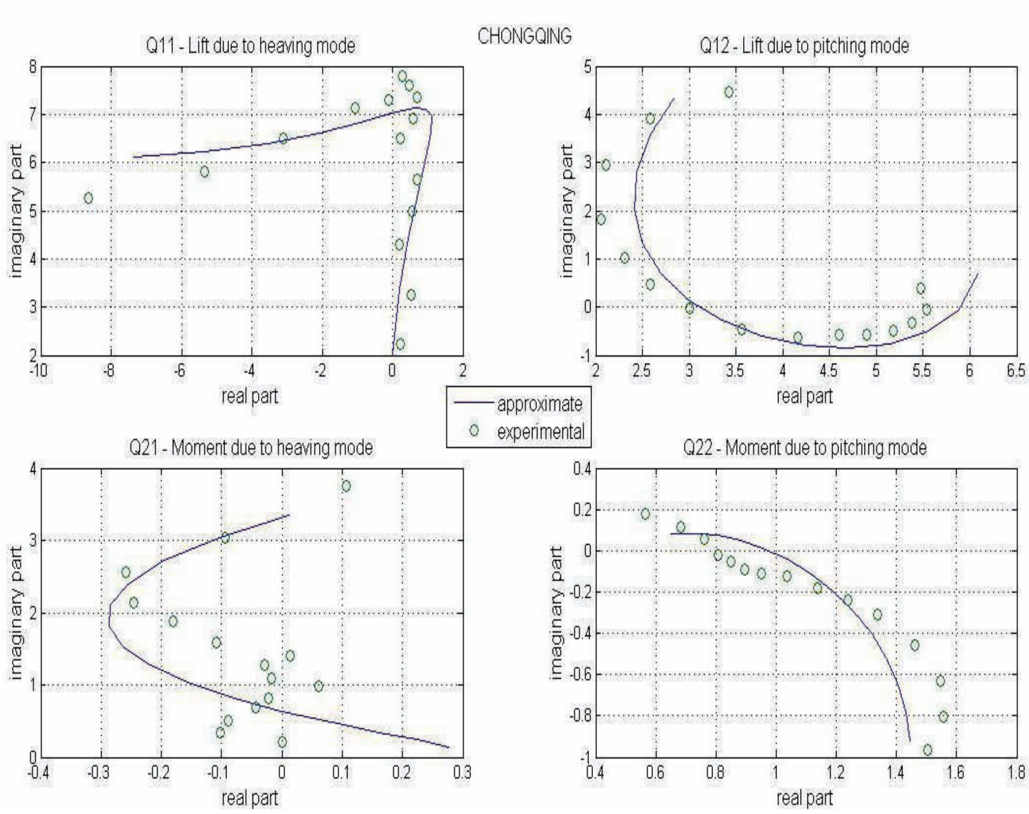

Figure 7 - Plots of C unsteady aerodynamic data

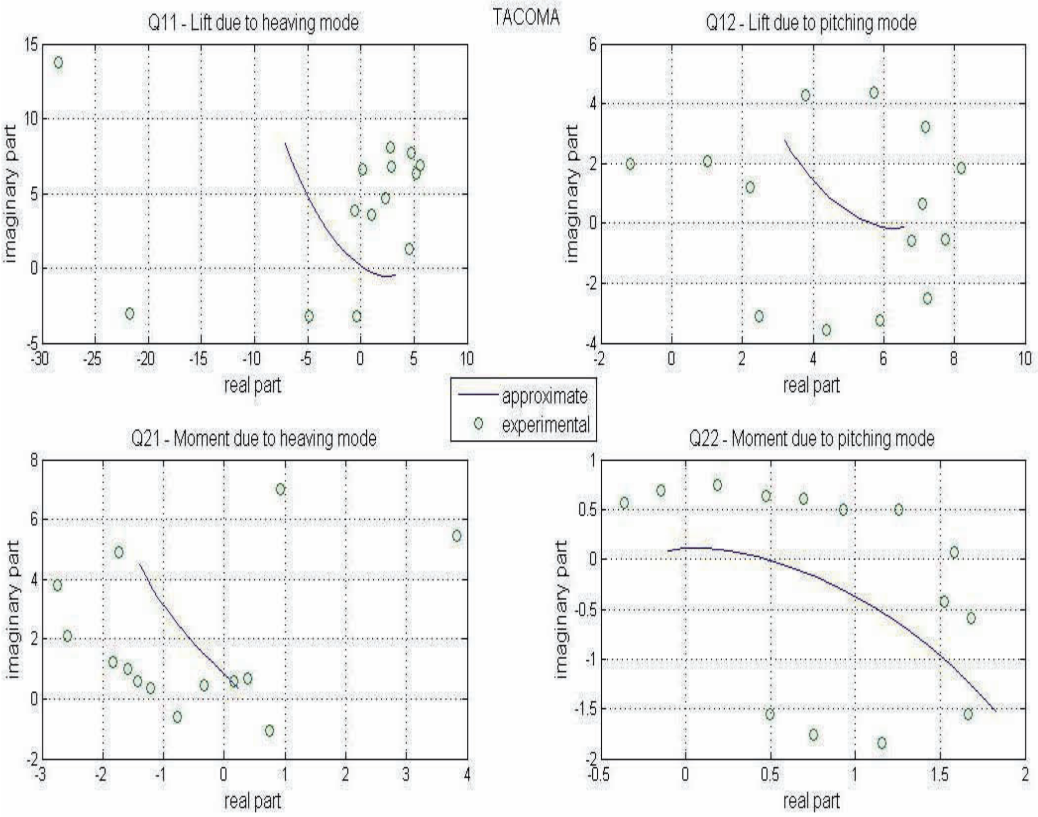

Figure 8 - Plots of TC unsteady aerodynamic data

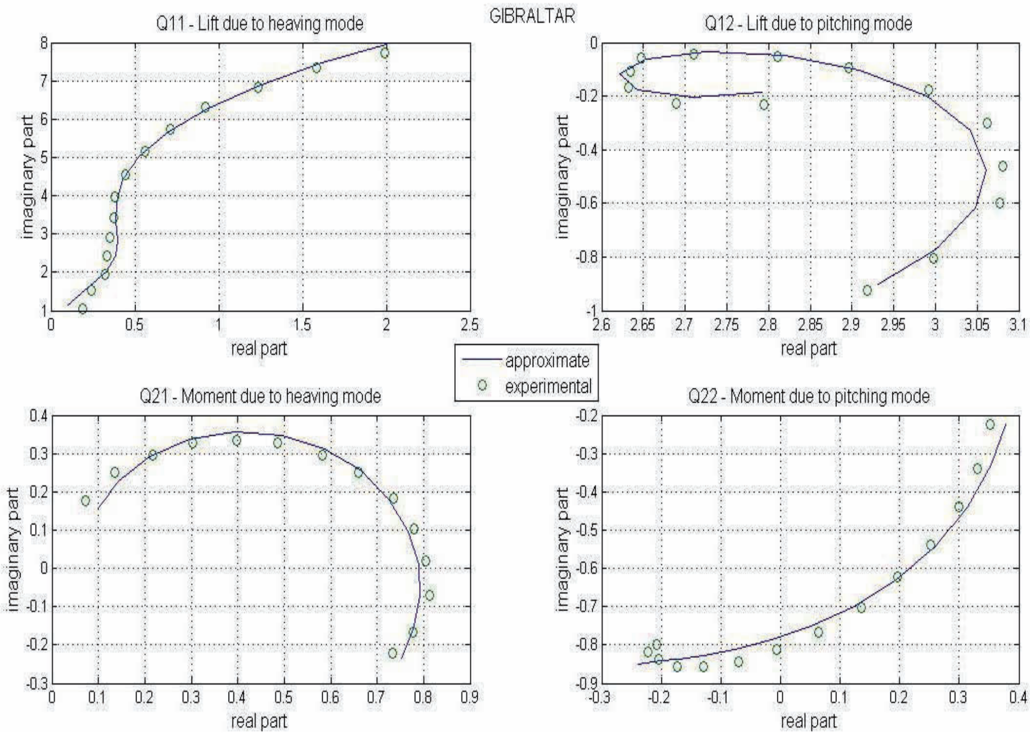

Figure 9 - Plots of G unsteady aerodynamic data

6 APPLICATIONS

6.1 Calculation of the critical velocity of a bridge

From (5) and (13), the following state-space system results:

$$\begin{bmatrix} \ddot{\mathbf{q}} \\ \dot{\mathbf{q}} \\ \dot{\mathbf{x}}_a \end{bmatrix} = \mathbf{A} \begin{bmatrix} \dot{\mathbf{q}} \\ \mathbf{q} \\ \mathbf{x}_a \end{bmatrix} \tag{17}$$

where the state matrix $\mathbf{A}$ reads:

$$\mathbf{A} = \begin{bmatrix} -\mathbf{M}^{-1}[\mathbf{C} - (\mathbf{B}/\mathbf{U})\mathbf{V}_f \mathbf{A}_1] & -\mathbf{M}^{-1}[\mathbf{K} - \mathbf{V}_f \mathbf{A}_0] & \mathbf{M}^{-1}\mathbf{V}_f \mathbf{D} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (U/B)\mathbf{E} & -(U/B)\mathbf{R} \end{bmatrix} \tag{18}$$

For 3 lag terms, the state-space equations form a square matrix with dimensions [7x7]. The state vector contains 4 structural terms $\dot{\mathbf{q}}$ and $\mathbf{q}$, as well as three new terms, known as aerodynamic states, represented by the vector $\mathbf{x}_a$. The addition of three lag terms results in the addition of three new aerodynamic states. The various terms of $\mathbf{A}$, for 3 lag terms (3 lambdas), in addition to equations (6) to (8), read:

$$A0 = \begin{bmatrix} A0_{11} & A0_{12} \\ A0_{21} & A0_{22} \end{bmatrix} \quad A1 = \begin{bmatrix} A1_{11} & A1_{12} \\ A1_{21} & A1_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D23 \end{bmatrix} \quad E^T = \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \end{bmatrix} \quad R = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Substituting the expressions above in (18), the following $\mathbf{A}$ matrix results:

$$A = \begin{bmatrix} -\frac{2c_b B\mu + UmA1_{1,1}}{2mB\mu} & -\frac{UA1_{1,2}}{2B\mu} & -\frac{2k_h B^2\mu + U^2 mA0_{1,1}}{2mB^2\mu} & -\frac{U^2 A0_{1,2}}{2B^2\mu} & -\frac{U^2 D_{1,1}}{2B^2\mu} & -\frac{U^2 D_{1,2}}{2B^2\mu} & -\frac{U^2 D_{1,3}}{2B^2\mu} \\ \frac{BUmA1_{2,1}}{2I_a\mu} & \frac{-2c_a\mu + BUmA1_{2,2}}{2I_a\mu} & \frac{U^2 mA0_{2,1}}{2I_a\mu} & \frac{-2k_a\mu + U^2 mA0_{2,2}}{2I_a\mu} & \frac{U^2 mD_{2,1}}{2I_a\mu} & \frac{U^2 mD_{2,2}}{2I_a\mu} & \frac{U^2 mD_{2,3}}{2I_a\mu} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{UE_{1,1}}{B} & \frac{UE_{2,1}}{B} & -\frac{U\lambda_1}{B} & 0 & 0 \\ 0 & 0 & \frac{UE_{12}}{B} & \frac{UE_{2,2}}{B} & 0 & -\frac{U\lambda_2}{B} & 0 \\ 0 & 0 & \frac{UE_{1,3}}{B} & \frac{UE_{2,3}}{B} & 0 & 0 & -\frac{U\lambda_3}{B} \end{bmatrix}$$

where m is the mass of the bridge deck per meter, I_α the torsional mass moment of inertia per meter, and $\mu = m/\rho B^2$ represents the dimensionless relation between the inertial forces of the bridge deck and the forces exerted by the fluid.

Introducing all variables in **A** and performing a complex eigenvalue analysis of the state matrix for increasing wind velocity, the velocity at which one eigenvalue has a positive real part is the critical one. The fluxogram in figure 10 shows the sequence to be followed.

- Read “Data2000”.
- Begin with the velocity $U = U_0$.
- Assemble the state matrix **A** .
- Calculate the eigenvalues of **A** with the program “Rootlocus.m”
- If one eigenvalue of **A** has a positive real part, print **A** and Ucritic.
- If no eigenvalue of **A** has a positive real part, increase U by ΔU and enter the loop again. MATLAB programs designed according to the Fluxogram 1 are presented in item 6.3.

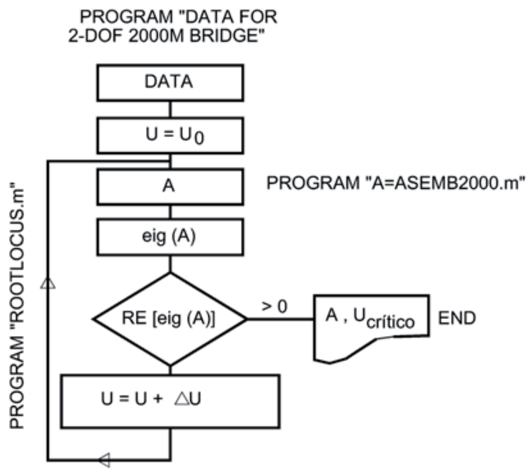

Figure 10- Fluxogram 1

6.2 Numerical example

In the example below, taken from Wilde and Fujino (1998), the critical velocity of a bridge with a 2000m span and streamlined cross section is determined. The bridge under study is the Akashi Kayko Bridge. The bridge structural properties are shown in the MATLAB program “Data 2000”, item (6.3). The critical velocity 10.21 m/s was verified by the present authors, considering a state matrix with dimensions [7x7] and 3 lag terms. The unsteady forces computed through the theoretical formulation of Theodorsen (1934) are shown in Tables 17 and 18. Plots of the system frequencies and damping ratios versus wind velocity are shown in figure 11.

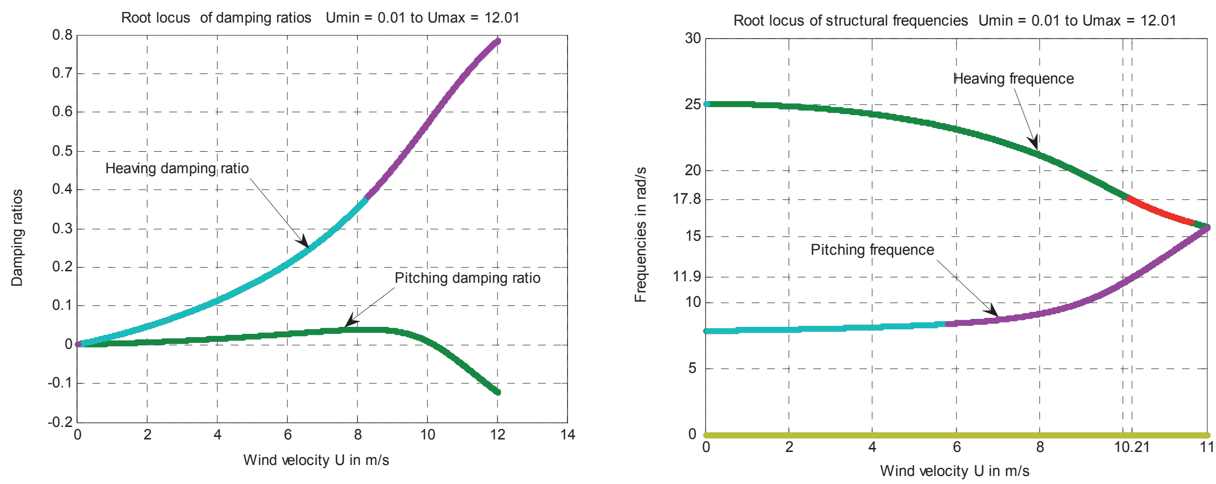

Figure 11 – Frequencies and damping ratios versus wind velocity

Theodorsen Derivatives (not taking into account the aerodynamic mass)									
	H2	H3	A2	A3		H1	H4	A1	A4
k	$=\frac{\pi(1+F+2G/k)}{8k}$	$=\frac{2\pi(F-kG/2)}{8k^2}$	$=\frac{-\pi(1-F-2G/k)}{32k}$	$=\frac{\pi(F-kG/2+0*k^2/8)}{16k^2}$	k	$=\frac{2\pi F}{4k}$	=-0.25*π(0+2*G/k)	$=\frac{\pi F}{8k}$	=-0.125*π*G/k
0.187	-0.5858	16.9460	-1.1956	4.2365	0.186	6.2311	1.5920	1.5578	0.3980
0.281	0.5304	6.9882	-0.5672	1.7471	0.279	3.7985	1.0231	0.9496	0.2558
0.374	0.7678	3.7344	-0.3329	0.9336	0.372	2.6752	0.7128	0.6688	0.1782
0.468	0.7909	2.3062	-0.2222	0.5765	0.466	2.0445	0.5244	0.5111	0.1311
0.561	0.7540	1.5623	-0.1616	0.3906	0.559	1.6474	0.4019	0.4119	0.1005
0.654	0.7015	1.1256	-0.1247	0.2814	0.652	1.3752	0.3171	0.3438	0.0793
0.748	0.6483	0.8498	-0.1006	0.2124	0.745	1.1797	0.2567	0.2949	0.0642
0.841	0.5989	0.6646	-0.0838	0.1661	0.838	1.0324	0.2120	0.2581	0.0530
0.934	0.5544	0.5337	-0.0717	0.1334	0.931	0.9170	0.1777	0.2292	0.0444
1.027	0.5148	0.4382	-0.0625	0.1095	1.025	0.8248	0.1511	0.2062	0.0378
1.120	0.4796	0.3660	-0.0553	0.0915	1.118	0.7491	0.1299	0.1873	0.0325
1.214	0.4483	0.3101	-0.0496	0.0775	1.212	0.6865	0.1129	0.1716	0.0282
1.308	0.4204	0.2660	-0.0450	0.0665	1.305	0.6337	0.0990	0.1584	0.0248
1.402	0.3955	0.2307	-0.0411	0.0577	1.398	0.5882	0.0875	0.1471	0.0219
1.496	0.3734	0.2022	-0.0379	0.0506	1.492	0.5488	0.0778	0.1372	0.0195

Table 17 - Theodorsen Derivatives (0.187 ≤ k ≤ 1.496)

Theodorsen - Unsteady aerodynamic forces								
	Q11		Q12		Q21		Q22	
K	2K^2*H4	2K^2*H1	2K^2*H3	2K^2*H2	2K^2*A4	2K^2*A1	2K^2*A3	2K^2*A2
0.373	0.4418	1.7293	4.7486	-0.1641	0.1105	0.4323	1.1871	-0.3350
0.559	0.6388	2.3718	4.4010	0.3340	0.1597	0.5930	1.1003	-0.3572
0.745	0.7912	2.9695	4.1808	0.8596	0.1978	0.7424	1.0452	-0.3727
0.931	0.9099	3.5471	4.0334	1.3833	0.2275	0.8868	1.0084	-0.3886
1.117	1.0035	4.1140	3.9306	1.8971	0.2509	1.0285	0.9827	-0.4066
1.304	1.0791	4.6794	3.8557	2.4032	0.2698	1.1699	0.9639	-0.4271
1.490	1.1403	5.2402	3.7999	2.8990	0.2851	1.3101	0.9500	-0.4496
1.676	1.1908	5.8000	3.7573	3.3861	0.2977	1.4500	0.9393	-0.4740
1.863	1.2332	6.3639	3.7239	3.8678	0.3083	1.5910	0.9310	-0.5000
2.049	1.2690	6.9280	3.6973	4.3438	0.3172	1.7320	0.9243	-0.5273
2.237	1.2996	7.4961	3.6757	4.8167	0.3249	1.8740	0.9189	-0.5558
2.423	1.3257	8.0619	3.6580	5.2887	0.3314	2.0155	0.9145	-0.5854
2.609	1.3482	8.6283	3.6433	5.7580	0.3371	2.1571	0.9108	-0.6158
2.796	1.3679	9.1984	3.6310	6.2244	0.3420	2.2996	0.9077	-0.6469
2.983	1.3852	9.7703	3.6207	6.6857	0.3463	2.4426	0.9052	-0.6785

Table 18- Theodorsen Unsteady Aerodynamic Forces

6.3 MATLAB programs DATA2000, ASSEMBLE and ROOTLOCUS

```
Data 2000
ro_a = 0.125; Bd = 0.2927; mh = 0.191; I_a = 0.0019345; delta_h = 0.007;  delta_a = 0.006;
ksi_h = delta_h/(2*pi); ksi_a = delta_a/(2*pi);
ome_h = 7.88;   ome_a = 25.06;  Md = [(mh)*Bd 0 ; 0 I_a];
Cd = [2*ksi_h*ome_h*mh*Bd 0; 0 2*ksi_a*ome_a*I_a ];
Kd = [ome_h*ome_h*mh*Bd 0; 0 ome_a*ome_a*I_a];
lamb1=0.6950568E-01;lamb2=0.4365911E+00;lamb3=0.1417495E+01;
A0  = [0.1523923E+01  0.3526492E+01 ;
0.3809271E+00  0.8815787E+00];
A1  = [0.3148492E+01  0.2364804E+01 ;
0.7871435E+00 -0.1962783E+00];
Dd  = [0.3836242E+01  0.3353295E+01  0.3177278E+01;
0.9738113E+00 0.8394293E+00  0.7933566E+00];
Ed  = [-0.7550497E-03 -0.1007897E+00 -0.3031127E+00;
0.2121847E-01 0.1980580E+00  0.1438997E+00];
miBd = 0.5*ro_a*Bd; Vf = miBd*[-1 0; 0 Bd]; A1g=Bd*Vf*A1;A0g = Vf*A0;
Dg =[ Vf*Dd]; Fg = -diag([lamb1/Bd; lamb2/Bd; lamb3/Bd]); Gg = [Ed/Bd];

% Program asemb2000.m
functionAp = asemb2000(U,Bd,A1g,A0g,Dg,Fg,Gg,Ms,Cs,Ks)
A1s = U*A1g;  A0s = U*U*A0g;   Ds = U*U*Dg;
Fs = U*Fg;  Gs = U*Gg;
% Assembling state matrix
Ap = [inv(Ms)*(-Cs+A1s) inv(Ms)*(-Ks+A0s) inv(Ms)*Ds;
eye(2,2) zeros(2,2) zeros(2,3);
zeros(3,2) Gs' Fs];

%  Rootlocus - Search for the flutter wind speed
clear
data2000
U=1;                  %
flag=1;               %
while flag==1           %   Loop to find flutter wind speed
   U=U+0.01;        %
A = asemb2000(U,Bd,A1g,A0g,Dg,Fg,Gg,Md,Cd,Kd);
test = real(eig(A));
for i=1:7
if test(i) > 0
flag=0;
Uc=U;          % Critical flutter wind speed
Ac=A;
DampA = damp(Ac);
end
end
end
disp(['Critical wind speed  Uc = ', num2str(Uc), 'm/s']);
Ac
damp(Ac)
```

6.4 Results of the complex eigenvalue analysis

Notation	Variable	Result
U	Critical velocity	10.22 m/s
Ac	State matrix for 10.22 m/s	$\begin{matrix} -3.099 & -2.3148 & -114.18 & -120.53 & -131.12 & -114.61 & -108.59 \\ 6.5175 & -1.673 & 110.13 & -373.13 & 281.54 & 242.68 & 229.36 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.026364 & 0.74087 & -2.4269 & 0 & 0 \\ 0 & 0 & -3.5192 & 6.9155 & 0 & -15.244 & 0 \\ 0 & 0 & -10.584 & 5.0244 & 0 & 0 & -49.494 \end{matrix}$
K	Reduced frequency $K = \frac{B.w}{U}$	(0.293 m x 17.7rad/s) /10.22 m/s = 0.507
		Eigenvalue / Damping / Freq. (rad/s) 5.70e-003 + 1.77e+001i / -3.22e-004 / 1.77e+001 5.70e-003 - 1.77e+001i / -3.22e-004 / 1.77e+001 -1.16e+000 / 1.00e+000 / 1.16e+000 -6.42e+000 / 1.00e+000 / 6.42e+000 -7.90e+000 + 1.01e+001i / 6.17e-001 / 1.28e+001 -7.90e+000 - 1.01e+001i / 6.17e-001 / 1.28e+001 -4.86e+001 / 1.00e+000 / 4.86e+001

Table 19- Results

Note that for a positive real eigenvalue (5.70e-003) the damping ratio approaches zero (-3.22e-004) and the critical frequency is 17.7 rad/s.

6.5 Structural properties of bridges and critical velocities

Data 1, 2, 6, 8 were reported by Thiesemann (2008) and used here to calculate the critical velocity of various bridges. Data W represents the scaled model of the proposed Akashi Strait Bridge, Wilde and Fujino (1998). Comparisons of critical velocities with RFA are shown in Table 20.

Data 1: Tacoma Narrows properties according to Starossek(1992); Data 2: Great Belt Bridge properties in construction, Walther (1994); Data 6: Properties of the main span of the Great Belt Bridge in construction, Taylor,Vezza (1999); Data 8: Properties of the Gibraltar Bridge.

Abbreviation		TC	GB	GB	G	Akashi
	Units (SI)	Data 1	Data 2	Data 6	Data 8	Data W
b	[m]	5.95	15.5	15.5	30	0.14635
ω_h	[rad/s]	0.84	0.62	0.622	0.383	7.54
ω_α	[rad/s]	1.11	1.17	1.71	0.509	24.504
ε	[-]	1.32	1.88	2.76	1.33	3.25
μ_m	[-]	61	19.7	24.1	6.50	23.17
r	[-]	0.77	0.71	0.67	1.20	0.6876
m	[kg/m]	8500	17800	22740	39500	1.9100
θ	[kg.m2/m]	177730	2173000	2470000	26.7E06	0.01934
ξ_h	[-]	0	0	0.002	0.0030	0.0011
U_{cr}/k_{cr} (Th.)						10.2/0.254
U_{cr}/k_{cr} (Rectangle 1:8)						6.7 / 0.916
U_{cr}/k_{cr} (FlatPlate)						8.9/ 0.29
U_{cr}/k_{cr} (GB)-CFD			40.2/ 0.37			
U_{cr}/k_{cr} (GB)-Exp.			41.3/ 0.37			
U_{cr}/k_{cr} (GB) - RFA			37.2/0.42			
U_{cr}/k_{cr} (GB) - CFD				73.0/ 0.26		
U_{cr}/k_{cr} (GB) - Exp.				70.2/ 0.28		
U_{cr}/k_{cr} (GB) - RFA				71.7/ 0.28		
U_{cr}/k_{cr} (TC) - CFD		7.4/0.89				
U_{cr}/k_{cr} (TC) - Exp.		7.4/0.88				
U_{cr}/k_{cr} (TC) - RFA		9.3/0.70				
U_{cr}/k_{cr} (G) – CFD					34 / 0.40	
U_{cr}/k_{cr} (G) - Exp.					43.9 / 0.33	
U_{cr}/k_{cr} (G) – RFA					48.2 / 0.3	

Table 20 - Structural properties of bridges and corresponding critical velocities/frequencies

7 NOTATION

RFA	Values obtained with the Rational Function Approximation of the unsteady aerodynamic data.	
Exp.	Values obtained experimentally	
CFD	Values obtained by Computer methods of Fluid mechanics	
U_{cr}	Critical velocity - Flutter velocity	
k_{cr}	Reduced frequency	$b\,\omega\,/\,U_{cr}$
b	Semi-chord	
ω_h	Circular frequency of heaving	
ω_α	Circular frequency of pitching	
ε	Ratio of pitching and heaving frequencies	ω_α/ω_h
μ_m	Relative mass	$m\,/\,\pi\rho b^2$
r	Relative radius of gyration	$\frac{1}{b}\sqrt{\frac{\theta}{m}}$
$m\,,\,\theta$	Mass and rotational mass moment of inertia per unit length	
ξ_h, ξ_α	Damping ratios (Lehrsches Dämpfungsmaβ)	
ρ	Specific mass of air	1.225 kg/m3

Table 20 was used to compare the critical velocity of the scaled model of the proposed Akashi Strait Bridge, e.g. see Wilde and Fujino (1998), using unsteady aerodynamic data derived from Theodorsen derivatives and experimental derivatives reported by Starossek (2009) for cases R (Rectangle 1:8) and P (Flat Plate 1:25).

8 ACKNOWLEDGEMENTS

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