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ANALYSIS OF CONCRETE-TIMBER COMPOSITE BEAMS

Nilson T. Mascia[a], Nádia C. S.Forti[b], Júlio Soriano[c] and Elias A. Nicolas[d]

[a]*Departamento de Estruturas,Faculdade de Engenharia Civil,Arquitetura e Urbanismo,Universidade Estadual de Campinas, ,Caixa Postal 6021, Campinas-SP,Brazil, nilson@fec.unicamp.br http://www.fec.unicamp.br*

[b]*Departamento de Estruturas,Faculdade de Engenharia Civil,Arquitetura e Urbanismo,Universidade Estadual de Campinas,Caixa Postal 6021, Campinas-SP,Brazil, nadiacazarm@yahoo.com.br http://www.fec.unicamp.br*

[c]*Departamento de Construções Rurais,Faculdade de Engenharia Agrícola,Universidade Estadual de Campinas, Av. Candido Rondon, 501, Campinas-SP,Brazil, julio.soriano@feagri.unicamp.br http://www.feagri.unicamp.br*

[d]*Departamento de Madeiras, Instituto Tecnológico de São Paulo, Av. Prof. Almeida Prado, 532,São Paulo-SP,Brazil, eliasnic@ipt.br http://www.ipt.br*

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Abstract. Composite structures, which are formed by a combination of materials with different mechanical properties, are an alternative solution for conventional structures used in construction. These structural arrangements seek to obtain a reduction in construction costs while also maintaining the architectural structural safety performance and environmental benefits. The integration of a composite structure depends, in general, on the efficiency of the connection, which can be rigid or flexible. This system is responsible for transmitting the shear longitudinal force at the interface of two materials along the length of the beam and, at the same time, for preventing their vertical detachment. This study contributes to the analysis of the mechanical behavior of composite beams built in concrete and timber, emphasizing the determination of vertical displacements in three ways: via resolution of the analytical equations of equilibrium, via a two-dimensional finite element program, and by the equation of the principle of virtual work. The latter approach was developed from researches on the theory of structures, expanding its concepts to composite beams. The proposed formulation has proved to be consistent and its results are according to experimental data.

1 INTRODUCTION

The development of timber projects has improved the knowledge of mechanical properties of materials and of the systems of connections in these structures. In turn, concrete structures are of current practice since concrete is a material of well-defined physical-mechanical properties. Nevertheless, the use of timber and concrete in composite structures is not yet widespread, though it is becoming more and more common in civil construction.

These composite structures, consisted of materials with different mechanical properties, may be offered as an alternative to structures currently used in construction. This solution tries to achieve reductions in construction costs and also by maintaining the structural safety with beneficial architectural and environmental performance.

The integration of a composite structure is generally due to the efficiency of the connection system, which may be rigid or flexible. This system transmits the longitudinal shear force at the interface of two materials combined along the length of the beam and prevents their vertical detachment.

In this context, this study aims to analyze the mechanical behavior of timber-concrete composite structures with a view on vertical displacement and considering: the analytical resolution of the equilibrium equations, the two-dimensional finite element methods of and the equation of the principle of virtual work (PVW). The last analysis, PVW, was designed from research on structure theory, with an application of concepts in composite beams. This formulation satisfies the equilibrium equations, being consistent and obtaining results that are according to experimental data. It is also important to notice that this formulation can be integrated with finite element codes and it only requires one-dimensional elements.

2 MODELS FOR THE ANALYSIS OF COMPOSITE BEAMS

The mathematical models to represent the behavior of composite structures generally proposed in literature, as presented by Ahmadi and Saka (1993) and Girhammar and Gopu (1993), are addressed on the basis of equilibrium equations and energy conservation principles. In view of the complexity that involves the behavior of structures with composite sections, some simplifications will be assumed and commented next for a general approach of the results.

2.1 Stevanovic model for timber-concrete composite structures calculation on the basis of equilibrium equations principle

Stevanovic (1996) used the theory of elasticity to determine the internal forces in timber-concrete composite structures, following the basic ideas:

- the timber and concrete are considered isotropic elastic materials in an axial direction, and this applies to Hooke´s law;
- the Navier-Bernoulli hypothesis is valid, i.e. plane sections remain plane and perpendicular to the section axis after deformation;

- timber (or wood used in general) and concrete have equal vertical displacements at all connections points;
- although the connectors are discrete, they are seen as equivalent continuous connections with elastic constant.

The slip between the materials is represented by the ratio of shear stress on the surface of the connection modulus T_s and slip modulus K, a parameter obtained in laboratory tests.

From equilibrium and compatibility conditions at the connection interface between timber and concrete, according to Figure 1 and Figure 2 (in which index c indicates concrete and w stands for wood), and considering the displacement w for the timber-concrete composite beam results in the following differential equation of fourth order:

$$w^{iv} - \alpha^2 w^2 = \frac{\alpha^2 M_x}{EI_\infty} - \frac{M_x^{ii}}{EI_0} \quad (1)$$

with: $\alpha^2 = K\left(\frac{1}{A_b E_b} + \frac{1}{A_d E_d} + \frac{r^2}{EI_0}\right); \beta = \frac{Kr}{EI_0}; EI_\infty = \frac{EI_0}{\alpha^2 - \beta r}; EI_0 = E_b I_b + E_d I_d.$

In Eq. (1) $E_b I_b$ and $E_d I_d$ are the bending stiffnesses of the sections of concrete and wood, respectively; EI_∞ represents the flexural stiffness y for total composition and EI_0 represents the stiffness of a non-composite section. E represents the elasticity modulus and I the inertia. Internal forces are written by N, T and M (its second derivative M^{ii}) indicating the normal, the shear and the bending moment respectively. R is the reaction of support; r is the distance among the centers of gravity of the flange and the web.

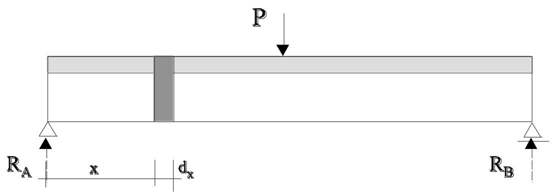

Figure 1: Static schematic of the composite structure

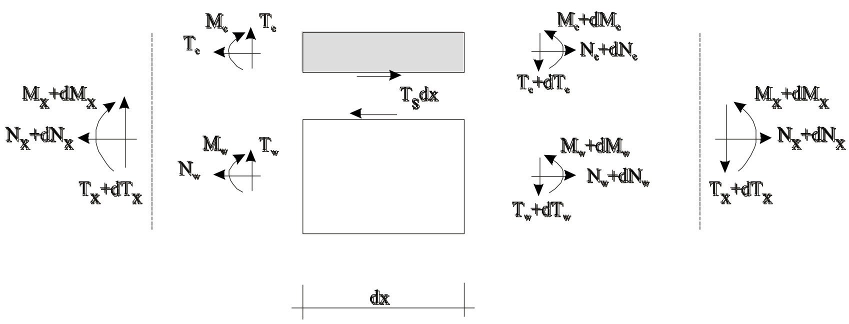

Figure 2 - Representation of the internal efforts in concrete and wood

In that sense, this method searches the general solution of Eq. (1), which is written as:

$$w = a_1 \sinh(\alpha x) + a_2 \cosh(\alpha x) + a_3 + a_4 + w_p \qquad (2)$$

in Eq. (2): a_1 until a_4 are constants that depend on the boundary conditions (conditions of supports), and w_p is a solution that is particularly dependent on external loading.

2.2 Numerical models of composite beams

The finite element method is a technique for the resolution of problems of the approximate contour value. The method involves partitioning the domain into a finite number of elements (finite elements), and by using variational concepts, involving the construction of an approximation of the solution on these elements.

A finite element analysis generally consists of the following steps:

- construction of a variational formulation for the problem. In elasticity, minimizing the functional of the strain energy leads to the variational formulation, which is equivalent to the formulation of the principle of virtual works;
- definition of the geometry of the problem, its domain and control conditions;
- selection of the area of approach: definition of functions, whether they are polynomial or trigonometric, and order of interpolation;
- application of Galerkin method for the space of functions used;
- solving the algebraic system *[R] {u} = {F}*, in which *[R]* is the stiffness matrix of the structure, *F* is the vector of force and *u* is the multiplier vector of adopted functions;
- analysis of results.

2.3 Modeling via program SAP2000

Soriano (2001) and Mascia and Soriano (2004) present a computer simulation of a timber-concrete composite beam using a finite element program, SAP2000 (2000). Due to the characteristics and dimensions of beams and panels tested, the use of shell elements was chosen to represent the pieces of concrete and wood. The meshes of elements for the concrete flanges were set on horizontal planes and retracted half the thickness of the flange; the thickness attributed to each element is the actual one of the piece.

The pieces of wood were mesh-modeled in the vertical plane of the beam, which thickness corresponds to that of the piece of wood. Based on the dimensions of the finite element, the program automatically considers the weight of the structure itself.

The metallic connectors were represented by linear elements with the diameters of the nails and screws used in the structures. Thus, the connection system is kept discreet and the spacings are the actual distances used. The nodes of the linear element unify the top surface of the timber beam to the concrete flange.

To ensure one of the basic assumptions of the composite structure, both nodes of a single bar are restricted so as to have the same displacement in vertical direction.

The effect of nonlinearity of the material is not considered in the program used for modeling. From such simplification, differences are expected to be observed between the results of tests and those obtained from the modeling as they occur, for example, with concrete, which will adopt a plastic behavior for a given level of stress.

To validate the results of this modeling, several simulations were performed; among those, for instance, it was believed that the connectors represented a perfect connection with infinite stiffness. Another simulation was that of the lack of connection system, i.e. the flange of concrete would be superimposed on the wood and be free to slip on a piece of wood.

The identified sections for the states of stress analysis and deformation in each structure were located in sections positioned one third from the supports in order to avoid the central region of the structures, where effects of concentrated load certainly are. The concentrated force was considered as an equivalent distributed force for the elements in the slab near the central region of the beam. Thus, in the laboratory, it was possible to simulate the actual application of force through a steel plate.

The data used for the preparation of timber-concrete model implemented in the program SAP 2000 is described in Table 1. Soriano (2001) used two beams and their results will be compared with the results of that program, with K= 15464 N/mm, the slip modulus. To assess the value of K, a model of specimen was adopted that consisted of a central concrete element unified by metal connectors and two pieces of wood, subjected to compression and shear connectors. The species of Brazilian wood used was Cupiúba (*Goupia glabra*).

Properties	Concrete	Wood
Section (mm2)	12000	7500
Inertia (mm4)	1.6x106	14.063×10^{6}
E (N/mm2)	19300	14700
Compression Strength (MPa)	22	57.47

Table 1 - Properties of composite structure components

The connectors, screws of 3/8" (0.95 mm), were simulated through the beam elements, joining a section of concrete to the wood in some areas. However, a connection continuous behavior was sought. Therefore, the rigidity of beam elements were chosen in order to simulate the same rigidity of the slip modulus obtained in the laboratory.

For calculation purposes, it is considered a uniformly distributed load which corresponds to the weight of the beam q = 0.36 N/mm, and a concentrated force P applied in the middle of the range were considered. Figure 3 and Figure 4 display details of beams used by Soriano (2001).

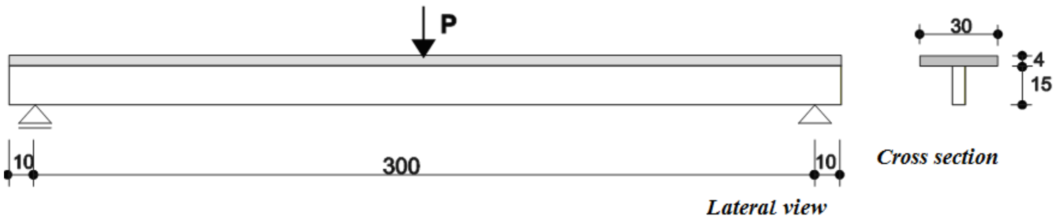

Figure 3 – Dimensions of Composite Beam (cm)

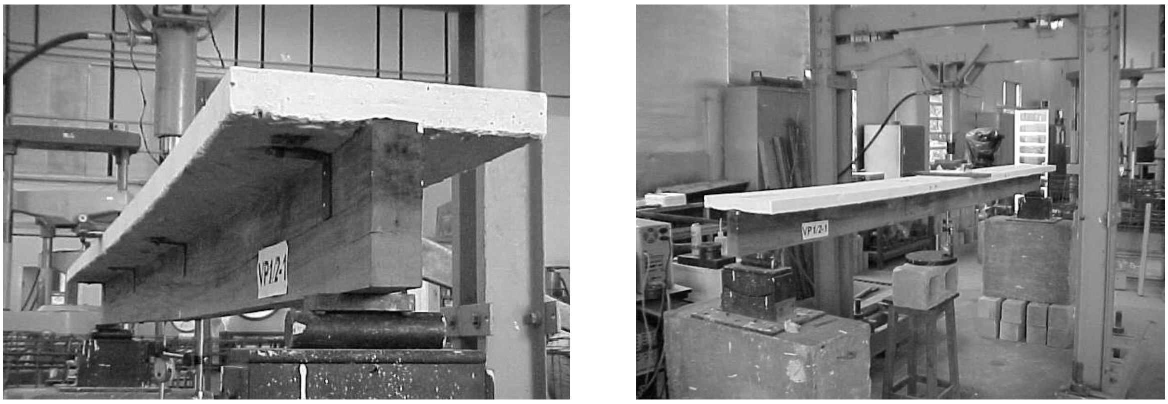

Figure 4 – Details of composite beam

Figure 5 highlights the mesh used to calculate the displacements of the composite beam. The finite elements used to construct the mesh were the projections of the beam in the vertical plan, so that forces T_s (slip) between the flange and web (as in Figure 2) were eccentric to the gravity center of the section of the piece, that is, simulating the real conditions of the tests of the composite beam.

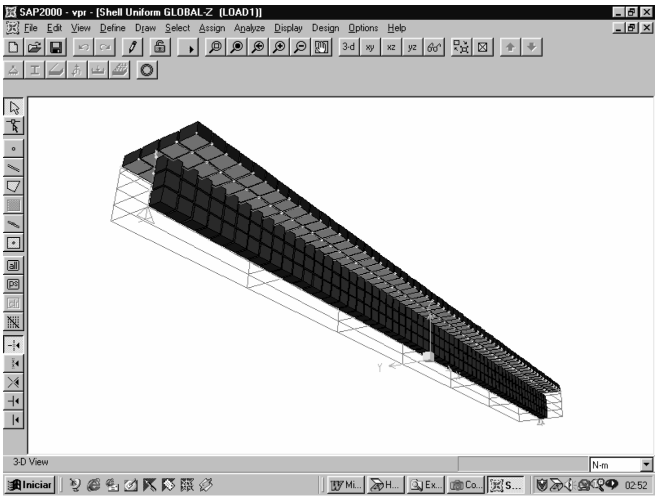

Figure 5- Mesh used to characterize the mixed beam in SAP 2000

2.4 Application of principle of virtual work (PVW) in composite beams

The principle of virtual work is a powerful technique for the resolution of several problems in engineering, especially in the mechanics of solids.

The equation of the principle of virtual work for a status of equilibrium forces and an independent state of the displacements compatible with the supports of the structure is written by:

$$\int_V \sigma_{ij}\,\delta\varepsilon_{ij}\,dV = \int_A T_i\,\delta u_i\,dA + \int_V F_i\,\delta u_i\,dV \tag{3}$$

in Eq. (3): σ_{ij} are the stresses in the structure, $\delta\varepsilon_{ij}$ are the strains compatible with the virtual displacements δu_i, T_i are the superface forces in A (area) e F_i body forces in V (volume).

The left side of Eq. (3) represents the virtual work of internal forces (TV_{int}) and right side of the virtual work of the external forces (TV_{ext}).

Two well-known theories for the beam analysis are Euler-Bernoulli´s and Timoshenko´s, Saada (1974).The difference between the two theories lies in the fact that Euler-Bernoulli theory does not consider the shearing force strain energy, whereas Timoshenko theory makes use of the shear strain to calculate the displacements. This study will deal with beams according to Euler-Bernoulli theory in two dimensions.

Euler-Bernoulli theory acknowledges that normal cross sections in relation to the axis of the beam before deformation remain plane and orthogonal in relation to it after deformation.

The displacement vector u can be expressed by Eq. (4):

$$\vec{u} = \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \tag{4}$$

with: $u_x = u - \theta y, u_y = v, \theta = dv/dx$,where u_x and u_y are displacements in the directions of the axis of the beam x and perpendicular to this axis y e θ is the slope of the cross section.

The tensor of infinitesimal deformation is defined by Eq. (5) as:

$$\nabla u = \begin{bmatrix} u' - yv'' & v' \\ v' & 0 \end{bmatrix} \tag{5}$$

with: $\varepsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right) = \begin{bmatrix} u' - yv'' & 0 \\ 0 & 0 \end{bmatrix}$ and: $\varepsilon = [\varepsilon_x] = [u' - yv'']$.

The tensor of stress is written as: $\sigma = [\sigma_x] = [E\,\varepsilon_x]$.

It should be observed that the hypothesis of the isotropy is considered for both concrete and wood, specifically in the direction of the beam axis.

Structural strains in a bar element are normal force, shear and bending moment, i.e.: $N = \int_A \sigma_x dA = EAu'; V = \int_A \tau_{xy} dA = 0; M = \int_A y\sigma_x dA = -EI v''$.

Virtual work of internal forces of a bar is equal to the virtual work of the actions and, based on Eq. (3), and can be written as:

$$TV_{int} = TV_{ext} \Rightarrow \int_0^L (EA\,\delta u' u' + EI\,\delta v'' v'') ds = \int_0^L q_i \delta u_i ds + P_i \delta u_i \qquad (6)$$

Eq. (6) considers actions of vertical force concentrated and distributed on the beam. The generalization to moments and horizontal forces can be found in Forti (2004) and Mascia, Forti and Soriano (2006).

2.5 Formulation of the composite beam

The approach adopted to be used for the formulation of a mixed structure is the principle of virtual work. The composite beam is considered as two independent beams (one made of concrete and one of wood) connected. The strain energy of the structure will be given as the sum of the strain energies of two beams with the strain energy of the connectors and thus, the virtual work of internal forces of the whole is the sum of the three individual internal virtual works. Thus, it will have Eq. (7):

$$TV_{int}^{C} + TV_{int}^{W} + TV_{int}^{S} = TV_{ext} \qquad (7)$$

in which: S ,W and C indicate: connector, wood and concrete.

It should be noticed that the parts of composite beam, which present different material behaviors, are displayed in Eq. (7).

The connection between the two beams (concrete and wood) will be treated as continuum and the relation stress versus slips as being linear.

As a hypothesis, it is assumed that the vertical displacements of the two beams are equal, and consequently also are its derivatives. As the approach follows that of Euler-Bernoulli beams, the derivative of the vertical displacement is equal to the slope of the cross section, and the two beams have the same slope.

The shear stress of the connectors yields to:

$$T_S = K \Delta u \qquad (8)$$

in Eq. (8): K is the slip modulus, Δu is the relative displacement between the lower fiber concrete beam and the upper fiber wooden beam. In Figure 6, u_{cinter} and u_{winter} are the total displacements on contact surface between the parts of concrete and wood.

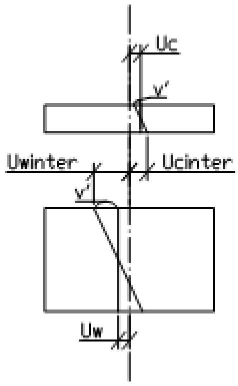

Figure 6 - Kinematics of a composite beam

According to Figure 6 Δu can be written by Eq. (9):

$$\Delta u = u_c - u_w + v'r \tag{9}$$

with: $r = (h_c + h_w/2)$, in which h stands for the heights of the parts of wood and concrete. Thus, the virtual internal work of the connector is worth according with Forti (2004) by Eq. (10):

$$TV_{int}^{S} = \int_0^L T_s \delta u ds = \int_0^L K \Delta u \delta u ds \tag{10}$$

Considering the Rayleigh-Ritz method, which is a technique for the resolution of approximate variational formulations, such as the PVW, the approximate solution is given by:

$$sol(x) = \begin{Bmatrix} \sum_{i=1}^{N} \alpha_i^{v} \varphi 1_i(x) \\ \sum_{i=1}^{N} \alpha_i^{u^C} \varphi 2_i(x) \\ \sum_{i=1}^{N} \alpha_i^{u^W} \varphi 3_i(x) \end{Bmatrix} \tag{11}$$

The indices v, u^C and u^W in Eq. (11) are the variables of the problem: vertical displacement, horizontal displacement of the average fiber concrete beam; horizontal displacement of the average fiber wooden beam and $\varphi 1(x)$, $\varphi 2(x)$ and $\varphi 3(x)$ are approximants functions, which can be equal or not.

The function $\varphi(x)$ must have some properties in order to ensure the quality of approximation. The first property concerns the regularity of $\varphi(x)$. For the approximation of horizontal displacements, the function $\varphi(x)$ must be continuous throughout the area. For the approximation of vertical displacement, it is necessary that the functions and their first derivatives are continuous. The restriction of the derivative is required to ensure the slope (derived from the vertical displacement) to be continuous throughout the area.

Besides the restriction of regularity, it is necessary that the $\varphi(x)$ satisfy the conditions of the outline of the structure, respecting its connections. The boundary conditions imposed in the simulations were: null vertical displacement at the supports; null horizontal displacement in the middle of the structure. Due to the symmetry of the simulated structure and loading, imposing a null displacement in the middle of the horizontal beams was chosen.

With the imposition of such boundary conditions, avoiding any kind of rigid body motion, which would make the stiffness matrix singular and hinder the resolution of the algebraic system.

Once chosen the functions $\varphi(x)$, replaced the functions u_c, u_w and v for Eq. (12):

$$\sum_{i=1}^{N} \alpha_i \varphi_i(x) \tag{12}$$

and the functions: δu_c , δu_w and δv for $\varphi(x)$.

Replacing the problem variables for the tentative functions $\varphi(x)$, the system of equations from PVW can be written in a matrix format as:

$$[R]\{\alpha\} = \{F\} \tag{13}$$

in Eq. (13) $[R]$ is named stiffness matrix, α the solution of linear system and F the force vector.

It should be noticed that the stiffness matrix can be divided into three terms: the concrete beam, the wooden beam and the connection according Eq. (14):

$$[R_{ij}] = [R_{ij}^{C}] + [R_{ij}^{W}] + [R_{ij}^{S}] \tag{14}$$

The solution of the linear system builds the approximation of the solution of PVW equation and, with that, it is possible to calculate the displacement of the beam.

2.6 Choice of approximant functions $\varphi(x)$

For the simulations of this study, sine functions were used as an approximation of the solution (Mathematica,1997). Sine functions are continuous, just like their derivatives. Furthermore, the conditions of null vertical displacement boundary of the support and zero horizontal displacements in the middle of the structure were imposed.

The functions outlined below (Eq. (15) and Eq. (16)) were used to approximate the vertical displacement. They are continuous across the field, just like their derivatives. Moreover, they satisfy the conditions for the contour to null at x = 0 and x = L.

$$ShapeV[i] = Sin[\pi x(2i-1)/L] \tag{15}$$

The following functions were used to approximate the vertical displacement:

$$ShapeU[i] = Sin[\pi(x - L/2)(2i-1)/L)] \tag{16}$$

They are continuous across the field, just like their derivatives. Moreover, they satisfy the boundary conditions to be null at $x = L/2$.

3 RESULTS AND DISCUSSIONS

The results obtained from the proposed formulation (PVW) were verified with the equations of equilibrium given by Stevanovic (1996). Results obtained from the analytical resolution of this formulation were compared directly with the results of the PVW. The comparison was made via diagrams of the two solutions to the cases of uniformly distributed loading and loading of the shear concentrated force in the middle of the range.

Another analysis in this study concerns the use of SAP 2000. Table 2 shows the experimental values of Soriano (2001) with the analysis undertaken in this study (SAP 2000, PVW, and Stevanovic).

Load (kN)	Exp.vp-1	Exp.vp-2	SAP2	PVW	Stevanovic
0	0	0	0	0.00	0.00
5	4.74	4.63	6.3	7.38	7.38
10	9.64	9.37	11.8	13.73	13.73
15	14.71	14.38	17.3	20.08	20.08
20	21.20	19.68	22.8	26.44	26.44
25	26.68	25.17	28.3	32.79	32.79
30	32.81	32.74	33.8	39.14	39.14

Table 2 - Results of vertical displacement (mm) to 1/3 of the beam span

Figure 7 displays the values of Table 2.

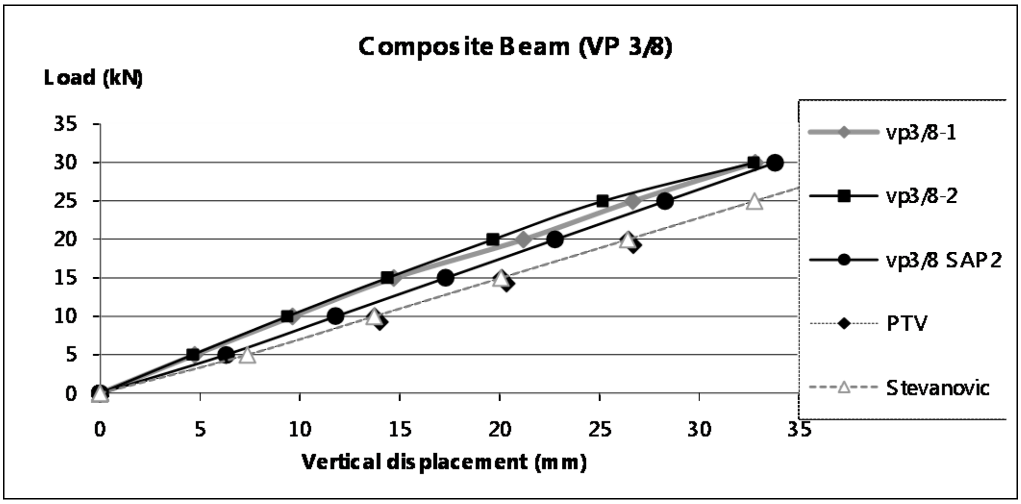

Figure 7 - Results of the vertical displacement 1/3 of the beam span

Observations on the legend of Figure 7:

• VP 3/8 -1, VP 3/8 -2: beams of concrete and wood connected with screws, 3/8" (0.95 mm) tested in the laboratory;

• VP 3/8 SAP2: the connectors were simulated using the elements of the beam, joining a section of concrete to wood in some areas. However, a manner was sought out for the continuous connection. For that the rigidity of the beam elements was chosen in order to simulate the same rigidity of the slip modulus obtained in the laboratory;

• PTV (PVW): the formulation proposed in this study, from the principle of virtual work. This curve almost coincides with the curve of Stevanovic;

• Stevanovic: the resolution of the analytical equilibrium equations of the composite beam, as the formulation proposed by the author.

To evaluate the results in Table 2 the statistical procedure based on the statistical program Minitab was used and according to the statistical concepts of small samples, as shown by Ryan and Joiner (1994). Thus, it is possible to follow the following analysis:

• μ_1 is the average of the 1st sample and μ_2 is the average of the 2nd sample. To test whether these two samples belong to the same universe, the following hypothesis was used: H_o: $\mu_1 = \mu_2$ vs. H_1 : $\mu_1 \neq \mu_2$.Calculating the significance through Eq. (17):

$$t = (\bar{x}_1 - \bar{x}_2)/\sqrt{s_1^2/n_1 + s_2^2/n_2} \leq t_\phi \ (p\%) \tag{17}$$

in which: $\bar{x}_1$: estimate of 1st sample average; $\bar{x}_2$: estimate of 2nd sample average; s_1: standard deviation of the 1st sample; s_2: standard deviation of the 2nd sample; n_1: number of the elements in the 2nd sample; n_2: number of the elements in the 2nd sample; t_ϕ (P%): value of the Student't-distribution with $P\%$ of confidence; $P\%$: confidence level adopted. For DF = 11(degrees of freedom) e $P\%$ = 95% the sample has been $t_\phi(95\%) = 1.796$.

Nevertheless, to check whether the averages of the samples are statistically equivalent, i.e. if the range of difference of averages μ_2 , μ_1 contains zero, is determined the following range:

$$\Delta_- = (\bar{x}_1 - \bar{x}_2) - t^* \sqrt{s_1^2/n_1 + s_2^2/n_2} \leq \mu_2 - \mu_1 \leq (\bar{x}_1 - \bar{x}_2) + t^* \sqrt{s_1^2/n_1 + s_2^2/n_2} = \Delta_+ \tag{18}$$

where in Eq. (18) t^* is the corresponding value for $P\%$ confidence.

Analyzing the data of Tabela 3 for the experimental analysis, with more critical emphasis on the test results, the study concludes that the results meet the statistical hypothesis adopted.

	Δ_-	Δ_+	$\lvert t^* \rvert$	Result
Exp.vp3/8-2 x SAP2	-16,0	11,9	0,32	Ok!
Exp.vp3/8-2 x PTV	-19,9	10,3	0,70	Ok!
SAP x PVW	-18,1	12,6	0,39	Ok!

Tabela 3 – Two sample test

It is observed that, in general, the differences between the experimental results of displacement and the SAP and Stevanovic (PTV) were, on average, 13.21 and 31.69%.

4 CONCLUSIONS

This study contributes to the analysis of composite structures and proposes a variational formulation based on the principle of virtual work. This principle has been used in almost all fields of structural engineering.

The results were fully adequate and in comparison with SAP suggest that the possibility of connection is still higher than the discrete. However, this analysis showed a deficiency. It was observed that the results vary with the size of the bars used to simulate the connection of composite beam. This is due to the bars are articulated on one end and rigid in another. On the rigid end, the bending moment is transmitted besides the expected horizontal forces. As the value of the moment is proportional to the length of the bar, the longer the bar the greater the error.

The analytical and numerical analysis, although calibrated with each other, had differences with the results of the tests. A possible contribution to these differences is the value of *K*, measurement of the slip modulus, which presents laboratory experimental results as a coefficient of variation of 15%.

The proposed formulation satisfies the equations of balance and is consistent, as are their results in relation to laboratory data.

The integration of this formulation in finite elements permits a greater dissemination of calculation procedures and composite beams behavior, which would be a catalyst for the employment of such structure.

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