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ON THE VIBRATION CONTROL OF BEAMS USING A MOVING ABSORBER AND SUBJECTED TO MOVING LOADS

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Abstract: In this work, the Euler-Bernoulli linear beam theory is used to study the vibrations of simply supported beams subjected to moving loads and controlled by a moving vibration absorber. The beam is considered as a linear elastic continuous system and the vibration absorber is described as a linear spring-mass-damper system moving with a constant velocity along the beam. A modal expansion with five modes is used to model the lateral displacements of the beam and the Galerkin method is used to obtain a set of equations of motion which are, in turn, solved by the Runge-Kutta method. The obtained results show the importance of position and velocity of the damper on the vibration control of beams with moving loads.

1 INTRODUCTION

The study of bridge oscillations and control is a problem that has been the object of interest of engineers and scientist over the last century (Museros and Martinez-Rodrigo, 2007; Yang *et al*, 1997). For example, Den Hartog (1956) has derived the optimum parameters of the absorber for suppressing the dynamic response of the single degree-of-freedom spring-mass system. Here only a few investigations will be cited.

Greco and Santini (2002), by using an extension of the complex mode superposition method, analyzed the dynamic problem of a continuous beam with two end rotational viscous dampers under a single moving load. They concluded that the damper's effectiveness is strongly dependant on the load velocity and proved that, in the relevant range of velocities, a considerable reduction of the dynamic response of the beam is to be expected if the damper's constants are selected properly.

Wu (2006) proposed the use of helical absorber to reduce the vibrations of beams subjected to moving loads. The finite element method was used to model the beam. The damper was located in the middle of the beam, taking into account the spring mass. To study the behavior of the beam, the governing equations were reduced to the first modal coordinate and, following Den Hartog's approach (Den Hartog, 1956), this simplified model was used to obtain optimal values for the stiffness and damping ratio of the absorber.

The possibility of reduction of the resonant vibration of simple beams under moving loads by increasing the structural damping with passive energy dissipation devices was evaluated by Museros and Martinez-Rodrigo (2007). The authors used a linear viscous damper (FVDs) to connect the main beam, which carries the loads, to an auxiliary beam placed underneath the main one. The results show that the resonant response of the main beam can be drastically reduced with this type of device and that proposed methodology has potential applications for reduction of the response of railway bridges subjected to the transit of high-speed trains.

Recently, Thompson (2008) used a continuous damped mass–spring system added to a beam to attenuate the propagation of structural waves and to reduce the radiated noise while Samani and Pellicano (2009) analyzed the effectiveness of a dynamic vibration absorber applied to a simply supported beam excited by moving loads. The performance of both linear and nonlinear dampers was analyzed. The performance of the dynamic dampers in vibration reduction was estimated through the maximum amplitude of vibration and by the amount of energy dissipated by the dynamic damper.

In this work, the Euler-Bernoulli linear beam theory is used to study the vibrations control of simply supported beams subjected to moving loads and controlled by a fixed or moving absorber. The beam is considered as a linear elastic continuous system and the absorber is described as a linear spring-mass-damper system moving with a specified velocity along the beam. A modal expansion with five modes is used to model the lateral displacements of the beam and the Galerkin method is used to obtain a set of discretized equations of motion which are, in turn, solved by the Runge-Kutta method. The initial results show the importance of position and velocity of the damper on the vibration control of the beam and can be used for engineers to optimize the position of vibration absorbers.

2 PROBLEM FORMULATION

Consider a simply supported elastic beam with length L, Young's modulus E, inertia I, distributed mass m and damping coefficient c subjected to a moving load $F(x,t)$ with velocity V_L as shown in Figure 1. The beam is connected to an absorber represented by a small mass m_2, a linear spring with stiffness k_2 and a linear viscous damper with damping coefficient λ. It is assumed that the absorber can either be fixed or move along the beam with a velocity V_A.

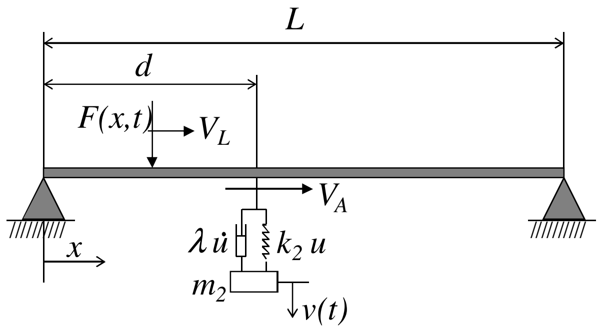


Figure 1 - The controlled beam model

In this work the mathematical formulation will follow that previously presented by Samani and Pellicano (2009). The partial differential equation of motion governing the flexural behavior of a simply supported beam using the linear Euler-Bernoulli theory can be found in the work of various authors such as Yang *et al.* (1997), Greco and Santini, 2002; and Muserosa and Martinez-Rodrigo (2007) and is written as:

$$EI\frac{\partial^4 y}{\partial x^4}+m\frac{\partial^2 y}{\partial t^2}+c\frac{\partial y}{\partial t}+\left[k_2 u+\lambda\frac{\partial u}{\partial t}\right]G(x,t)=F(x,t); \qquad (1)$$

$$m_2\frac{\partial^2 v}{\partial t^2}-k_2 u+\lambda\frac{\partial u}{\partial t}=0 \qquad x\in(0,L)\;\; t>0; \qquad (2)$$

where $y(t)$ represents the lateral displacement field of the beam; $v(t)$ is the absolute position of the mass m_2; d is the absorber position and $u(t) = y(d, t)-v(t)$.

In Eq. (1) the term $\left[k_2 u+\lambda\frac{\partial u}{\partial t}\right]G(x,t)$ is the force exerted by and $G(x, t)$ is given by:

$$G(x,t)=\delta(x-d) \text{ For a fixed absorber;} \qquad (3)$$

$$G(x,t)=\delta(x-V_A t)\,H\left(\frac{L}{V_A}-t\right) \text{ For a moving absorber;} \qquad (4)$$

where $x = d$ represents the location of the damper in the beam at time t; V_A is the absorber velocity; δ is the Dirac delta function which defines the location of the dynamic damper and $H(t)$ is the Heaviside function.

The external force $F(x, t)$ is a moving load given by:

$$F(x,t)=\delta(x-V_L t)\,H\left(\frac{L}{V_L}-t\right). \qquad (5)$$

The boundary conditions for the simply supported beam and initial conditions for the problem are given by:

$$y(0,t)=0; \quad y(L,t)=0; \quad \frac{\partial^2 y}{\partial x^2}(0,t)=0; \quad \frac{\partial^2 y}{\partial x^2}(L,t)=0; \qquad (6)$$

$$y(x,0)=0; \quad \frac{\partial y}{\partial t}(x,0)=0; \qquad (7)$$

The attached mass is small compared to the beam mass. In this work, the lumped mass of the absorber is taken to be 5% of the total mass of the beam (Wu, 2006).

The equations of motion of the system represented by Eqs. (1) and (2) are analyzed after projecting the partial differential Eq. (1) into a complete orthonormal basis (Samani and Pellicano, 2009). The eigenfunctions of the simply supported beam with no attachments can be used as interpolating functios. They are given by:

$$\phi(x)=\sum_{r}\sin\left(\frac{r\pi x}{L}\right) \quad r = 1, 2, 3 \ldots. \tag{8}$$

The natural frequency of the beam for the r^{th} mode is given by:

$$\omega_r = (\pi r)^2 \sqrt{\left(\frac{EI}{mL^4}\right)} \tag{9}$$

Finally, the transverse vibration of the beam can be assumed as (Museros and Martinez-Rodrigo, 2007; Samani and Pellicano, 2009):

$$y(x,t) = \sum_{r=1}^{\infty} A_r(t)\,\phi_r(x), \tag{10}$$

where the $A_r(t)$ are the unknown functions of time and $\phi_r(x)$ is given by Eq. (8).

Substituting Eq. (10) into Eqs. (1) and (2), applying the Galerkin method and using the orthonormality conditions, the following system of equations is obtained:

$$\begin{aligned}&\frac{mL}{2}\ddot{A}_p(t) + \xi_p\omega_p mL\,\dot{A}_p(t) + \frac{\omega_p{}^2 mL}{2}A_p(t) + \left\{k_2\left[\sum_{r=1}^{\infty}A_r(t)\,\phi_r(D) - v(t)\right] + \right.\\ &\left.\lambda\left[\sum_{r=1}^{\infty}\dot{A}_r(t)\,\phi_r(D) - \dot{v}(t)\right]\right\}\phi_p(D)\,G(t) = F_o\phi_p(V_L t)\,H\left(\frac{L}{V_L} - t\right)\end{aligned} \tag{11}$$

$$m_o\ddot{v}(t) - k_2\left[\sum_{r=1}^{\infty}A_r(t)\phi_r(D) - v(t)\right] - \lambda\left[\sum_{r=1}^{\infty}\dot{A}_r(t)\phi_r(D) - \dot{v}(t)\right] = 0\,; \tag{12}$$

where $\dot{A}_p(t) = dA_p / dt$ and:

$$D = d \ \text{ and } \ G(t) = 1 \ \text{For fixed absorber,} \tag{13}$$

$$D = V_A t \ \text{ and } \ G(t) = H\left(\frac{1}{V_A} - t\right) \ \text{For moving absorber.} \tag{14}$$

2.1 Absorber optimal parameters

The dynamic amplification factor for the principal system with an added classic absorber as shown in Figure 1, is given by (Den Hartog, 1956; Ávila, 2002; Wu, 2006):

$$R = \sqrt{\frac{(\alpha^2 - \beta^2)^2 + (2\xi_2\,\alpha\,\beta)^2}{C_1}} \tag{15}$$

and

$$\begin{aligned}C_1 &= \left[(\alpha^2 - \beta^2)(1 - \beta^2) - \mu\,\alpha^2\beta^2 - 4\,\xi_1\,\xi_2\,\alpha\,\beta^2\right]^2 \\ &+ \left[2\,\xi_2\,\alpha\,\beta(1 - \beta^2 - \mu\,\beta^2) + 2\xi_1\,\beta(\alpha^2 - \beta^2)\right]^2\end{aligned} \tag{16}$$

where ξ_1 and ξ_2 are respectively the damping ratio of the principal system and absorber; α is the ratio of the natural frequencies of the absorber and the principal system; μ is the ratio of the masses of the absorber and the principal system and β is the ratio of the frequency of the external excitation and the natural frequency of the principal system.

The system optimal parameters can be found by applying different control theories, in this work the Den Hartog (1956) theory will be used. Assuming zero damping for the principal system ($\xi_1 = 0$), it is possible to find the classical expressions obtained by Den Hartog (1956) for the optimum tuned mass and the optimal damping ratio which are given respectively by:

$$\alpha_{ot} = \frac{1}{1+\mu} \tag{17}$$

$$\xi_{2ot} = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{18}$$

The generalized modal mass and stiffness of the principal system, $\tilde{m}_1$ e $\tilde{k}_1$, necessary for the calculation of the optimal parameters are given by:

$$\tilde{m}_1 = \int_0^1 \sin(\pi x)\, m_1 \sin(\pi x)\, dx \tag{19}$$

$$\tilde{k}_1 = \omega_o^2\, \tilde{m}_1 \tag{20}$$

where m_1 is the mass and ω_o is the natural frequency of the principal system and μ is written as:

$$\mu = \frac{m_2}{\tilde{m}_1} \tag{21}$$

Thus, the absorber stiffness k_2 and the damping coefficient of the viscous damper λ are, respectively, given by:

$$k_2 = \alpha^2 \omega_o^2 m_2 = \alpha_{ot}^2 \frac{\tilde{k}_1}{\tilde{m}_1} m_2 \tag{22}$$

$$\lambda = 2\xi_2\, m_2 \sqrt{\frac{k_2}{m_2}} = 2\xi_{2ot} \sqrt{k_2\, m_2} \tag{23}$$

3 NUMERICAL RESULTS

For the numerical analysis, consider a beam with Young's modulus $E = 206.8$ G*Pa*, mass density $\rho = 7820$ Kg/m^3, the cross-sectional area 0.03 *m* x 0.03 *m*, $\xi_p = 0$ ($p = 1, 2, \ldots$) and $F_o = 9.8$ *N*. For the beam, three different lengths are used in the analysis and were chosen only to extend the length of Samani and Pellicano (2009). They are shown in Table 1 together with the associated system parameters.

	L (*m*)	m_1 (*Kg*)	ω_o (*rad/s*)	m_2 (*Kg*)	$\tilde{m}_1$ (*Kg*)	$\tilde{k}_1$ (*N/m*)	k_2 (*N/m*)	λ (*Ns/m*)
L4	4	28.152	27.471	1.4076	14.076	10622.92	877.92	12.98
L5	5	35.197	17.582	1.7595	17.595	5438.93	494.45	10.89
L6	6	42.228	12.209	2.1114	21.114	3147.24	286.11	9.076

Table 1 – Parameters of the beam and absorber with $\mu = 0.1$; $\alpha_{ot} = 0.909$ and $\xi_{2ot} = 0.18464$.

3.1. Fixed absorber and moving load

To check the accuracy of the present model, consider the system shown in Figure 1 with a fixed absorber ($V_a(t) = 0$ and $d = 0.5L$) and an external force with constant amplitude and constant velocity $V_L(t) \neq 0$. Figure 2 shows the maximum displacement at the mid-span of the L4 beam as a function of the velocity of the moving load. The results compare well with those obtained by Samani and Pellicano (2009). Figure 3 shows the maximum displacement at the mid-span of the beam L4 with and without damper as a function of the load velocity. The maximum displacement occurs for a velocity $V_L = 21.2$ *m/s*. The inclusion of the absorber reduces the maximum displacement of the beam up to 5%.

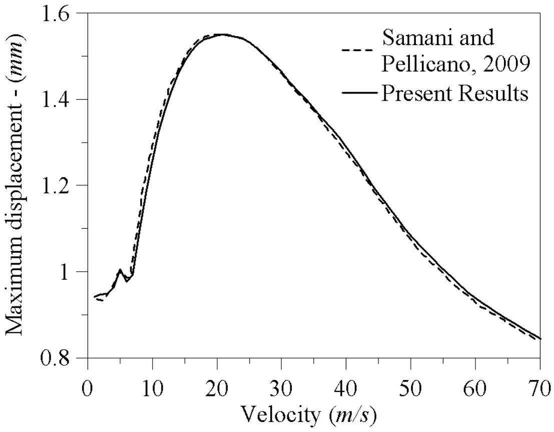


Figure 2 - Beam subjected to moving load and connected to linear fixed damper. Maximum displacement at mid-span of the L4 beam as a function of the load velocity.

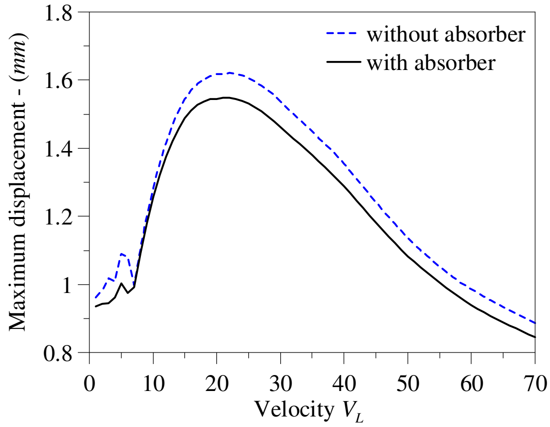


Figure 3 - Comparison maximum displacement at mid-span of the beam ($x = 0.5L$). Beam L4 with and without absorbers.

Figure 4 shows the influence of the beam length on the mid-span displacement of the beam without absorber. It is possible to observe that the maximum displacement occurs for different load velocities: for the L4 beam the maximum displacement occurs for $V_L = 21.2$ *m/s*, for the L5 beam at $V_L = 16.7$ *m/s* and for the L6 beam at $V_L = 13.9$ *m/s*.

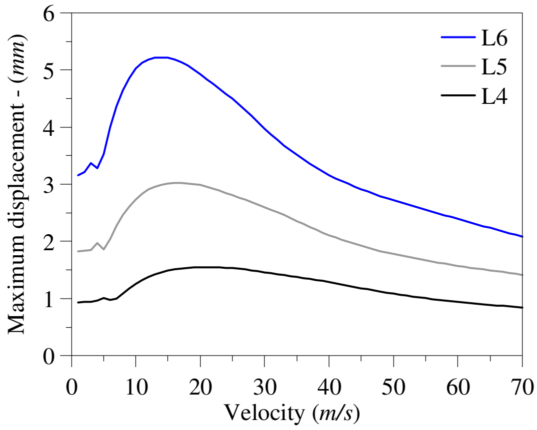
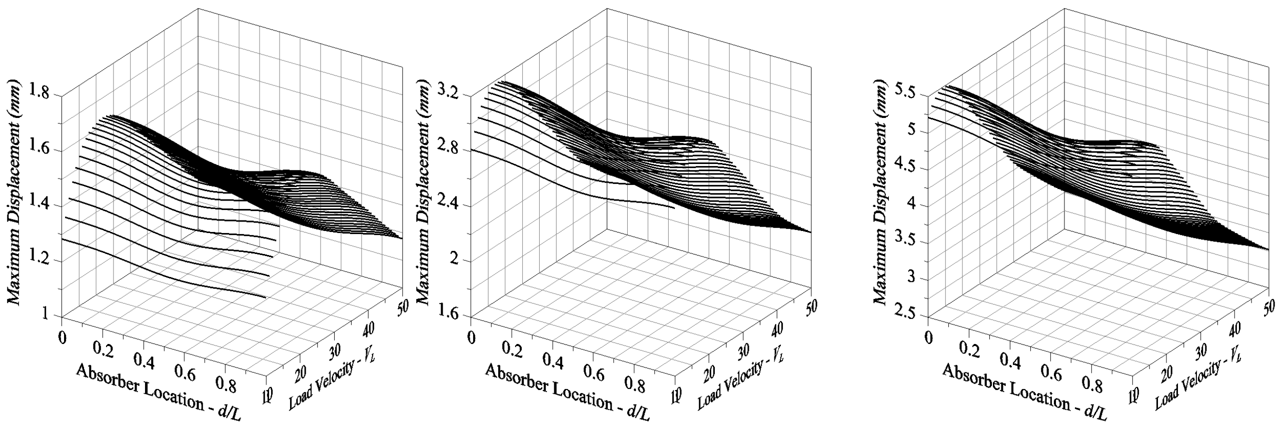


Figure 4 - Comparison of the displacement at mid-span of the beams lengths shown in Table 1.

The optimization of the dynamic damper is focused on the minimization of the maximum beam displacement. Then, the stiffness, the viscous damping coefficient and the location of the dynamic absorber can be varied to find the optimum values.

Figure 5 shows the maximum displacement at mid-span of the beam ($x = 0.5L$) against load velocity (V_L) considering different absorber positions (d). As can be observed, the damper position has an important influence on the beam response. The displacement variation due to the load velocity can be also observed in Figure 6, where a projection of Figure 5 on $y(0.5L, t)$ *versus* V_L is displayed. The dispersion of results at each velocity value shows the variation of the displacement with the absorber location.



(a) L4 beam (b) L5 beam (c) L6 beam

Figure 5 - Maximum displacement at mid-span of the beam ($x = 0.5L$) as a function of the absorber location (d/L) and load velocity ($10 \leq V_L \leq 50$). (a) L4 beam, (b) L5 beam and (c) L6 beam.

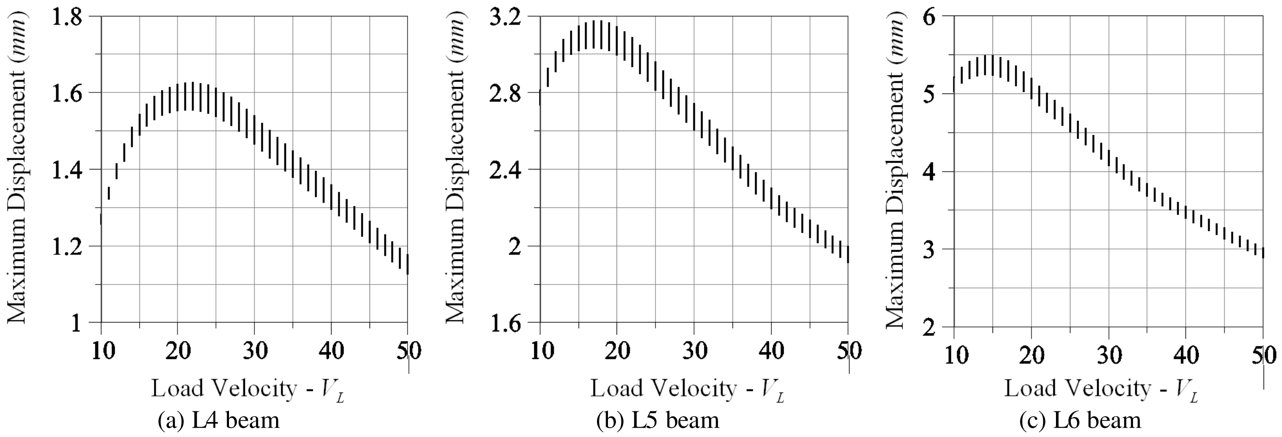


Figure 6 - Maximum displacement at mid-span of the beam ($x = 0.5L$) *versus* load velocity (V_L). (a) L4 beam, (b) L5 beam and (c) L6 beam.

Figure 7 displays another projection of Figure 5 where the displacement of the beam at $x = 0.5\ L$ is shown as a function of the absorber position (d/L). For any velocity and beam length, the best absorber position is located in the range of $0.4 < d/L < 0.6$.

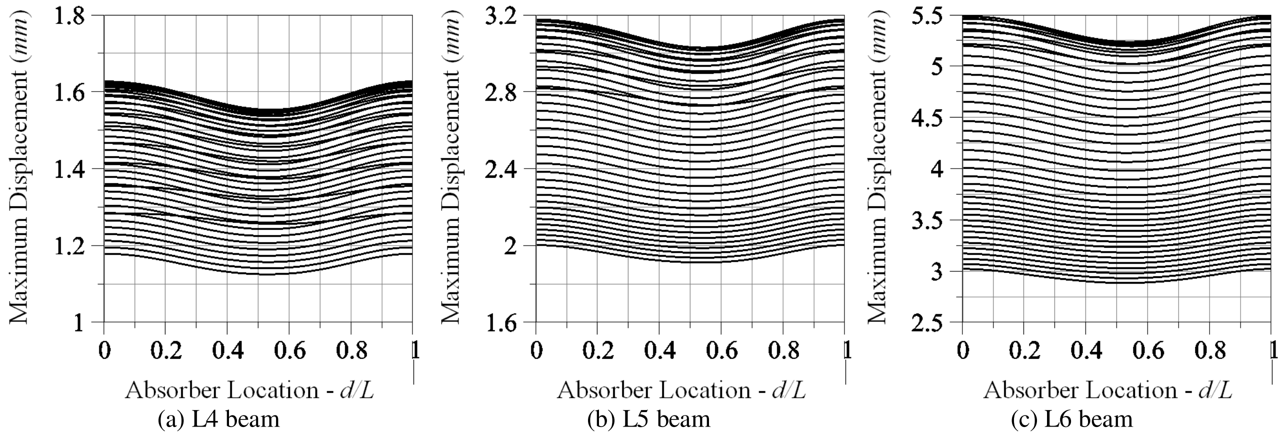


Figure 7 - Maximum displacement at mid-span of the beam ($x = 0.5L$) as a function of the absorber location (d/L). (a) L4 beam, (b) L5 beam and (c) L6 beam.

The absorber position was varied considering the load velocities that generate the maximum displacements at the beam (L4, $V_L = 21.2$ *m/s*; L5, $V_L = 16.7$ *m/s*; L6, $V_L = 13.9$ *m/s*). The results are displayed in both Table 2 and Figure 8. In Table 2 the parameter $l_{máx}/L$ indicates the position at which the maximum displacement occurs. As can be observed, the best absorber position is located close to the mid-span of the beam.

	V_L (*m/s*)	$l_{máx}/L$	d/L
L4	21.2	0.525	0.540
L5	16.7	0.520	0.544
L6	13.9	0.533	0.543

Table 2 – Location of maximum beam displacement and optimal absorber position.

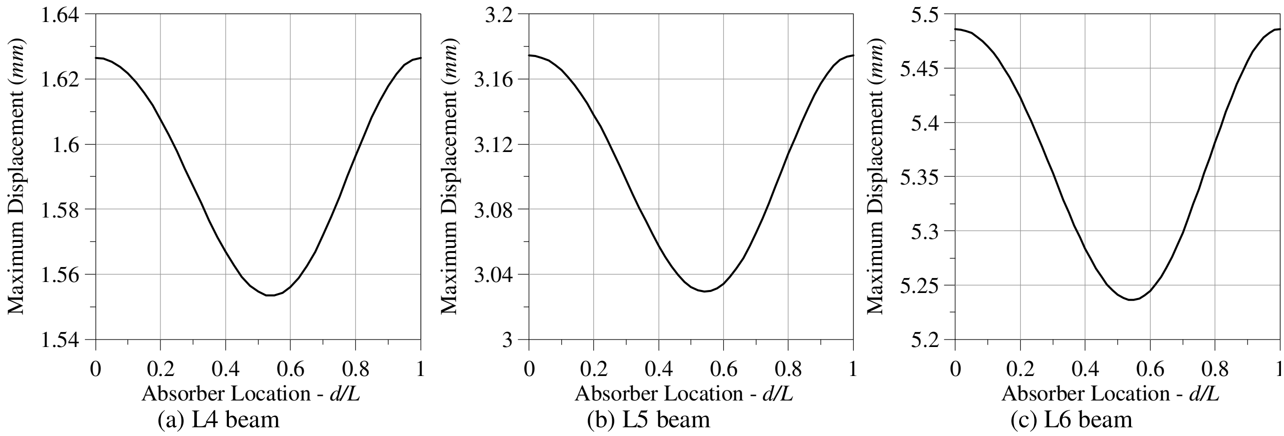


Figure 8 – Maximum beam displacement with varying absorber position. (a) L4 beam, (b) L5 beam and (c) L6 beam.

Figure 9 displays the beam coordinate where the maximum displacement occurs considering different absorber positions ($0 < d < L$ varied with increments of 0.1 m) and different load velocities varying from 10 *m/s* to 50 *m/s* with increments of 1 *m/s*. From all absorber positions and load velocities combinations, this Figure displays only the maximum obtained values and they fit between $x/L = 0.38$ and $x/L = 0.54$. This means that the maximum beam displacement occurs approximately in mid-span of the beam.

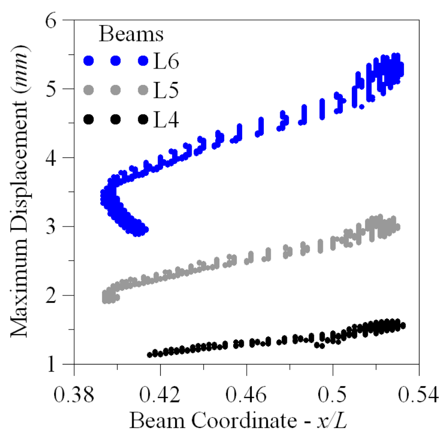


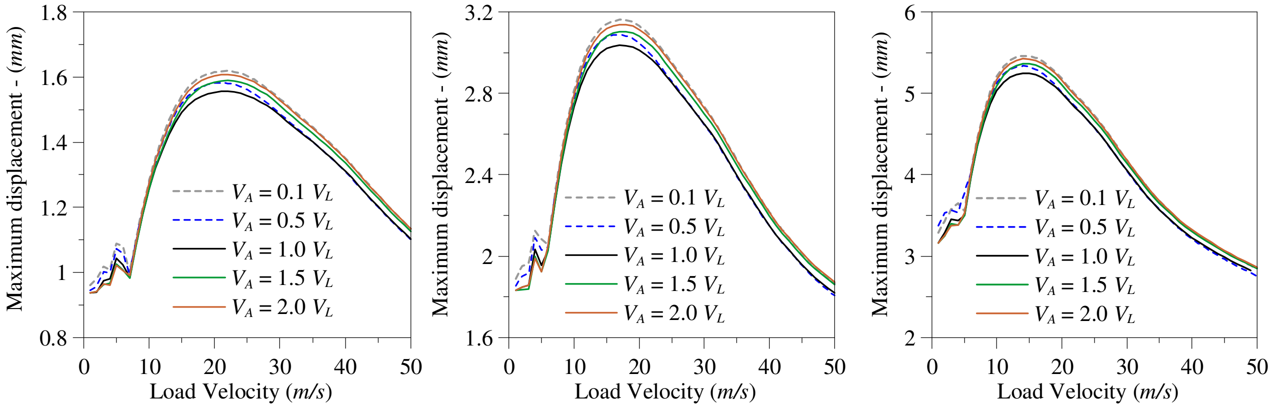
Figure 9 – Maximum beam displacement and coordinate at which it occurs for different absorber position ($0 \leq d \leq L$) and load velocity varying in the interval 10 *m/s* $\leq V_L \leq$ 50 *m/s*.

3.2. Moving absorber and moving load

Consider now the system of Figure 1, with a moving load and a moving absorber with velocity $V_a(t)$ = constant. Figure 10 shows the maximum displacement at the mid-span of the beam for the three considered models as a function of the load velocity V_L for five different values of the absorber velocity V_A. The maximum displacement curves are rather similar but the maximum in shape, being a function of both the load and absorber velocity.

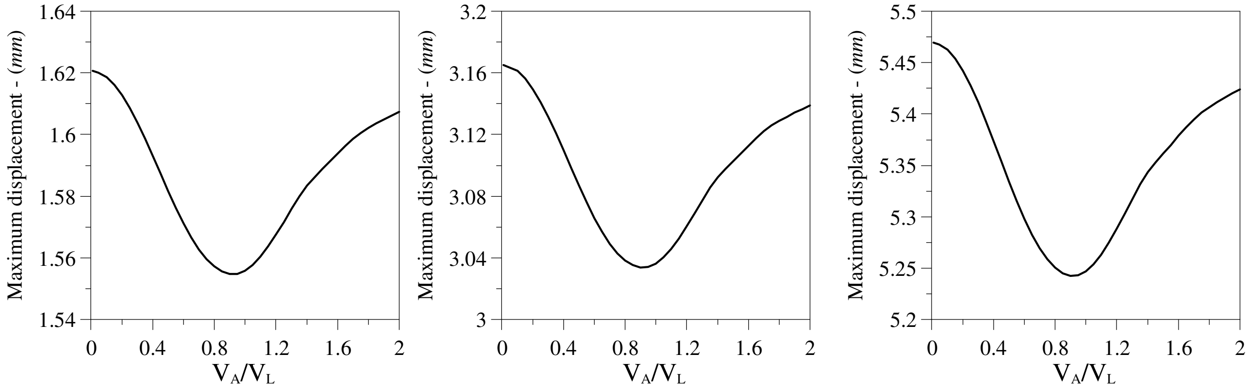
Figure 11 displays the variation of the maximum displacement at the mid-span of the beam for varying absorber velocity and for a load velocity that generates the maximum displacement in Figure 10. The maximum displacement varies with the absorber velocity and, in most cases the maximum reduction is achieved for ratios V_A / V_L around *0.9* and, this ratio should be due to the maximum energy absorption with at almost the same velocities. Figure

12 shows a comparison of the maximum displacements displayed in Figure 11 but normalized in relation to the maximum of the beam when $V_A / V_L = 0.9$ and, as can be observed, the obtained results show that all beams display the same normalized behavior.



(a) L4 beam (b) L5 beam (c) L6 beam

Figure 10 - Maximum displacement at mid-span of the beams ($x = 0.5L$) versus Load velocity (V_L). (a) L4 beam, (b) L5 beam and (c) L6 beam.



(a) L4 beam (V_L = 21,5 *m/s*) (b) L5 beam (V_L = 17,2 *m/s*) (c) L6 beam (V_L = 14,3 *m/s*)

Figure 11 – Maximum displacement at mid-span as a function of V_A/V_L. (a) L4 beam, (b) L5 beam and (c) L6 beam.

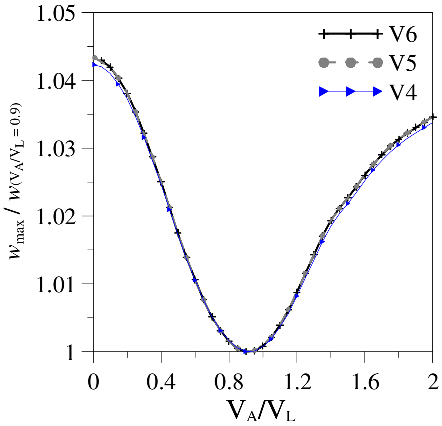


Figure 12 – Normalized maximum displacement at mid-span of the beam.

Figure 13 shows the position in the beam where the maximum displacement occurs as a function of x/L. Again, as seen in Figure 9, the maximum displacement occurs in a range located very close to the mid-span of the beam.

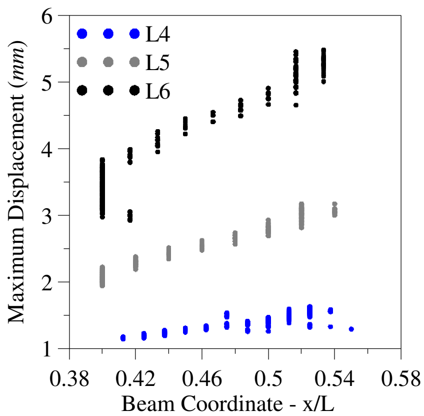


Figure 13 - Maximum displacement x beam normalized coordinate.

4 CONCLUSION

In this work, the Euler-Bernoulli linear beam theory is used to study the vibrations control of simply supported beams subjected to moving loads and controlled by a moving absorber. The beam is considered as a linear elastic continuous system and the absorber is described as a linear mass-spring-damper system moving with a defined velocity along the beam. The maximum vibration amplitudes of the beam depend on both the load and absorber velocity. The absorber velocity can be optimized to obtain the maximum displacement reduction and, as observed in results, for all combinations of both absorber and load velocities, the maximum displacement reduction achieved is for ratios V_A / V_L around *0.9*. These initial results show the great influence of a moving absorber on the beam vibrations and, depending on both the absorber velocity and load velocity, the maximum beam displacement occurs very close to the mid-span.

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