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APPLICATION OF A HIGH-ORDER FINITE VOLUME METHOD TO AEROACOUSTIC PROBLEMS OF INDUSTRIAL INTEREST

Sofiane Khelladi[b], Xesús Nogueira[a], Hakim El Hadjen[b], Ignasi Colominas[a], Farid Bakir[b] and Manuel Casteleiro[a]

[a]*Grupo de Métodos Numéricos en Ingeniería, Universidade da Coruña, Campus de Elviña, 15071 A Coruña, España, jparis@udc.es, http://caminos.udc.es/gmni*

[b]*Arts et Métiers ParisTech, 151 boulevard de l'Hôpital, 75013 Paris, France,*

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Abstract.

Nowadays, the resolution of Aeroacoustic problems using numerical methods (CAA) is currently a very active area of research, with great industrial interest. However, the simulation of the sound propagation is a very difficult numerical problem, and moreover, most of industrial applications present complex geometries. Thus, numerical methods successfully used in unstructured grids in aeronautics fail when applied to CAA. This is due to the excessive amount of dissipation and not enough spectral resolution of these schemes. In this paper we present the application of a high-order finite volume method based in Moving Least Squares (FV-MLS) to the simulation of acoustic wave propagation in complex domains. This work continues the work of the authors in the development of numerical methods to be used in the resolution of acoustic problems of engineering interest.

1. INTRODUCTION

Nowadays there is a need of decreasing the noise generation of industrial devices. Environmental problems and the interest in public health have increased the attention to the effects of noise-pollution. Thus, laws about the level of noise are currently more restrictive, and a low-noise-level emission is a key feature that can rule the success or the failure of a commercial product (for example air-conditioning machines, vacuum cleaners, wind turbines, etc). In this context, the resolution of Aeroacoustic problems using numerical methods (CAA) is currently a very active area of research, with great industrial interest.

The simulation of the sound propagation in the air is a very difficult numerical problem (Tam (1995)). High-resolution finite difference schemes or spectral methods have been traditionally used as the main methods for the resolution of aeroacoustic problems (Hu et al. (1996); Tam y Webb (1993)). The reason is the low-dissipation and excellent dispersion relation of this kind of numerical schemes. However, these schemes require structured grids to be used, and this is an important drawback for its application to complex geometries. Thus, the time needed to generate a block-structured grid for complex geometry problems is very often bigger than the time required for the numerical simulation. In this context, unstructured grids are preferred.

Finite volume methods have been very widely and successfully used in aerodynamics. However, the application of these methods in its most usual formulation (at most order two) to CAA on unstructured grids is not straightforward, due to the excessive dissipation and lack of resolution of the numerical method. Even though rising the order is not the only (nor probably the best) way to improve the resolution of the schemes, it is the most usual approach on unstructured grids, due to the difficulty in generalize the methods developed for structured meshes (Lele (1992)). But this is also not obvious. The main problem relies on the evaluation of high order derivatives.

The FV-MLS method (Cueto-Felgueroso et al. (2007); Nogueira et al. ((2009)) overcomes this difficulty by using Moving Least Squares (MLS) (Lancaster y Salkauskas. (1981)) to compute the gradients and successive derivatives on a finite volume framework, without the introduction of new degrees of freedom. This method has already been applied to test cases of CAA with excellent results (Nogueira et al. (2010b); Khelladi et al. (2010)). In this work we show the behavior of the FV-MLS method to the computation of acoustic wave propagation in complex geometries.

2. THE LINEARIZED EULER EQUATIONS

Most of aeroacoustic problems are linear, so it is possible to linearize Euler equations around a (mean) stationary solution $\boldsymbol{U}_0 = (\rho_0, u_0, v_0, p_0)$. Then, the Linearized Euler Equations written in conservative form are the following:

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{E}}{\partial x} + \frac{\partial \boldsymbol{F}}{\partial y} + \boldsymbol{H} = \boldsymbol{S} \tag{1}$$

being $\boldsymbol{S}$ a source term and

$$\boldsymbol{U} = \begin{pmatrix} \rho' \\ \rho_0 u' \\ \rho_0 v' \\ p' \end{pmatrix} \qquad \boldsymbol{E} = \begin{pmatrix} \rho' u_0 + \rho_0 u' \\ p' + \rho_0 u_0 u' \\ \rho_0 u_0 v' \\ u_0 p' + \gamma p_0 u' \end{pmatrix} \qquad \boldsymbol{F} = \begin{pmatrix} \rho' v_0 + \rho_0 v' \\ \rho_0 v_0 u' \\ p' + \rho_0 v_0 v' \\ v_0 p' + \gamma p_0 v' \end{pmatrix} \tag{2}$$

$$\boldsymbol{H} = \begin{pmatrix} 0 \\ (\rho_0 u' + u_0 \rho') \dfrac{\partial u_0}{\partial x} + (\rho_0 v' + v_0 \rho') \dfrac{\partial u_0}{\partial y} \\ (\rho_0 u' + u_0 \rho') \dfrac{\partial v_0}{\partial x} + (\rho_0 v' + v_0 \rho') \dfrac{\partial v_0}{\partial y} \\ (\gamma - 1)\, p' \boldsymbol{\nabla} \cdot u_0 - (\gamma - 1)\, u' \boldsymbol{\nabla} p_0 \end{pmatrix} \tag{3}$$

where the velocity is $\boldsymbol{\nu} = (u, v)$, ρ is the density, p the pressure and $\gamma = 1{,}4$. Subscript $_0$ is referring to mean values and $'$ indicates perturbation quantities around the mean. In case of an uniform mean flow, $\boldsymbol{H}$ is null.

3. NUMERICAL METHOD

A method based on the application of Moving Least Squares to compute the derivatives on a finite volume framework (FV-MLS) (Cueto-Felgueroso et al. (2007); Cueto-Felgueroso y Colominas (2008); Nogueira et al. (2010a)) has been used to discretize the Linearized Euler Equations (1). In order to increase the order achieved by the method, a Taylor expansion of the variable is performed at the interior of each cell. Next, the approximation of the higher order derivatives needed to compute the Taylor reconstruction are obtained by a Moving Least Squares approach.

Thus, if we consider a function $\Phi(\boldsymbol{x})$ defined in a domain Ω, the basic idea of the MLS approach is to approximate $\Phi(\boldsymbol{x})$, at a given point $\boldsymbol{x}$, through a weighted least-squares fitting of $\Phi(\boldsymbol{x})$ in a neighborhood of $\boldsymbol{x}$ as

$$\Phi(\boldsymbol{x}) \approx \widehat{\Phi}(\boldsymbol{x}) = \sum_{i=1}^{m} p_i(\boldsymbol{x})\, \boldsymbol{\alpha}_i(\boldsymbol{z}) \Big|_{\boldsymbol{z}=\boldsymbol{x}} = \boldsymbol{p}^T(\boldsymbol{x})\, \boldsymbol{\alpha}(\boldsymbol{z}) \Big|_{\boldsymbol{z}=\boldsymbol{x}} \tag{4}$$

$\boldsymbol{p}^T(\boldsymbol{x})$ is an m-dimensional polynomial basis and $\boldsymbol{\alpha}(\boldsymbol{z})\Big|_{\boldsymbol{z}=\boldsymbol{x}}$ is a set of parameters to be determined, such that they minimize the following error functional:

$$J\left(\boldsymbol{\alpha}(\boldsymbol{z})\Big|_{\boldsymbol{z}=\boldsymbol{x}}\right) = \int_{\boldsymbol{y}\in\Omega_{\boldsymbol{x}}} W(\boldsymbol{z}-\boldsymbol{y}, h)\Big|_{\boldsymbol{z}=\boldsymbol{x}} \left[\Phi(\boldsymbol{y}) - \boldsymbol{p}^T(\boldsymbol{y})\boldsymbol{\alpha}(\boldsymbol{z})\Big|_{\boldsymbol{z}=\boldsymbol{x}}\right]^2 d\Omega_{\boldsymbol{x}} \tag{5}$$

being $W(\boldsymbol{z}-\boldsymbol{y}, h)\Big|_{\boldsymbol{z}=\boldsymbol{x}}$ a *kernel* with compact support (denoted by $\Omega_{\boldsymbol{x}}$) centered at $\boldsymbol{z} = \boldsymbol{x}$. The parameter h is the smoothing length, which is a measure of the size of the support $\Omega_{\boldsymbol{x}}$. In this work the following polynomial cubic basis is used:

$$\boldsymbol{p}(\boldsymbol{x}) = \begin{pmatrix} 1 & x & y & xy & x^2 & y^2 & x^2y & xy^2 & x^3 & y^3 \end{pmatrix}^T \tag{6}$$

which provides cubic completeness. In the above expression, (x, y) denotes the cartesian coordinates of $\boldsymbol{x}$. In order to improve the conditioning, the polynomial basis is locally defined and scaled: if the shape functions are going to be evaluated at $\boldsymbol{x}_I$, the polynomial basis is evaluated at $(\boldsymbol{x} - \boldsymbol{x}_I)/h$. Following (Cueto-Felgueroso et al. (2007)), the interpolation structure can be identified as

$$\widehat{\Phi}(\boldsymbol{x}) = \boldsymbol{p}^T(\boldsymbol{x})\boldsymbol{M}^{-1}(\boldsymbol{x})\boldsymbol{P}_{\Omega_{\boldsymbol{x}}}\boldsymbol{W}(\boldsymbol{x})\boldsymbol{\Phi}_{\Omega_{\boldsymbol{x}}} = \boldsymbol{N}^T(\boldsymbol{x})\boldsymbol{\Phi}_{\Omega_{\boldsymbol{x}}} = \sum_{j=1}^{n_{\boldsymbol{x}}} N_j(\boldsymbol{x})\Phi_j \tag{7}$$

where $\boldsymbol{M}$ is the moment matrix, $\boldsymbol{P}_{\Omega_{\boldsymbol{x}}} = \left(\boldsymbol{p}(x)_1 \cdots \boldsymbol{p}(x)_{n_{\boldsymbol{x}}}\right)$, $\boldsymbol{\Phi}_{\Omega_{\boldsymbol{x}}} = \left(\Phi x_1 \cdots \Phi x_{n_{\boldsymbol{x}}}\right)$and $\boldsymbol{W}(\boldsymbol{x}) = diag(W_i(\boldsymbol{x}))$ with $i = 1, ..., n_{x_I}$(Cueto-Felgueroso et al. (2007)). As in the finite element method, the approximation is written in terms of the MLS "shape functions" $\boldsymbol{N}^T(\boldsymbol{x})$. The derivatives of $\boldsymbol{N}^T(\boldsymbol{x})$ can be used to compute an approximation to the derivatives of the function. So, the gradient of $\widehat{\Phi}(\boldsymbol{x})$ is evaluated as

$$\boldsymbol{\nabla}\widehat{\Phi}(\boldsymbol{x}) = \sum_{j=1}^{n_{\boldsymbol{x}_I}} \Phi_j \boldsymbol{\nabla} N_j(\boldsymbol{x}) \tag{8}$$

The equation to be solved is the one resulting by the application of the finite volume discretization to equations (1):

$$A_I \frac{\partial \boldsymbol{U}_I}{\partial t} = \sum_{iedge=1}^{nedge_I} \sum_{iq=1}^{nq_I} \left[-\boldsymbol{\mathcal{F}} \cdot \boldsymbol{n}\right]_{iq} W_{iq} + \boldsymbol{S}_I \tag{9}$$

where $\boldsymbol{\mathcal{F}} = (\boldsymbol{E}_x, \boldsymbol{F}_y)$, A_I is the area of cell I, $nedge_I$ the number of cell edges, $\boldsymbol{U}_I$ and $\boldsymbol{S}_I$ are the average values of $\boldsymbol{U}$ and $\boldsymbol{S}$ respectively, over the cell I (associated to the cell centroid). W are the integration weights and iq is the index for integration points. $\boldsymbol{\mathcal{F}} \cdot \boldsymbol{n}$ is computed with a standard flux vector splitting technique (Toro (1999)).

The MLS approximation has been used to compute the derivatives for the reconstruction of variables at quadrature points at the edges by using a Taylor expansion until the fourth derivative. The resulting scheme is a fifth order method (Cueto-Felgueroso et al. (2007); Nogueira et al. ((2009)).

For time integration, we use the Implicit Differential Algebraic (IDA) solver (Hindmarsh et al. (2005)). As major feature, the time integration method is the variable-order (from 1 to 5) variable-coefficient Backward Differentiation Formula, with adaptive time-step, (Brenan et al. (1996)) for more details. For the solution of the linear system (21), an inexact Newton/Krylov subspace iterative corrections based on a scaled preconditioned GMRES solver is used. See Khelladi et al. (2010) for more details.

The particles needed for the application of the method are identified with the centroids of every cell of the grid. For boundary cells, we add nodes (ghost nodes) placed in the middle of the edge defining the boundary. The definition of the stencil for each cell is done at the beginning of the calculations. An exponential kernel has been used, defined in 1D as:

$$W(x, x^*, s_x) = \frac{e^{-\left(\frac{s}{c}\right)^2} - e^{-\left(\frac{dm}{c}\right)^2}}{1 - e^{-\left(\frac{dm}{c}\right)^2}} \tag{10}$$

with $s = |x - x^*|$, $d_m = max\left(|x_i - x^*|\right)$, $i = 1, ..., n_{x_*}$ $c = \frac{d_m}{2s_x}$, x^* is the reference point (the point around which the stencil moves, in this case the centroid of each cell, I), and s_x is a shape parameter. A 2D kernel is obtained by multiplying two 1D kernels:

$$W_j(\boldsymbol{x}, \boldsymbol{x}^*, s_x, s_y) = W_j(x, x^*, s_x) W_j(y, y^*, s_y) \tag{11}$$

The shape parameter s_x varies from 1 to 6 according to the applications. A complete analysis of the influence of the kernel parameters on the properties of the numerical scheme can be found in Nogueira et al. (2010a). Here, in 1 we show the dispersion and dissipation characteristics of the FV-MLS 3rd order scheme for different values of s_x.

In this work we have used the values of $s_x = s_y = 3$.

3.1. Boundary Conditions

Absorbing boundary conditions have been implemented by using grid stretching and a MLS-based filtering. Grid stretching transfers the energy of the wave into increasingly higher wave-number modes and the filter removes this high-frequency content. With this process the energy of the wave is dissipated. The filtering process is developed by the application of a MLS reconstruction of the variables, i.e:

$$\mathbf{\Phi}(\boldsymbol{x}) = \sum_{j=1}^{n\boldsymbol{x}_I} \Phi(\boldsymbol{x}) N_j(\boldsymbol{x}) \tag{12}$$

where Φ is a variable, $\mathbf{\Phi}$ is the filtered variable and N is the MLS shape function. This reconstruction is performed by using a kernel with different shape parameters that the used to the approximation of the variables. The value of these parameters determines the range of frequencies to be filtered.

4. NUMERICAL EXAMPLES

4.1. Noise generated for a monopolar source placed on a volute-like domain

Here we propose to solve the 2D Linearized Euler Equations (LEE)in complex geometry using the proposed formulation. A previous work has been done by the authors [38,39] concerning the application of FV-MLS to CAA problems on unstructured grids. We have shown that FV-MLS is very well adapted to solve LEE with a very good accuracy. Here we show the potential of this formulation for use with unstructured grids. In this first example, we simulate the propagation of acoustic waves originated for a monopolar acoustic source placed on a domain that mimics the geometry of the volute of a turbomachine. The complexity of the geometry is evident.

The monopolar source is defined as:

$$S_p = \frac{1}{2} e^{\left(-ln(2)\frac{(x-x_s)^2+(y-y_s)^2}{2}\right)} \sin(\omega t) \times [1,0,0,1]^T \tag{13}$$

where the angular frequency is $\omega = 2\pi/30$ and t is the time coordinate. The wave length is $\lambda = 30$ units. In figure 2 we plot the isosurfaces of the acoustic pressure for different times.

4.2. Noise generated for a several bipolar sources placed on a turbomachine-like domain

The nature of the noise generated in centrifugal turbomachines is bipolar. It is originated by the unsteady forces acting over the rotor blades. In the second example, we show the acoustic wave propagation generated in the rotation of 9 artificial bipolar sources, simulating a centrifugal turbomachine with a nine-blade rotor. This setup of the problem mimics the noise generated at the trailing edge of rotor blades in a centrifugal turbomachine.

The acoustic source is defined as:

$$S_p = \frac{1}{2} e^{\left(-ln(2)\frac{(x-x_s)^2+(y-y_s)^2}{2}\right)} \sin(\omega t) \times [0,n_x,n_y,0]^T \tag{14}$$

where the angular frequency is $\omega = 2\pi/30$ and t is the time coordinate. The wave length is $\lambda = 30$ units.

In figure 3 we plot the isosurfaces of the acoustic pressure for different times.

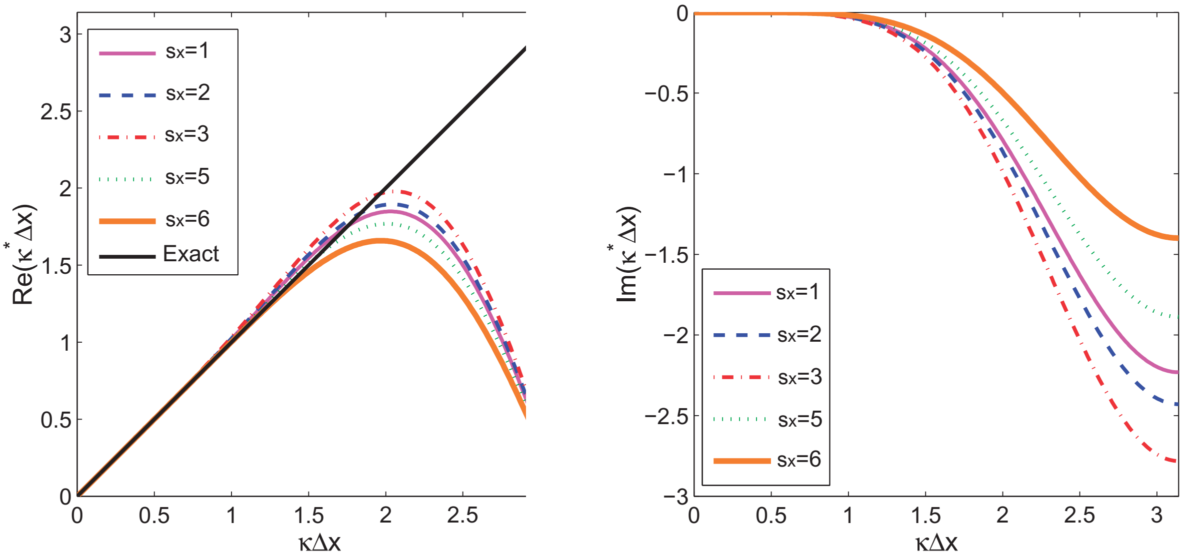


Figura 1: Dispersion and dissipation characteristics of the FV-MLS 3rd order scheme for different values of s_x

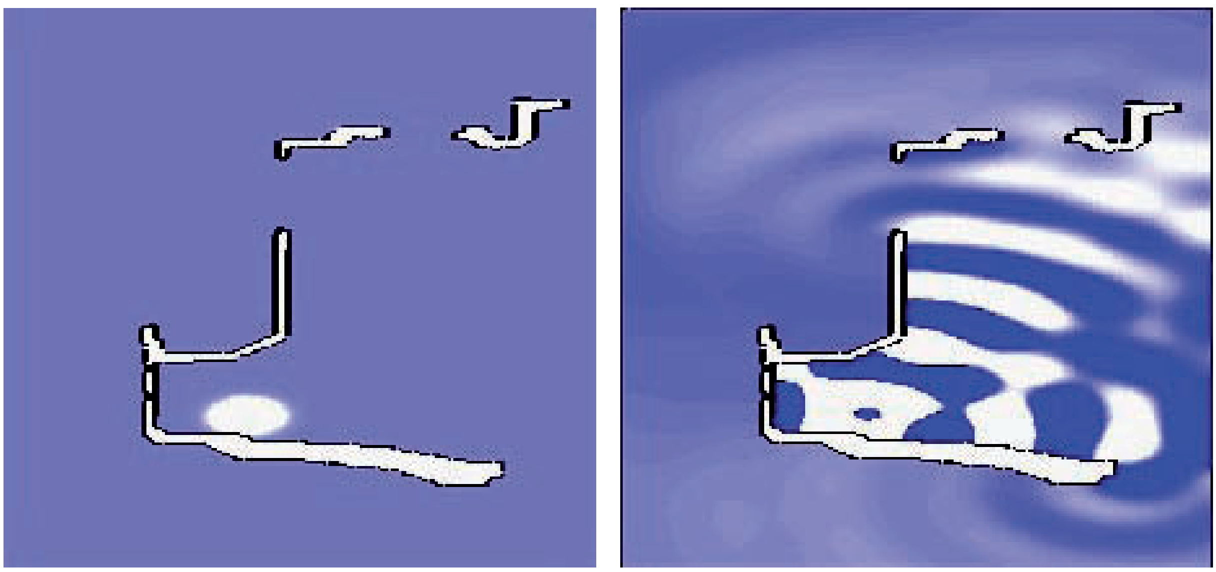


Figura 2: Propagation of acoustic waves on a volute-like domain. On the left we plot the acoustic pressure at $t = 0$ and on the right, we plot the acoustic pressure at$t = 8$

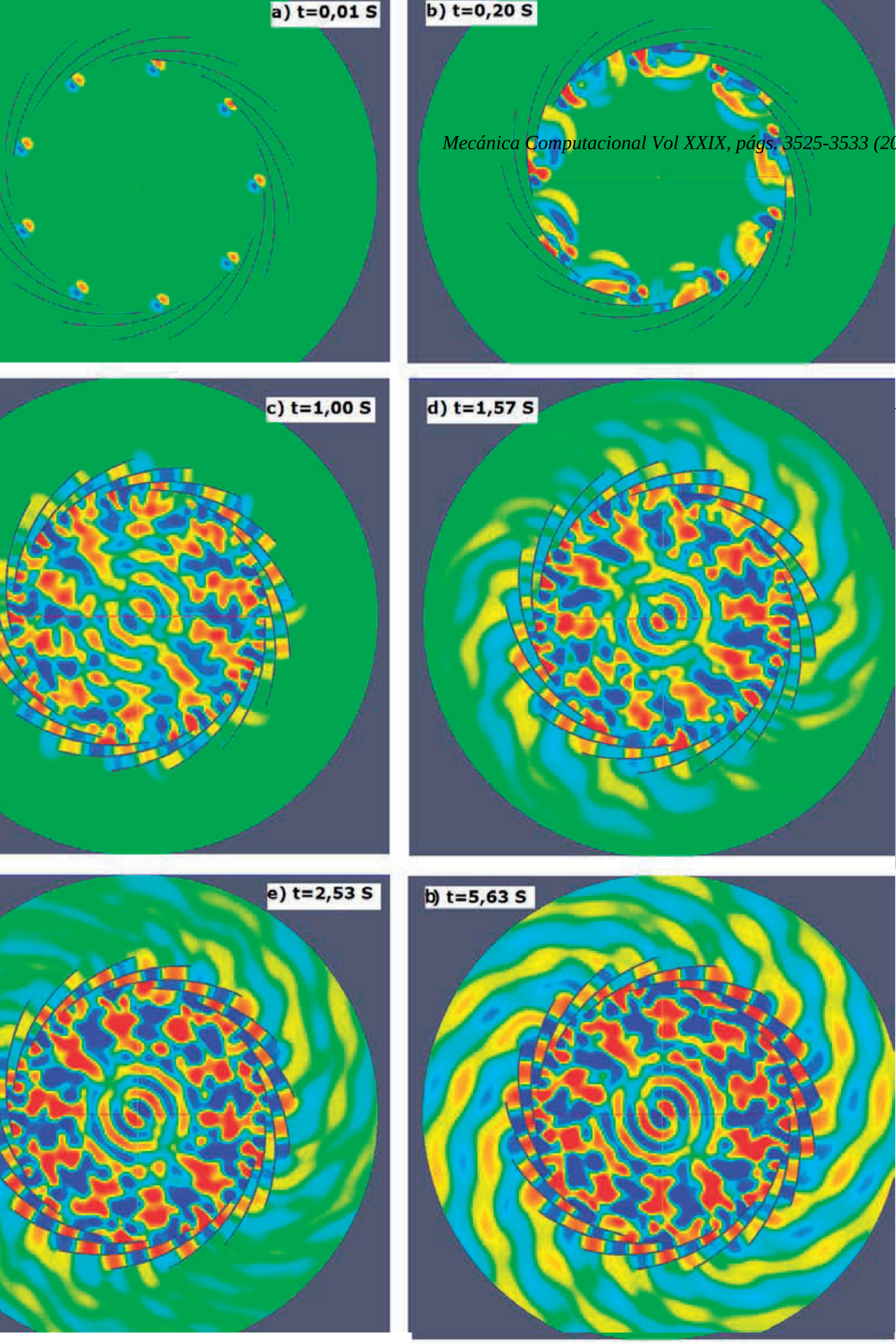
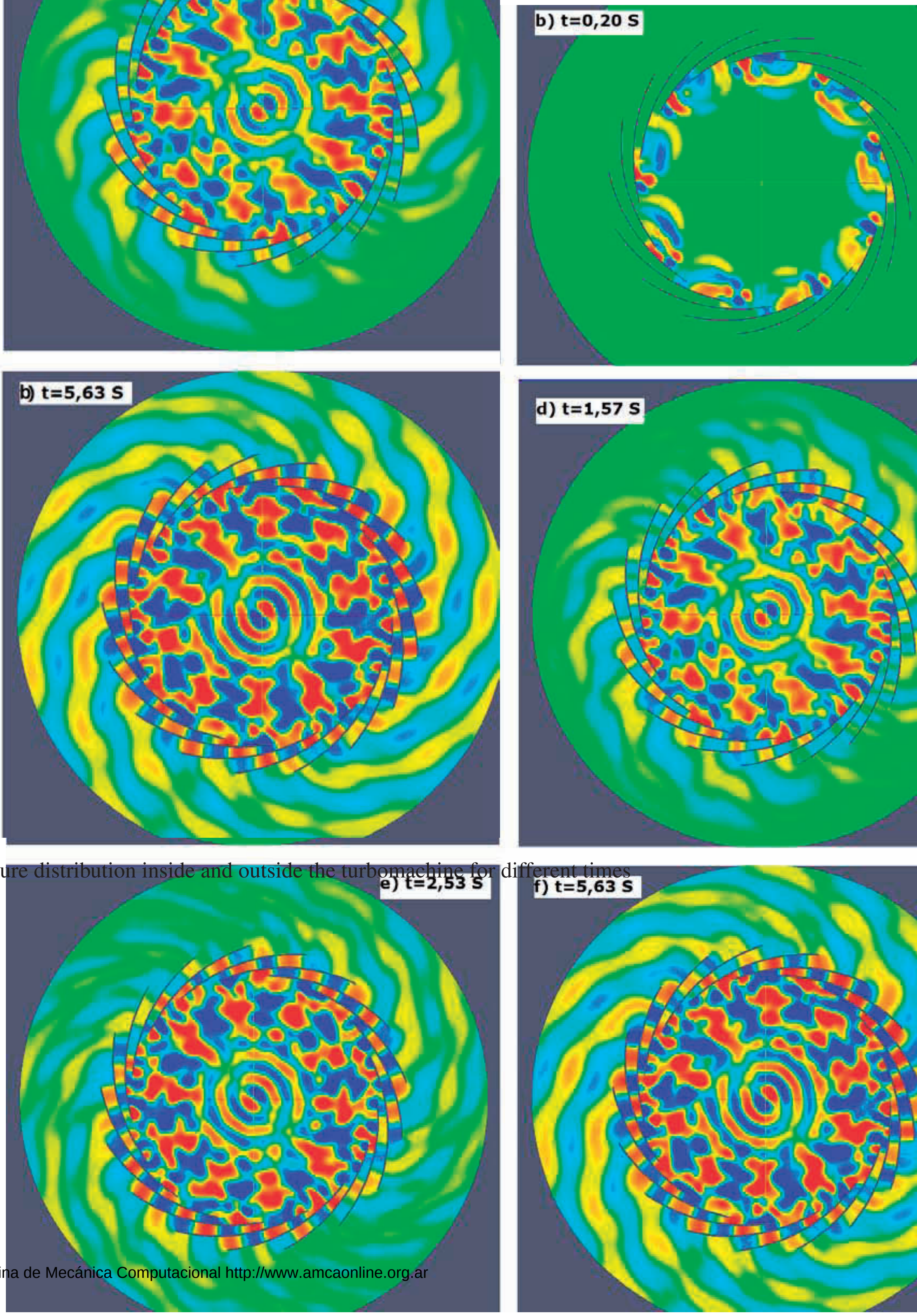


Figura 3: Pressure distribution inside and outside the turbomachine for different times



5. CONCLUSIONS

In this work we show the application of the FV-MLS method to the resolution of CAA problems in complex geometries. We present the results of acoustic pressure propagation in turbomachinery. Preliminary results are really excellent. The results encourage us to go further and use the FV-MLS scheme to compute the noise sources by solving the Navier Stokes equations in a LES simulation. Work is in progress in this direction.

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