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# TRANSIENT ANALYSIS OF SQUARE AND DEEP DRIVEN CAVITY FOR DIFFERENT RE NUMBERS BY LATTICE BOLTZMANN METHOD (LBM)

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**Abstract.** The main objective of the present study was the evolution mechanisms of driven cavity flows for high Re. It was found and proposed the definition of "vorticity channels" which are responsible for the system vorticity input from the walls, either by transport or by diffusion. We studied the vortex dynamic both in deep and square cavities, explaining the mechanism of binding of vortices that occurs repeatedly during the evolution to steady state. For deep cavities a phenomenon that was called "mirror phenomenon" was observed and that it occurs during the formation of the two vortices that appears in steady state. In conjunction with the above, the system circulation was studied for different Re numbers where it was found that for Re 10.000, the system allows vorticity to accumulate three times more than for Re 1.000. We also studied the periodic steady states that arise due to Hopf bifurcation for high Re numbers i.e., grater than 8.000 in the cavity, being the first study to submit a complete cycle of the period that occurs in a deep cavity for Re 8.000.

## 1 INTRODUCTION

Among the studies of lid-driven cavities, there are two important motivations. The numerical motivation since the lid-driven cavity work as a benchmark for new numerical methods, and the second motivation, the one which motivates our study, the fluid dynamics within the cavity as the Re is increased. Most of the studies focus their effort in the steady state of the cavity, but just a few study the flow evolution mechanisms or transients (Gustafson. and J.E., 1991). This is why was considered interesting to understand what is behind the flow evolution mechanisms until the steady state and the steady state by itself, considering that the steady state of the cavity for different Re (1.000 10.000) are similar while the evolution mechanisms are so different. With the aim of studying the flow dynamics within the cavity the lattice boltzmann method was used based on the equations proposed by Sheng (Sheng et al., 2008). The lattice boltzmann method was developed in the late 90ś as a derivation LGA. The main idea that governs the method is to build a simple kinetic model that replicates the macroscopic physics, and when going up to the continuum, it obeys the governing equations, i.e. moment equations (Navier- Stokes).The reason why the method was used lies in its easier implementation and the low computational cost it represents. One of the characteristics of the method used is that their primitive variables are the vorticity-stream function (Sheng C. and M., 2008). Under this approach we intended to understand in a better way what is behind the flow dynamics, because what characterizes the cavity flow is that the movement of the lower wall introduces a vorticity impulse inside the cavity which is transported either by advection or diffusion into the cavity.

## 2 GOVERNING EQUATIONS

The moment equation with the introduction of a turbulence model(Smagorinsky model) are

$$\frac{\partial \omega}{\partial t} + [\nabla \omega]\, v = \frac{\partial}{\partial x}\left(\nu_e \frac{\partial \omega}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_e \frac{\partial \omega}{\partial y}\right) \tag{1}$$

where $\nu_e = \nu_o + \nu_t$ being $\nu_t$ the viscosity based on the stress tensor.

The equations that govern the method were taken from (Sheng C. and M., 2008) and are described below: The flow evolution equation is descibed by:

$$g_k\left(\vec{x} + c\vec{e_k}\Delta t, t + \Delta t\right) - g_k\left(\vec{x}, t\right) = -{\tau_e}^{-1}\left[g_k\left(\vec{x}, t\right) - {g_k}^{eq}\left(\vec{x}, t\right)\right] \tag{2}$$

The equilibrium distribution function is described as

$$g_k\left(eq\right) = \frac{\omega}{5}\left[1 + 2.5\frac{\vec{e_k} \cdot u}{c}\right] \tag{3}$$

vorticity can be recovered

$$\omega = \sum_{k \geq 0} g_k \tag{4}$$

and $\tau$ as calculated as a function of Re

$$Re = \frac{5}{2c^2(\tau - 0.5)} \tag{5}$$

In order to recover the velocity field, the Poisson equation has to be solved. Shen[1] propose the next evolution equation

$$f_k\left(\vec{x} + c\vec{e_k}\Delta t, t + \Delta t\right) - f_k\left(\vec{x}, t\right) = \Omega_k + \Omega_k' \tag{6}$$

where $\Omega_k = -{\tau_\Psi}^{-1}\left[f_k\left(\vec{x}, t\right) - {f_k}^{eq}\left(\vec{x}, t\right)\right]$,$\Omega_k' = \Delta\xi_k D$ and $D = \frac{c^2}{2}\left(0.5 - \tau_\Psi\right)$. $\tau_\Psi$ is the relaxation time that can be arbitrarily chosen.

## 3 ALGORITHM

- **Step 1.** Wall vorticity calculation

$$\omega = \frac{7\Psi_w - 8\Psi_{w-1} + \Psi_{w-2}}{2\Delta n^2} \tag{7}$$

$$\omega = \frac{7\Psi_w - 8\Psi_{w-1} + \Psi_{w-2}}{2\Delta n^2} - \frac{3Uo}{\Delta n} \tag{8}$$

  The equations above arise from solving the poisson equation on the walls by a taylor expansion of order 2.

- **Step 2.** Velocity field calculation based on the streamfunction.
- **Step 3.** Equilibrium probabilities calculation
- **Step 4.** Collision probabilities calculation
- **Step 5.** Probability transport
- **Step 6.** Average vorticity calculation
- **Step 7.** To solve the poisson equation a loop is required wich compares the variation of $f_k$ because we need $\frac{d\Psi}{dt} =^2 \Psi + \omega$ equal to zero.

## 4 STEADY STATES

The flow has reached steady state when collisions and transport do not affect the probabilities of each node. The system energy graph was considered as a measure of the steadiness of the flow. Below is presented the steady state for Re 1.000 and Re 10.000. It is important to note that the Fig. 2 is a "picture" of the periodicity highly discussed in references (Gustafson. and J.E., 1991) and (Olivier., 1996) which is localized on the upper left vortex. As the Re is increased, a particular configuration at the steady state is always achieved, with some variations owing the Re number. For Re 1.000 the configuration consists of one big positive vortex located in the center of the cavity and two negative vortexes on the upper corners. Re 10.000 configuration is similar to Re 1.000 configuration but the appearance of a third negative vortex located in the left lower corner and the periodicity of the upper vortexes. At first sight, comparing both steady states (Figures 1 and 2), we can assume that the evolution paths or transients until the steady state have to be similar because the steady states are. Assumption that turns out to be false in the next section.

## 5 TRANSIENT BIFURCATION

During the simulation was observed that the vorticity and stream-function transients until steady state were completely different as the Re was increased, but as it is known the steady states of the flow inside the cavity are topologically similar (Figures 1 and 2) despite of the Re. Excepting the corners of the cavity where is located the periodicity for Re larger than 8017.6 (Auteri F. and L., 2002). In a way of illustrating this bifurcation, Figures 3 and 4 presents the vorticity transients of Re 1.000 and Re 10.000 until the mentioned steady state configuration is attained

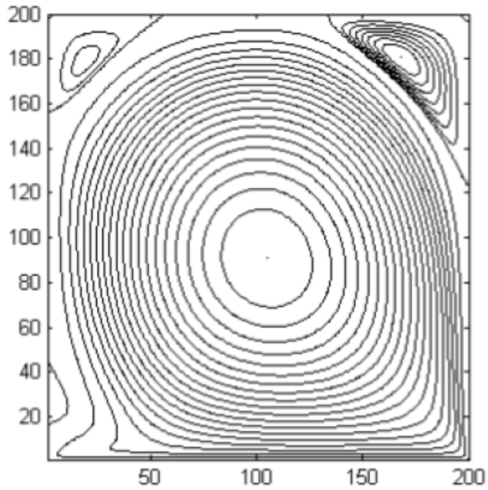


Figure 1: Steady state for Re 1.000

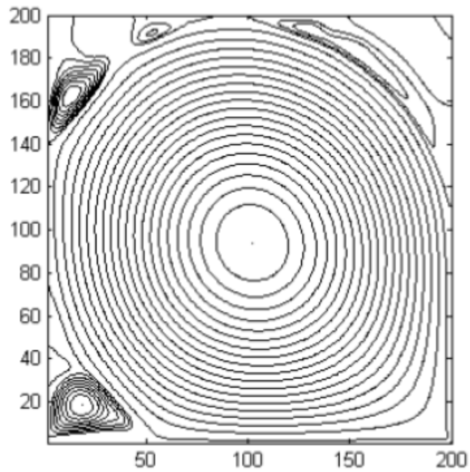


Figure 2: Steady state for Re 10.000

Qualitatively, transients develop in different ways depending the Re:

For Re 1.000 a positive vortex is created in the low right corner by the low wall movement. It is fed with positive vorticity by the vorticity impulse made by the low wall movement until it takes the whole cavity. The red band that covers the blue zone (Fig. 3 ) is a vortex (with negative vorticity) that through the evolution is cornered by the positive vortex offering no resistance owing the high viscosity. For Re 10.000 a positive eddy is created as in Re 1.000 case, but immediately it creates a negative vortex, owing to the low viscosity, coming from the right wall that has the enough strength (vorticity) to interact with the positive vortex, changing in size and form until the steady state. The above observations show that in fact a bifurcation exists. Observing the transients for different Re numbers in the interval [1.000,10.000] it is not clear before or after what Re number the bifurcation occurs, because the transients change "smoothly" as the Re increases. Another interesting fact is the presence of those red tubes (Fig. 4 ) that connects vortexes with the walls of the cavity. They are going to be named "Vorticity Channels" trough the rest of the study, but they are simply separate boundary layers which conserve their structure while transporting into the fluid.

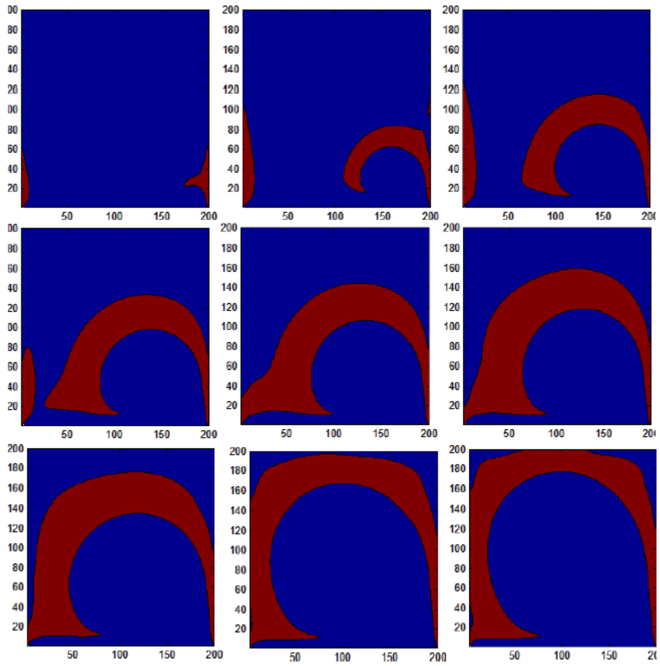

Figure 3: Flow transient for Re 1.000

## 6 FEEDING MECHANISMS

The cavity flow is a phenomenon characterized by an introduction of vorticity to the system at a constant rate. A vorticity impulse is created owing the no-slip condition on the moving wall which can be transported into the cavity by advection or diffusion. As it is known vorticity transport occurs by two forms: an advective $[\nabla\omega]\cdot v$ way and a diffusive way $\nu\Delta\omega$ (see transport equation). At the beginning of evolution, the vorticity impulse is transported purely by diffusion from the wall but as the flow evolves, the terms of the vorticity transport equation begin to have different weights being the diffusive term the most sensitive to Re variations. A channel is defined as an input of vorticity coming from the walls that feeds and creates vortexes inside the cavity. Despite the fact that vorticity channels are boundary layer we intended to rename them due to some nice characteristics they appear to have.

### 6.1 Channel creation and characteristics

Channels are created owing two phenomena: The first is the energy transformation that occurs in the moving wall i.e. translational energy is transformed into rotational energy, and the second is that a vortex, no matter what it sign is, creates channel that transport vorticity of the opposite sign. To make the latter fact clear, suppose a positive vortex is located near a wall. The vortex movement makes the particles between the wall and it begin to rotate due to the viscosity in the counter way creating a vorticity input, in this case negative vorticity input. Three interesting characteristics were observed. The first is that the channels transport vorticity into the cavity created in the walls and diffuse vorticity to the near channels in proportion to the vorticity gradient. The second one is that every positive channel surrounds or wraps a negative vortex and visceverza and finally that the thickness of the channels is a function of the Re number.

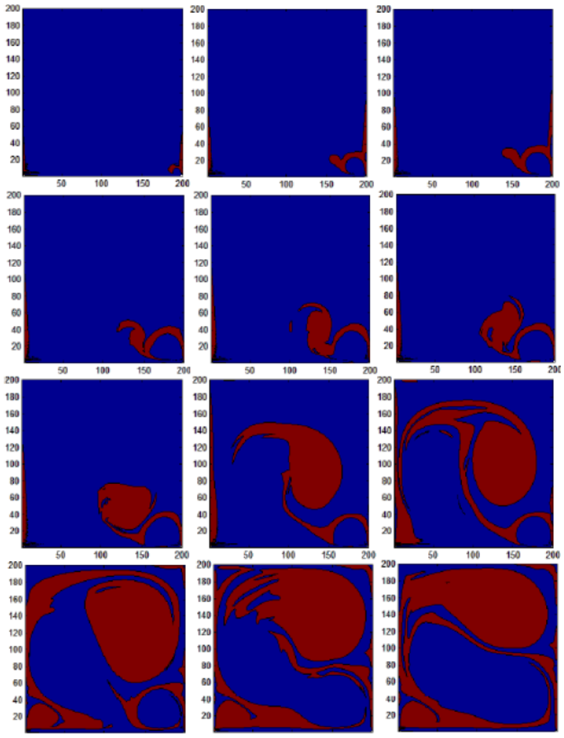

Figure 4: Flow transient for Re 10.000

### 6.1.1 Channels study for Re 1.000

In the transient presented on Fig. 3 can be seen from the beginning that a feeding channel appears from the right wall, that grows through the evolution until pasting with a channel originated from the left wall. It can be noted that the channels are thick due to the low Re number because the diffusive term has an important weight in the equation letting the vorticity spread into the fluid.

### 6.1.2 Channels study for Re 10.000

In Fig. 4 the channels are the red tubes and the red patches are configured vortexes. The interesting thing in the transient is that for Re 10.000 the negative vortex has enough strength to interact with the positive vortex until the negative vortex took the upper half of the cavity. This happens due to the low weight of the diffusive term in the equation allowing vorticity to accumulate without spreading into the cavity, also can be seen that the channels are thinner than Re 1.000 channels.

## 7 VORTEX BINDING

In order to explain the vortexes junction that happens through the evolution, which is illustrated in the Fig. 5, the transport equation in needed. In the transport equation, there are two terms that dictate the way vorticity is transported, the diffusive and advection term. For higher Re we can keep only the advection term (see transport equation). As the flow evolves can be seen, that the constant vorticity lines tends to be parallel to the streamfunction making the vorticity gradient and the speed vector orthogonal , in other words no transport due to $[\nabla \omega] \cdot v = 0$.

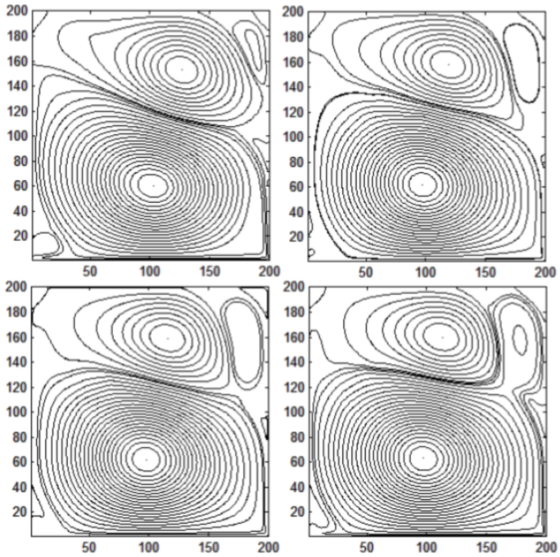


Figure 5: Vortex binding

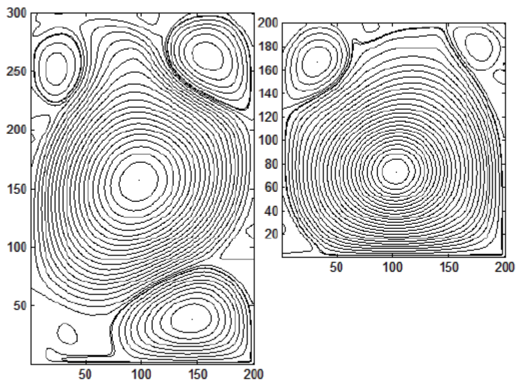


Figure 6: Mirror phenomena: Deep and square cavity comparison

## 8 DEEP CAVITIES STUDY

Due to the low computational cost LBM requires, was intended to study a deep cavity with aspect ratio 1.5 and Re 8.000. Observing the transients can be seen that similar vortex dynamics, with the square cavity, appear to take place but one interesting came to light. In the streamfunction transients was observed (Fig. 7 ) that after the positive vortex juncture take place (third frame Fig. 7) making one positive vortex on the lower half of the deep cavity, this vortex plays the role of a moving wall for the upper negative vortex injecting and producing the same dynamics that occurs on a square cavity near the steady state . This phenomenon can be seen in the Fig. 6 which illustrates that the upper half of the deep cavity is a reflection, based on a vertical axe, of the square cavity. The periodicity for Re 8.000 was also found for the deep cavity and is presented on Fig. 8.

## 9 SYSTEM CIRCULATION ANALYSIS

The motivation lying beneath the circulation study is to understand in a quantitative way what is happening in the fluid and how the accumulation of vorticity occurs for different Re. In Table 8 some interesting things can be observed. First the increase on circulation for Re 10.000

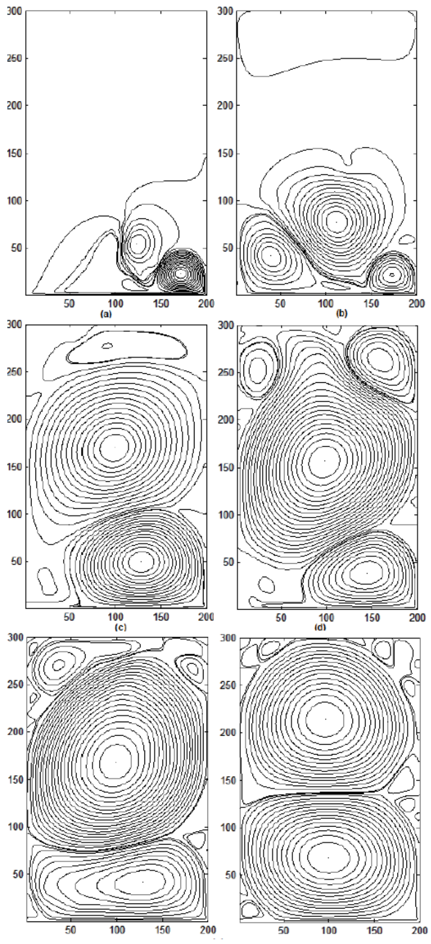

Figure 7: Vortex dynamics for Re 8.000

and Re 1.000 which is three time bigger, due to the fact as the viscosity decreases the system is able to accumulate more vorticity and secondly that Kelvinś theorem holds for the system. That is, as the negative circulation increases so does the positive vorticity, keeping the same difference through the evolution.

## 10 CONCLUSIONS

In the study was implemented a lattice boltzmann method algorithm for square and deep driven cavities with a good performance and interesting results. We renamed the boundary layers that appear between the vortexes and the walls as feeding channels acording to the role they play in the flow evolution and studied them for different Re. Deep driven cavities were also studied where we found a mirror phenomenon and present a cycle of the periodicity for

Table 1: System circulation for Re 1.000 and Re 10.000

| | Re 1.000 | Re 10.000 |
|---|---|---|
| | Max | Min |
| $\Gamma positivo$ | 48.52 | 83.5 |
| $\Gamma negativo$ | 23.8 | 60.67 |

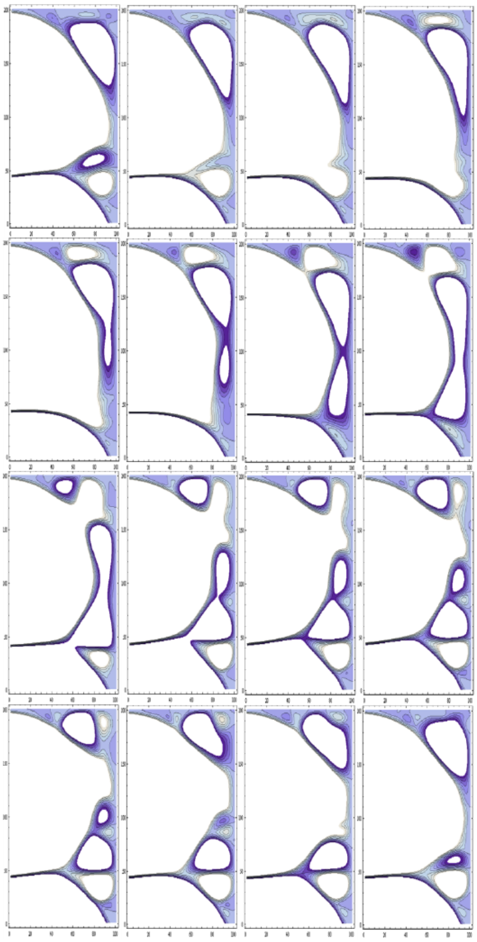

Figure 8: Periodicity for Re 8.000 on the upper right vortex

Re 8.000 Fig. 8. We believe that the study takes up a line of research that has been relegated and has not been given much importance, as are the evolutionary processes to steady state in both square and deep cavities, being these processes so interesting and full of questions, some of which were left open for future research.

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