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POLLUTANTS DISPERSION PROBABILISTIC ANALYSIS USING A FIRST ORDER RELIABILITY METHOD

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Abstract.

In this paper a First Order Reliability Method (FORM) is applied to the problem of pollutants dispersion on a straight beach. Even if this problem is governed by a system of nonlinear partial differential equations, an analytical solution was found for a simplified but representative case. This solution allows one to evaluate the concentration of the pollutants in a point distant from its source. Coupling this equation with a FORM it is then possible to evaluate the probability of the pollutant concentration being higher than a prescribed limit, assuming that the variables of the problem have Gaussian distributions. For validation, results given by FORM are compared to results of Monte Carlo simulation.

1 INTRODUCTION

In the coast of Paraná, environmental problems arising from inadequate human occupation have been observer for a long time. In the last years the pollution of coastal waters of Paraná by pathogens reached a critical level. An example of this situation were results of the balneability analysis of the coastal waters of Paraná in the season of 2008/2009 in which only nine of forty-three points were considered suitable for bathing.

In the state of Paraná the policy of balneability is based on the resolution 274/2000 of CONAMA (Conselho Nacional do Meio Ambiente) and works as follows: water samples are collected in the critical zones, the samples are analyzed for the presence of pathogens. If the presence of pathogens is below the maximum permissible concentration in eighty percent of the samples, then the area are suitable for swimming. Otherwise the area is interdicted in a neighborhood of 200 meters centered in the sample point.

The estimation of the size of banned area around a point of contamination is the initial motivation of the paper Solheid et al. (2010). That paper deals with analytical solutions which approximate the results of a more complex case, and it is shown that simple approximations are accurate enough for most practical applications.

Here we address the problem from a probabilistic point of view. In this case, the variables that rule the problem are assumed to be Gaussian random variables. We then use reliability analysis in order to evaluate the probability of the pollutant concentration being higher than prescribed limits in a given point along the shore.

2 ANALYTICAL SOLUTION

The probabilistic analysis is made for the problem of a load of pollutant entering the surf zone. Fig.1 outlines the situation studied in this paper, it represents a panoramic view of the beach. In this problem the waves always arrive from the same direction. This wave train is characterized by its amplitude, angle to the coast, velocity of propagation and frequency. The pollution load is generated by the arrival of a contaminated effluent (i.e. a river).

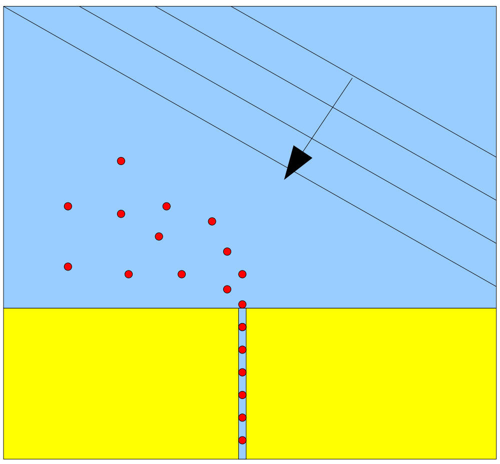

Figure 1: A straight beach subjected to a pollutant input.

The problem described above has a simple mathematical formulation in terms of the diffusion-advection equation, but the non-homogeneous parameters in the differential equation lead to difficulties in obtaining the analytical solution. To find some analytical solution it is necessary to make some simplifying assumptions. First, we assume that the only component of velocity is that parallel to the coast, developed by the formation of "longshore currents" by wave breaking (Higgins and Stewart, 1964). Besides, we assume that this velocity profile is uniform. We then assume that the load of pollutant is modeled as a line normal to the coast of constant pollutant concentration. With these assumptions we have the following mathematical formulation

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial y} = D\nabla^2 C - KC \qquad (y,x) \in (0,\infty), 0 < t < \infty); \tag{1}$$

$$\lim_{x\to\infty} C(x,y,t) = 0; \tag{2}$$

$$\lim_{y\to\infty} C(x,y,t) = 0; \tag{3}$$

$$C(x,0,t) = C_0; \tag{4}$$

$$C(x,y,0) = 0; \tag{5}$$

$$\frac{\partial C(0,y,t)}{\partial x} = 0; \tag{6}$$

where, C is the pollutant concentration; V is the velocity parallel to coast;D is the turbulent diffusion coefficient and K is the decay rate of the pollutant. Fig. 2 outlines the problem given by (1)-(6).

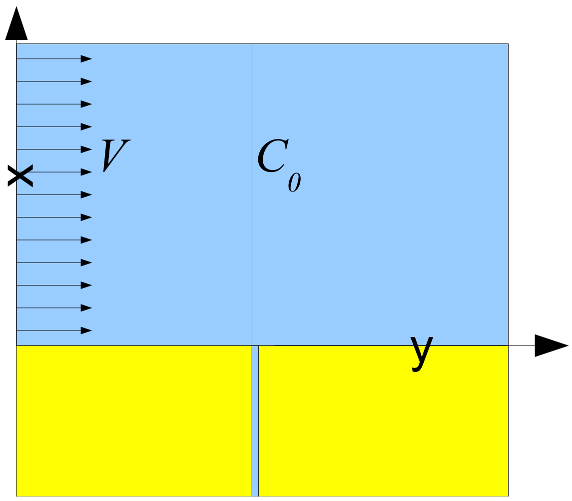

Figure 2: The physical model.

The problem (1)-(6) has the following solution

$$C(x,y,t) = yC_0 \exp\left[\frac{Vy}{2D}\right] \int_0^t \frac{\exp\left[-\left(\frac{V^2}{4D}\right)(t-\tau) + \frac{-y^2}{4D(t-\tau)} - K(t-\tau)\right]}{2\sqrt{D\pi(t-\tau)^3}} d\tau \tag{7}$$

The expression (7) reaches the permanent behavior when the time tends to infinity. The permanent expression is given by

$$C(y) = C_0 \exp\left[\frac{Vy}{D} - \sqrt{\frac{V^2}{D^2} + \frac{4K}{D}y}\right]. \tag{8}$$

The equation (8) was calibrated to fit the results of a complex case solved numerically. This problem is described in details by Solheid et al. (2010). With the calibrated solution one can use the FORM to carry the reliability analysis.

3 RELIABILITY PROBLEM

Here we assume that the pollutant concentration must be smaller than a given limit in order to guarantee and adequate use of the beach. That is, "failure" is assumed to occur when the pollutant concentration C in the measurement point y is bigger than a prescribed limit C_{max}. This constraint can be written as

$$C(\mathbf{x}, y) < C_{max}, \tag{9}$$

where the vector $\mathbf{x}$ is composed of the variables V, D, K and C_0. That is, all the variables that affect the pollutant distribution are grouped in a single vector. This vector is called design vector in the context of reliability analysis (Haldar and Mahadevan, 2000).

The condition from (9) can be rewritten as a failure function (Haldar and Mahadevan, 2000) as

$$g(\mathbf{x}, y) = C(\mathbf{x}, y) - C_{max}, \tag{10}$$

where $g(\mathbf{x}, y)$ is the failure function that depends on the point being studied y and the vector $\mathbf{x}$. Note that here the inadmissible event occurs when $g \geq 0$.

Assuming that the variables V, D, K and C_0 are actually Gaussian random variables, one can associate a mean value μ and a standard deviation σ to each of them. In this context, the vector $\mathbf{x}$ can be transformed to a normalized vector $\mathbf{X}$ by applying the following transformation rule (Haldar and Mahadevan, 2000; Melchers, 1999)

$$X_i = \frac{x_i - \mu_i}{\sigma_i}, \tag{11}$$

where μ_i and σ_i are the mean value and the standard deviation of the design variable x_i. Note that from now on we use capital letters for the variables on the normalized space and small letters form realization of such variables. That is, $\mathbf{x}$ is a realization of the normalized variable $\mathbf{X}$.

The failure function can now be written in terms of $\mathbf{X}$ and y, thus giving

$$G(\mathbf{X}, y) = C(\mathbf{X}, y) - C_{max}, \tag{12}$$

that is the failure function evaluated in the normalized space. The difference between g and G is that the first is evaluate in the real variables space while the latter is evaluated in the normalized space.

Note, from (11), that we have $\mathbf{X} = \mathbf{0}$ when $\mathbf{x}$ is composed of the mean values of the design variables. Besides, the vector $\mathbf{X}$ is measured in "standard deviation" units (Haldar and Mahadevan, 2000; Melchers, 1999).

In the First Order Reliability Method (FORM), the reliability of the system is evaluated as the smallest distance from $\mathbf{X} = \mathbf{0}$ to the surface given by $G(\mathbf{X}) = 0$. That is, we search for the point $\mathbf{X}^*$ that lies over the surface $G(\mathbf{X}) = 0$ that is closest to the origin $\mathbf{X} = 0$. The distance from the origin to this point is defined as the reliability index β (Haldar and Mahadevan, 2000; Melchers, 1999). The point $\mathbf{X}^*$ itself is called most probable failure point.

The geometrical interpretation for this definition is pictured in Fig. (3), considering two design variables x_1 and x_2. Note that in the real space, the mean value is located at an arbitrary $\mathbf{x}$. However, when we apply the transformation given by (11) the mean value is given by the origin of the normalized space. The failure function $g(\mathbf{x})$ is also transformed into $G(\mathbf{X})$. We locate the point $\mathbf{X}^*$ that is the closest to the origin that lies over $G = 0$, and this distance is taken as the reliability index β.

That is, we assume that the system is more reliable the more one needs to move (starting from the origin) in order to find a design vector $\mathbf{X}$ that leads to failure. A more rigorous definition of the FORM is presented by Haldar and Mahadevan (2000), Madsen et al. (1986) and Melchers (1999).

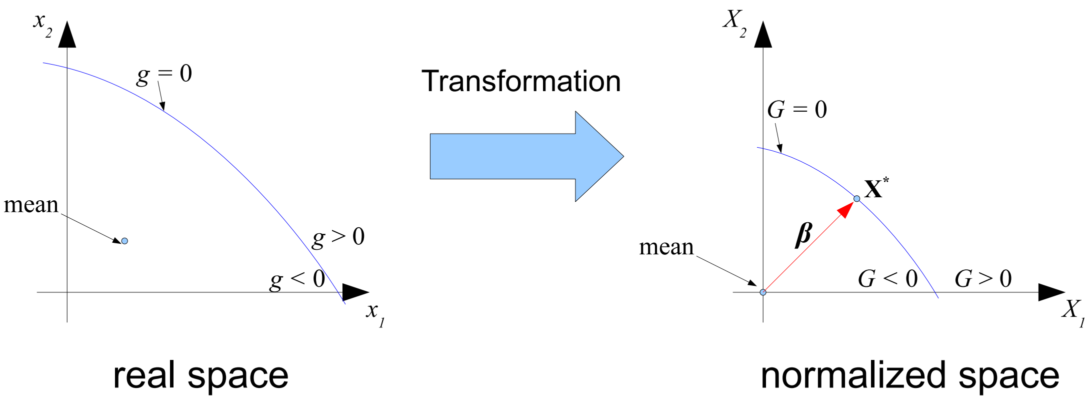

Figure 3: The failure function on the real space and the normalized space.

Based on these assumptions, the reliability index is given by the following minimization problem

$$\texttt{Find:} \quad \mathbf{X} \tag{13}$$

$$\texttt{that minimize:} \quad \beta = \sqrt{\mathbf{X}.\mathbf{X}} \tag{14}$$

$$\texttt{subject to:} \quad G(\mathbf{X}) = \mathbf{0}. \tag{15}$$

This problem can be solved by optimization techniques, but here it is solved by an specific purpose algorithm presented by Haldar and Mahadevan (2000).

After we find the reliability index β, the failure probability can be evaluated as (Haldar and Mahadevan, 2000; Melchers, 1999)

$$P_f = \Phi(-\beta), \tag{16}$$

where Φ is the standard Gaussian cumulated function.

4 RESULTS

The mean values are assumed here $C_0 = 785666$, $U = 0.0174$, $D = 0.2552$ and $K = 0.00014$. The standard deviations are assumed to be 10% of the mean values, since the authors do not have information about the variation of such quantities. The concentration is monitored at $y = 750m$ and the maximum concentration allowed is $C_{max} = 1000$. This problem is solved using both the FORM approach and Monte Carlo Simulation for 100,000 samples.

The MPP obtained using FORM is presented in Tab. 1. The reliability index given by FORM is 3.586, while The reliability index given by Monte Carlo Simulation is equal to 3.808. However, the FORM took about $0.088s$ to run, while the Monte Carlo Simulation took about $13.114s$.

The difference between the reliability indexes given by FORM and Monte Carlo Simulation is about 5%, and it can be said that the results agree for most practical purposes. However, the FORM took much less time to obtain the results.

Space	C_0	V	D	K
Real	811334.7	0.0204	0.2581	9.63E-5
Normalized	0.327	1.1731	0.113	-3.121

Table 1: Most probable failure point.

5 CONCLUSION

The approach proposed here allows one to evaluate the probability of the pollutant concentration being bigger than allowable limits. It's also possible to evaluate the most probable failure point, that puts in evidence the likely configuration of the system that leads to pollutant concentrations higher than the allowable limits.

One important aspect is that Monte Carlo Simulation takes more time to obtain the same results. This is a drawback when evaluation of the failure function is computationally demanding. Even if this is not the case here, this can happen when the analytical solution depends on the evaluation of some special function (Solheid et al., 2010) or when the reliability analysis is applied directly to a numerical model.

The importance of the proposed approach is that the variables involved in the pollutant dispersion problem are expected to present significant variations. Deterministic analyses are not able to take into account these aspects, and thus some kind of probabilistic analysis can lead to a more rational analysis. As discussed previously, balneability is defined by studying a few samples, but does not consider probabilistic aspects. This kind of analysis is not able to take into account the pollutant concentration variations that may arise from wave energy variation, for example. That is, the approach is not rigorous when applied to different coastal regions that may be subject to different wave distributions over the year. This puts in evidence the necessity of studying the problem addressed here from the probabilistic point of view.

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