

## AN OPTIMIZED REINFORCED CONCRETE PAVEMENT: ANALITIC AND NUMERICAL PERFORMANCE STUDY

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**Abstract.** This work reports the design of a particular pavement system composed by pre-casted reinforced concrete plates. This system, made by pre-stressed hollow-core plates, allowed the design of a durable, mobile and high-performance pavement system under heavy industrial loads. The structural and by geotechnical behavior of the plates, was performed both for numerical FEM tools and analytic solution (thin plate theory). The stress and displacement obtained by analyses allowed the design for the plates. By this way, the construction of a precast system for 652ton off-road trucks operation, located inside a very-dense industrial plant became possible in a very short time.

## 1 INTRODUCTION

Among the various technological developments concern the construction of concrete railways, the adoption of precast concrete plates has made great progress over time. The use of reinforced concrete precast panels have as main advantages the quality and speed in implementing of the pavement, and allow reduction in the amount of material, equipment and labor deployed in site (Bruggeling and Huyghe 1991; Wilde et al. 1999).

The performance of reinforced concrete pavement can be increased by adopting the concept of the pretension (Lin and Burn, 1981). The introduction of compression prior to live load may be obtained by a rational way if it will be allied to the concept of precast. The pretension leads to panels with larger extensions and hence lower density of joints, prevention of premature cracks in the concrete and sections more economical. (Corley et al. 1980). Hargett (1968) reported the construction of a constructive system composed of precast concrete deck supported by a granular sub-base layer covered by sand. On the plates of concrete is applied layer of asphalt concrete. Also according Hargett (1968) that system proved to be inefficient, since constructive difficulties caused by connection between the plates and premature cracks occurred in the asphalt layer. In Nishizawa (1994) is described a system consisting of paving slabs in precasted concrete of variable dimensions in plan, supported by mechanically stabilized sub-base, which showed satisfactory results. Kumakura et al. (1994) proposed a system for load transfer between concrete slabs, composed of prestressed steel bars every 0.5 meters. Through this mechanism it was possible to obtain full transfer of loads between panels and precise union between them. Chisato et al. (1997) reported research involving the study of the influence of the elements of transfer of load on pre-manufactured panels with varying dimensions. From extensive research on the performance of different systems pre-existing manufactured by the 80's, Cable et al. (1995) developed new and important concepts involving the design and construction of prestressed concrete pavements. In Merritt et al. (2000) presents a new constructive system composed of prestressed plates in situ. This system was initially tested in the laboratory and then in real conditions of use, giving excellent results (Merritt et al., 2003). Fort Miller (2007) presents a system composed of prestressed plates aimed to apply in situations of heavy traffic (Lepree, 2002) and when speed in the pavement execution was necessary. Chang et al. (2004) present an extensive comparative study between the main techniques of precast pavement systems in reinforced concrete, analyzing the technical and economic features of these systems, both in the execution of repairs and in the development of new paved roads.

Although widely used like slabs of floors buildings and bridges (Huyghe, 1991; Lin and Burns, 1981), the authors don't found reports or technical research about hollow-core panels applied at structural pavement. This work aims to investigate the structural and geotechnical behavior for a hollow-core plate system made by structural concrete. The pavement will receive a very high load due the traffic of off-road trucks with total weight about 652ton. The behavior of pavement will be investigated by considering a single isolated plate, since the designed system don't presents transfer load between plates. It will be obtained the analytical solution for hollow-core plate, based on bending problem for retangular plate on elastic foundation, considering Kirchhoff simplifications. Numerical modeling will be conduced by a parallel Finite Element Method (FEM) considering the plate and soil elastic materials without discontinuous. The validation of FEM code will be conduced by comparing the numerical and analytical solutions. Finally, according the results obtained by reinforced concrete design, it will be showed that the pavement system, composed by hollow-core pre-casted plates offers excellent structural performance due heavy moving loads and allow implementation of large

expanses of pavement in a quickly way with competitive cost.

## 2 ANALITICAL SOLUTION

For the determination of deformation and stress in any kind of structure, it required the use of physical and mathematical models representing the behavior of materials and the structure itself. Due to the complexity of the rheology of the materials and geometry of the structure, are adopted simplifications that allow the solution of the problem. To determine the efforts that direct or indirect loads are acting over concrete slabs supported by the ground, several theories were developed. The methods of scaling of concrete pavements are based on the equations developed by Westergaard (1939), which published articles on determination of displacements, stresses and bending moments in thin elastic plates, supported on dense liquid. Other authors also proposed equations for the purpose of establishing values for the efforts in concrete slabs, especially the graphs of Pickett and Ray (1951).

Considering a thin rectangular plate resting on a elastic foundation on which acts a concentrated load with varying position, according figure 1.

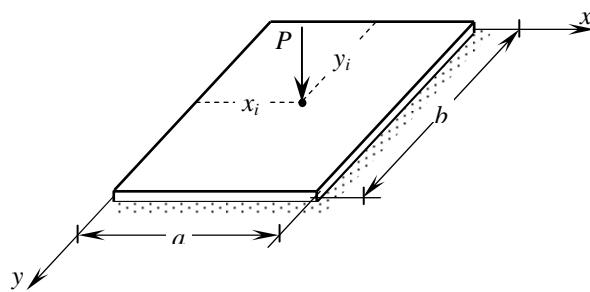


Figure 1: Plate under elastic foundation

For a isotropic thin plate, the differential equation for vertical displacements, is given by

$$\nabla^4 w + kw/D - q/D = 0 \quad (1)$$

where  $\nabla^4$  is the biharmonic operator,  $k$  the Winkler modulus,  $q$  the magnitude of distributed force and  $D$  flexural stiffness given by  $D = Eh^3/12(1-\mu^2)$ , witch  $E$ ,  $h$  e  $\mu$  are elastic modulus, plate thickness and Poisson ratio, respectively.

For the plate showed above, whose edge are free, the corresponding bending moments and internal shear forces should be zero. For example, in the case of isotropic thin plate, the boundary conditions are given by.

Along edges  $x=0$  and  $x=a$ , we should take for both sides:

$$w_{xx} + \mu w_{yy} = 0 \quad (2a)$$

$$w_{xxx} + (2 - \mu) w_{xxy} = 0 \quad (2b)$$

Already on edges  $y=0$  e  $y=b$ , we should take for both sides:

$$w_{yy} + \mu w_{xx} = 0 \quad (2c)$$

$$w_{yyy} + (2 - \mu) w_{xxy} = 0 \quad (2d)$$

where the indexes represents partial derivations.

As a solution for this kind of problem (Eq. 1 and 2), it is possible to adopt the following function

$$w = f_1 \cos \frac{2\pi x}{a} + f_2 \cos \frac{2\pi y}{b} + f_3 \cos \frac{2\pi x}{a} \cos \frac{2\pi y}{b} + f_4 \quad (3)$$

where  $a$  and  $b$  are the lengths of the plate in  $x$  and  $y$  directions, respectively. The parameters,  $f_1$ ,  $f_2$ ,  $f_3$  and  $f_4$  are the unknowns of problem.

Thus, in order to satisfy all the boundary conditions of this problem, it is possible to obtain two relations between the unknowns:

$$f_1 = \beta_1 f_3 \quad (4a)$$

$$f_2 = \beta_2 f_3 \quad (4b)$$

with

$$\beta_1 = -\frac{1}{\mu} (\mu + \lambda_1^2) \quad (4c)$$

$$\beta_2 = -\frac{1}{\mu} (\mu + \lambda_2^2) \quad (4d)$$

where

$$\lambda_1 = \frac{a}{b} \quad (4e)$$

$$\lambda_2 = \frac{b}{a} \quad (4f)$$

and by this way, expression (3) can be written as:

$$w = f_3 \left( \beta_1 \cos \frac{2\pi x}{a} + \beta_2 \cos \frac{2\pi y}{b} + \cos \frac{2\pi x}{a} \cos \frac{2\pi y}{b} \right) + f_4 \quad (5)$$

To obtain the unknowns of the problem, it is necessary to apply the energy of system theory. The potential energy for deformation is defined by:

$$U = \frac{D}{2} \iint [w_{xx}^2 + w_{yy}^2 + 2\mu w_{xx} w_{yy} + 2(1-\mu) w_{xy}^2] dx dy + \frac{k}{2} \iint w^2 dx dy \quad (6)$$

and external forces energy given by:

$$V = \iint q dx dy + P w \Big|_{(x,y)_i} \quad (7)$$

Based on the Variational Principle, when the condition of stable equilibrium is required, we have that the total potential energy ( $\Pi = U - V$ ) must have a minimum value, that is,  $\delta\Pi = 0$ . Therefore, considering the function (5) inside expressions (6) and (7), we arrive at two equations, function only of unknowns  $f_3$  and  $f_4$ , as follows:

$$Af_3 - \frac{\partial V}{\partial f_3} = 0 \quad (8)$$

$$Bf_4 - \frac{\partial V}{\partial f_4} = 0 \quad (9)$$

with

$$A = 8\pi^4 ab \left[ \frac{\beta_1^2}{a^4} + \frac{\beta_2^2}{b^4} + \frac{1}{2} \left( \frac{1}{a^2} + \frac{1}{b^2} \right)^2 \right] + \frac{kab}{2D} \left( \beta_1^2 + \beta_2^2 + \frac{1}{2} \right) \quad (10a)$$

and

$$B = kab \quad (10b)$$

The partial derivates presents at equation (8) e (9) will depend on the point of application of concentrated load, if only this is considered.

For critical points  $(0,0)$ ,  $(a,0)$ ,  $(0,b)$  e  $(a,b)$  the values of partial derivates that appear in both expressions above are equal, and given as:

$$\frac{\partial V}{\partial f_3} = P(\beta_1 + \beta_2 + 1) \quad (11a)$$

$$\frac{\partial V}{\partial f_4} = P \quad (11b)$$

At  $(a/2,0)$  point, the values are:

$$\frac{\partial V}{\partial f_3} = P(-\beta_1 + \beta_2 - 1) \quad (12a)$$

$$\frac{\partial V}{\partial f_4} = P \quad (12b)$$

while for  $(0,b/2)$  point, these derivates given by:

$$\frac{\partial V}{\partial f_3} = P(\beta_1 - \beta_2 - 1) \quad (13a)$$

$$\frac{\partial V}{\partial f_4} = P \quad (13b)$$

At center, that is,  $(a/2,b/2)$  point, we have:

$$\frac{\partial V}{\partial f_3} = P(-\beta_1 - \beta_2 + 1) \quad (14a)$$

$$\frac{\partial V}{\partial f_4} = P \quad (14b)$$

So, with the aid of expressions (11-14) together with equations (8) and (9), it is calculated the unknown  $f_3$  and  $f_4$ . These values are then replaced in expression (5), were it is obtained

the deflection of the plate.

### 3 NUMERICAL MODELLING BY FEM

The finite element method (MEF) is one of the most widely considered computational tools used in solving physical problems. Initially developed aimed at analyzing problems of structural engineering Aeronautics is currently widely used in all areas of Science. The basic idea of numerical analysis by MEF is the subdivision of the field of physical problem with subdomains where the behaviours of the problem is easily represented. It is not intended to calculate the displacement of all points in the subdomains of the structure, such as analytical solutions. In a first instance are calculated only the field variable of some points which are named the nodes of model. Thus, the analyses in these individual subdomains are made and then they are interconnected, simulating the physical problem as a whole. This way it's performed an approximation of the physical response of the real problem, which is more or less close to the exact solution, depending, among other factors, the number of subdomains used in the representation of the physical problem.

### 4 APPLICATION

#### 4.1 Validation example

The first example has for objective to compare the analytical procedure described in this work and the numerical solution obtained by the parallel MEF code developed by authors. It was analyzed several plates with variable aspect ratio  $a/b$  (see figure 1), by fixing the first dimension ( $a$ ) and them varying de second length ( $b$ ), reaching aspect ratios of 1,5 to 3,5. The other parameters of problems were kept constants. The analytical and numerical displacements were plotted for the center, edge and corner of plates. Figure 2 presents the response for the normalized displacement  $w/w_0$  ( $w_0$  correspond to the displacement for aspect ratio equal unit)

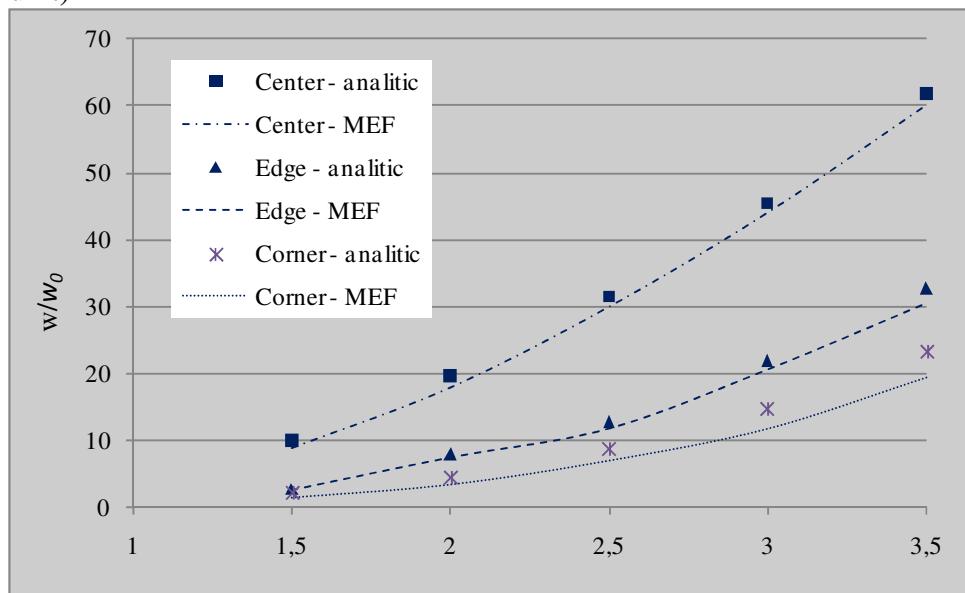


Figure 2: Analytical x Numerical solutions

## 4.2 Industrial application

A system for ore discharging consists of a floor over which off-road trucks operate. This floor has fifteen slabs of concrete structural placed over a existing concrete wall with sixteen meters high (figure 3). These plates have dimensions of 12x4 meters in plan and are partially supported in the existing wall, resulting in a cantilever with 2,75 meters (Figure 4).

The ore discharging system consist a dense and operating industrial area were the execution of concrete works are not feasible, so the slabs that form the operating floor must be performed outside the final position and subsequently transported. In addition, on these floor is acting off-road trucks of 652 tons weight, which requires high capacity of panels.

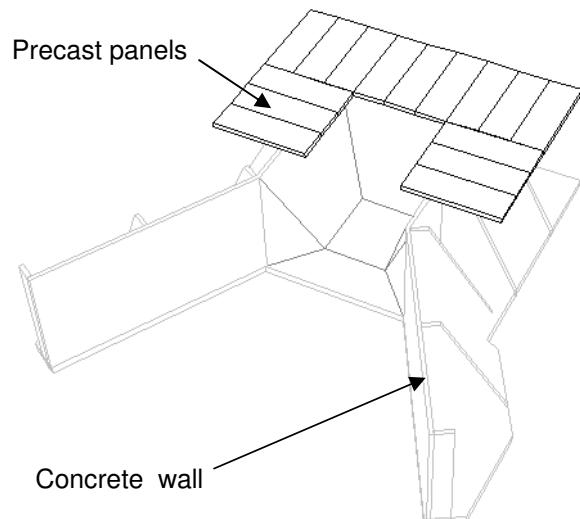


Figure 3: Ore discharging geometry

It was used to assemble precast panels hollow-core prestressed concrete since these elements combine high structural performance and lightness, allowing its transport and quick assembly. Figure 5 shows transversal section of the plates, made by structural concrete class C40, with  $f_{ck}=40MPa$ , initial modulus of elasticity  $E_{ci}=35GPa$  and Poisson coefficient  $\mu=0,20$ . The steel CA-50 used for the passive reinforcing of concrete structures has resistance characteristic  $f_yk=500MPa$  and modulus of elasticity  $E_s=196GPa$ . For pretension was adopted CP-190RB12,7 steel, with tensile strength  $f_y=187,3kN$  and modulus of elasticity  $E_a=195Gpa$ . The soil was formed of an initial layer of 16 m thick composed of compacted embankment ( $E_{e1}=1MPa$ ,  $\mu_{e1}=0,4$ , frictional angle  $\phi_{e1}=15^\circ$ , cohesion  $c=0,4 MPa$ ) followed by a layer of compact sand thickness 6 meters ( $E_{e2}=6MPa$ ,  $\mu_{e2}=0,45$  and  $\phi_{e2}=30^\circ$ ). Under the compact sand, there is a layer in very hard sand. The level of the water table was not found. For the determination of reinforcement bar and concrete sections of the panels, Brazilian standard NBR-6118 (2003) and NBR-7197 (1989) was adopted, with concrete cover  $d=3cm$  and cracks opening  $w_k=0,2mm$ .

The figure 6 shows the mesh (generated by a commercial pre-processing software) of the model considered in the soil simulation like a semi-infinite space. The model was discretized by approximately 0,6 million 3D lagrangean elements, totaling 26 million degrees of freedom. The analyses were conducted by a parallel processing according domain decomposition concept, where the mesh was partitioning in four subdomain, which were processed in independent CPU's, adopting the Conjugated Gradient with Jacob Precondition (PCG)

algorithm in the resolution of the resultant algebraic system. In figure 7 it is shown the traffic load model considered. The Figure 8 is presented the displacements envelopment obtained from the numerical analysis, considering the traffic load model operating in several critical positions on the panel. Figure 9 presents the resulting envelopment for the tensile stress at panel. From envelopment of tensions were determined both the sections of passive bar reinforcement ( $0,27 \text{ cm}^2$ ) and pretension bar (7 bars CP-190RB12,7). (Figures 8 and 9 are generated by commercial pos-processing software)

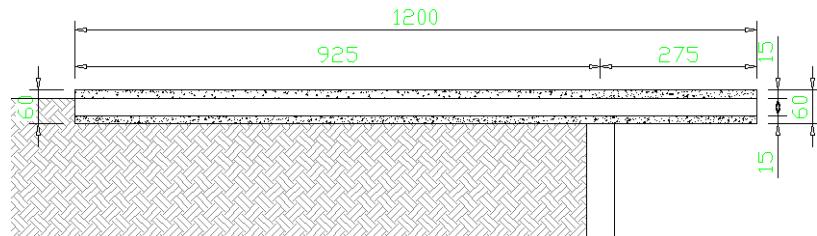


Figure 4: Longitudinal section

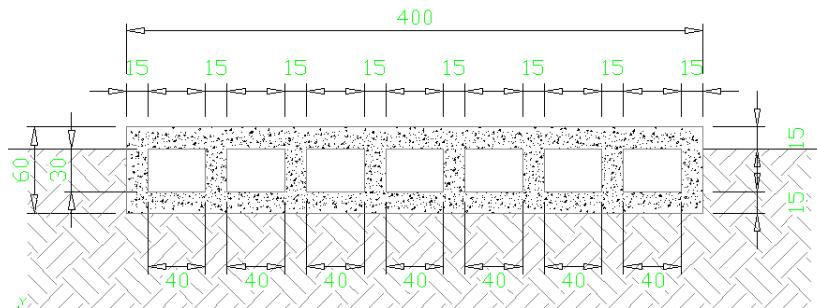


Figure 5: Transverse section

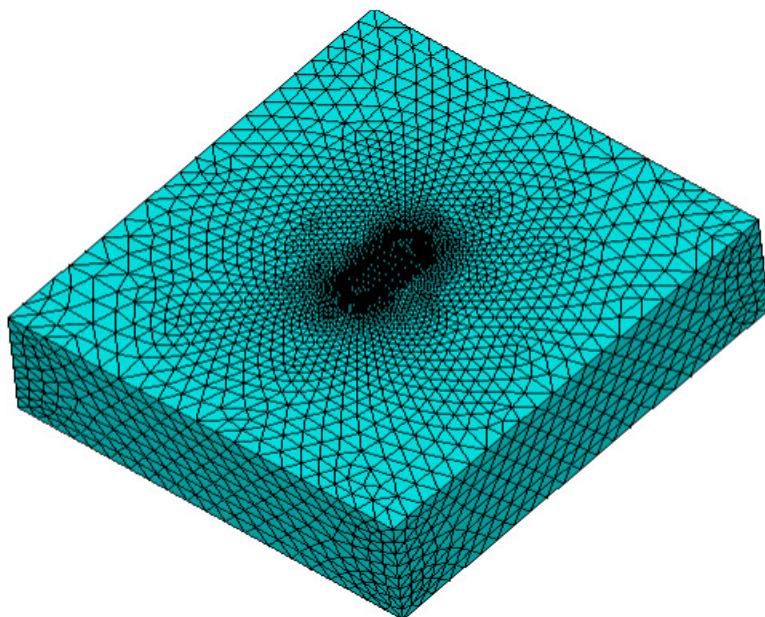


Figure 6: Superior view of 3D mesh

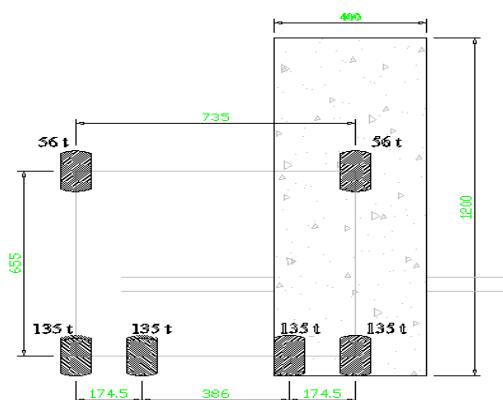


Figure 7: Traffic load model (centimeter, tons)

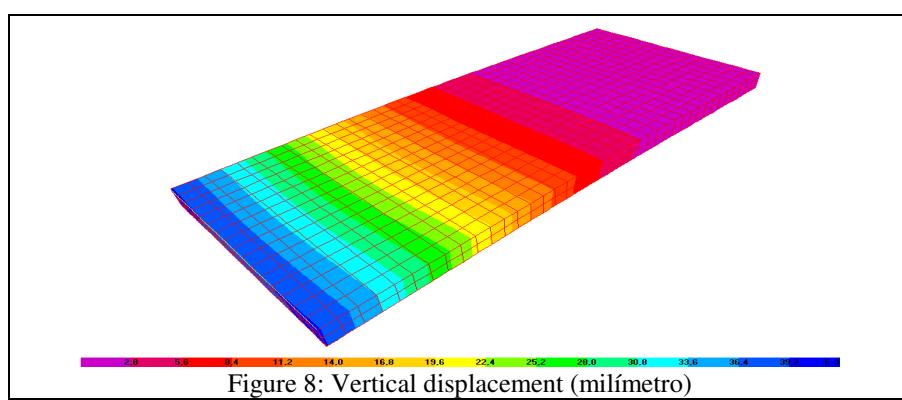


Figure 8: Vertical displacement (milímetro)

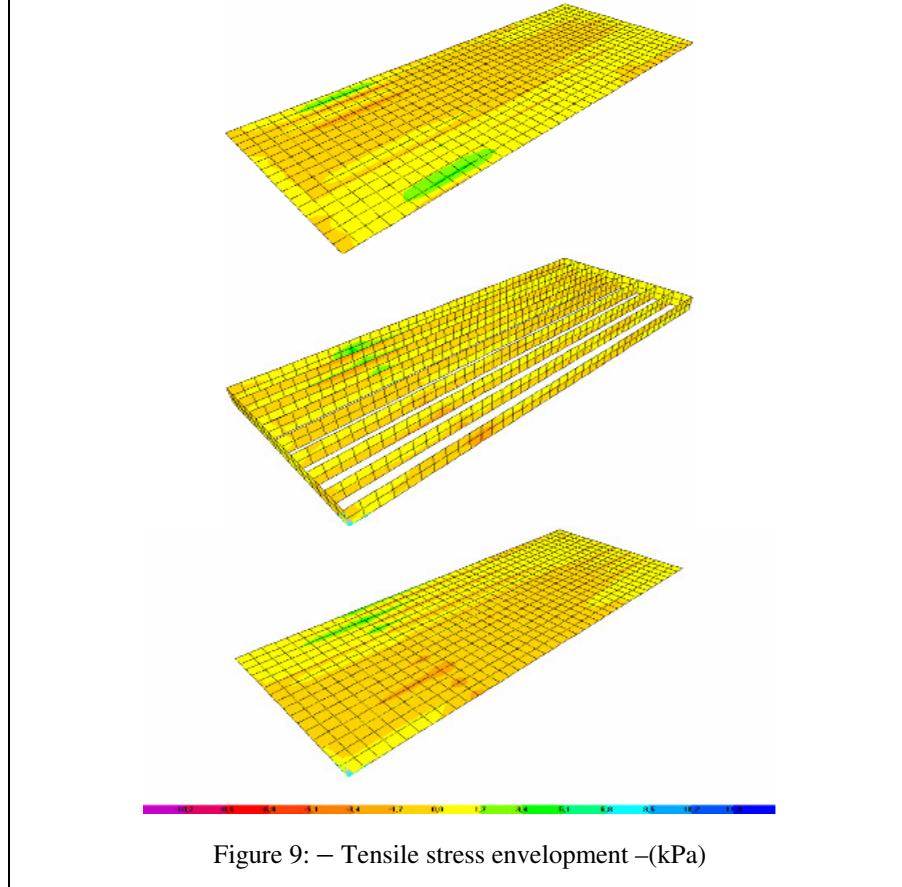


Figure 9: – Tensile stress envelopment -(kPa)

## 5 CONCLUSIONS

Techniques of prestress ally the concept of precast of structural elements in reinforced concrete was applied to industrial pavement with the purpose of reducing costs and increasing construction speed. As the constructive system was formed by hollow-core slab panels, although widely applied to structures of bridges and buildings, no application concerning paving has been reported in technical literature.

This paper presented a case study, in which precast panels formed by hollow-core prestressed panels were adopted as the rational solution for construction of industrial pavement where trucks with very high weight will operate. This structural solution in the industrial area with high traffic volume, presented themselves as the logical choice because it allowed the manufacture of boards outside the region of high density operations.

Numerical modeling was performed by algorithm based on parallel Finite Element Method (FEM), resulting in responses of tension and displacement of the panels. From these responses, the design was conducted according to applicable Brazilian standards. It was shown that these plates allowed the construction of a pavement system with the purpose of receiving high loads of trucks, and thus absorbs very high efforts. Finally it was shown that this system offers excellent structural performance by moving loads and allows implementation of large expanses of pavement quickly and rational, may be better studied and possibly applied to highways.

## 6 ACKNOWLEDGEMENTS

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