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ANT COLONY OPTIMIZATION MODEL FOR COMPOSITE LAMINATE DESIGN

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Abstract. A composite material is the result of the aggregation of two or more distinct materials to form a new one with specific properties. Designing composite materials is difficult because it involves designing their geometry and composition, and therefore, the possibilities for creating composite materials are almost unlimited. Algorithms based in swarm intelligence are showing better results to solve combinatorial optimization problems. In this paper we propose an ant colony optimization model for the design of composite materials. Finally, the propose method is applied to a real case showing results that are promising.

1 INTRODUCTION

A composite material is the result of the aggregation of two or more distinct materials to form a new one with specific properties. The agglomerate material is known as the matrix, and the rest are the reinforcement materials that may be made up of continuous fibers, short fibers or particles (Barbero, 1999). When the design is good, the new material adopts the best properties of its constituents, and sometimes even some that none of these possess. The properties that the design of a composite material aims to improve on are strength, stiffness, toughness, lightness, thermal insulation, etc. Of course, not all of them can be improved simultaneously, so the design objective is to obtain a new material that offers the best possible adaptation to the required specifications. Although composite materials, such as the addition of straw to mud or the use of laminated wood, have been used since ancient times, the real boom has only happened recently with the development of materials based on a fiber reinforced resin matrix (Duratti et al, 2002).

However, this type of material is difficult to develop, because the design of a new composite material involves designing both, the geometry of the element and the configuration of the material itself so as to best exploit the qualities of the constituent materials.

The design process for a laminate starts with the definition of the problem to be solved and the specifications to be met by the element to be designed. From this information, a series of solutions are generated by a synthesis process that is usually supported by the expertise of the designer. Potentially viable solutions are subsequently analysed to test their effectiveness. It's common to make the analysis of viable solutions using ANSYS (Ansys Multiphysics, Release 11.0), a finite element modelling based software package that is used usually in industry.

Traditionally, the design work, both synthesis and analysis has been carried out using empirical knowledge based methods (Grosset et al, 2006). This is partly because the number of possible combinations of composites is almost unlimited and also because characterization by experimentation is very expensive.

Since the 1990's different design systems have been proposed which aim to overcome these limitations. These proposals have involved approaches ranging from traditional techniques such as classical nonlinear optimization procedures combined with finite element modelling (FEM) (Huang et al, 2005), through generic task methods and case-based reasoning (Lenz, 1997), to modern artificial intelligence techniques (Adams et al, 2003; Falkenauer, 1998; Soremekun et al 2001).

At present, one technique that is proving rather efficient in a large number of situations is the Ant Colony Optimization (ACO) algorithm. In this paper we propose an Ant Colony Optimization model for the design of Composite Materials and Structures. The paper is organized as follows: section 2 relates the analysis of symmetric laminates and theory of Ant Colony Optimization algorithms; section 3 presents a model for the designing and optimization of symmetric laminates based on an ACO; in section 4, this model is applied to a real application; finally, conclusions are presented.

2 BACKGROUND

In this section we introduce the composite material design and the Ant Colony Optimization algorithm.

2.1 Symmetric Laminates Analysis

In this section we consider the mechanical analysis of a symmetric laminate to determine the numbers of laminas that break.

A unidirectional reinforced lamina is made up of a set of very thin fibres, aligned in a particular direction, and embedded in a polymeric matrix that supports them. The relative amounts of the two components are expressed as the fibre and matrix volume fraction, V_f and V_m, which are dimensionless quantities and must satisfy that $V_f+V_m=1$. V_f is usually between 0,3 and 0,7. For the lamina in the 1-2 plane (see Figure 1), a plane stress state is defined by setting:

$$\sigma_3=0;\ \tau_{23}=0;\ \tau_{31}=0 \tag{1}$$

The characteristics of a lamina are defined by their elastic constants: Young's modulus for the principal directions, E_1 and E_2; stiffness modulus, G_{12}; and Poisson ratio, υ_{12}. These constants are in turn dependent on the properties of the fibre and the matrix: Young's modulus, E_f and E_m; Poisson coefficients, υf and υm; as well as the shape and size of the fibres, their distribution and volume fraction, Vf, etc.

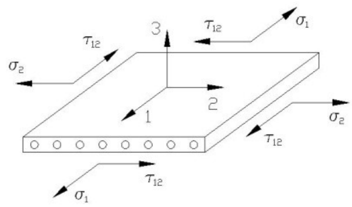


Figure 1. Stresses on a lamina subjected to a state of plane stress.

Using Equation (1), strain-stress relations can be formulated as

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{pmatrix} . \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} \tag{2}$$

where

$$S_{11} = \frac{1}{E_1}; S_{22} = \frac{1}{E_2}; S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{\nu_{21}}{E_2}; S_{66} = \frac{1}{G_{12}} \tag{3}$$

Different failure criteria are used to determine whether a lamina can withstand specific stress conditions without breaking (Barbero, 1999). In this paper we have selected the Tsai-Wu tensor failure criterion (Stephen, 1971), that postulates that a failure surface in six-dimensional stress space exists in the form

$$F_i.\sigma_i + F_{ij}.\sigma_{ij} = 1 \quad i,j = 1\ldots6 \tag{4}$$

where in F_i and F_{ij} are strength tensors of the second and fourth rank, respectively, and the usual contracted stress notation is used $(\sigma_4 = \tau_{23}, \sigma_5 = \tau_{31} \text{ and } \sigma_6 = \tau_{12})$. In the case of an orthotropic lamina under plane stress conditions, we have the relation

$$\left(\frac{1}{x}-\frac{1}{x'}\right).\sigma_1+\left(\frac{1}{y}-\frac{1}{y'}\right).\sigma_2+\frac{1}{x.x'}.\sigma_1^2+\frac{1}{y.y'}.\sigma_2^2-\sqrt{\frac{1}{x.x'.y.y'}}.\sigma_1.\sigma_2+\frac{1}{s^2}.\tau_{12}^2=1 \tag{5}$$

where *x, x′, y, y′* and *S* are, respectively, the ultimate tensile and compressive strength in the fibre direction, the ultimate tensile and compressive transversal strength, and the shear strength. In order to determine if a lamina breaks, we establish a breakage coefficient P_k, defined as:

$$P_k=\left(\frac{1}{x}-\frac{1}{x'}\right).\sigma_1+\left(\frac{1}{y}-\frac{1}{y'}\right).\sigma_2+\frac{1}{x.x'}.\sigma_1^2+\frac{1}{y.y'}.\sigma_2^2-\sqrt{\frac{1}{x.x'.y.y'}}.\sigma_1.\sigma_2+\frac{1}{s^2}.\tau_{12}^2 \tag{6}$$

The laminate does not break if $P_k < 1$, and breaks otherwise. Besides, if P_k is approximate to 1, the lamina *i* is working at full capacity.

Now, we can define the optimization problem as minimize the number of laminas n. The set of design variables is expressed as a vector $S=(F,M,VF,\theta_1,\ldots,\theta_m)$. The optimization problem with appropriate constraints can be expressed as:

$$\begin{aligned}
&\min n(s)\ \textit{such that}:\\
&n \geq 2\ (\textit{number of laminas})\\
&P_k \leq 1\ (\textit{Tsai-Wu Coefficients})\\
&V_f \in [0.3, 0.7]\ \textit{in}\ 0.1\ \textit{intervals}\ (\textit{Volume Fraction})\\
&\theta_i \in [-80°, 90°]\ \textit{in}\ 10°\ \textit{intervals}\ (\textit{Orientation of the laminas})
\end{aligned} \tag{7}$$

2.2 Ant Colony Optimization

The Ant Colony Optimization (ACO) algorithm is inspired by the behavior of real ants when they cooperate to search for food from their nest to the food sources. Ants communicate using pheromone, which is a chemical substance produced by ants and is deposited to their trails. The traditional ACO algorithm consists of a population of n ants, where each ant consists of two modes, the forward mode and the backward mode. Initially, all ants are placed on a randomly selected node and all pheromone trails are initialized with an equal amount of pheromone. All ants on their forward mode choose the next node based on pheromones and some heuristic information. This process continues until ants have visited each node once and hence constructs one solution. An iteration is finished when all ants have constructed their solutions using a probabilistic decision rule. After each iteration, all ants proceed to their backward mode in order to deposit pheromone and update their trails. They retrace their solutions and deposit pheromone according to their solution quality on the corresponding trails. However, before adding any pheromone, a constant amount of pheromone is deduced from all trails due to the pheromone evaporation. Reducing the pheromone values enables the algorithm to forget bad decisions made in previous iterations (Mullen, 2009; Dorigo, 2004). After evaporation, all ants deposit pheromone to the corresponding trails of their tour until reach the final objective. The Algorithm 1, show the framework of a generic ACO algorithm.

```
input: An instance of a combinatorial optimization problem
InitializePheromoneValues()
while (termination conditions not met) do
   for j=1, . . . ,Q.Ants do
      ConstructSolution()
      if (is a valid solution) then
         LocalSearch(solution)
```

```
      end if
   end for
   ApplyPheromoneUpdate()
end while
output: best solution found
```

Algorithm 1. Framework of a generic ACO algorithm.

3 PROPOSED ACO MODEL

This section presents an Ant Colony Optimization model for the design of symmetric laminates. In order to develop this model, we propose the following function, a fiber/matrix space generation, new ant and pheromone update (see Algorithm 2).

3.1 Fiber Matrix Space Generator

In this subsection we present the solutions space generator, where this space is built in fiber/matrix combinations. Figure 2 shows how to build the solutions space. Level 0, establish the fiber/matrix combination while the other levels show the different laminates (level 1 represents laminate 1) with the possible fiber angle orientation. In our case, angle values from -80° to 90° in 10° increment on every. The limitation in the solutions space is due to the maximum level defined in the initialization function.

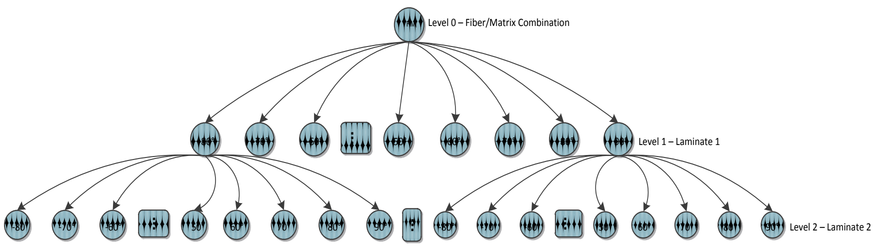


Figure 2. Solutions space generation diagram.

The general proposal model structure is showed in figure 3. Each fiber/matrix combination generated a thread that represents an ant colony and creates the maximum ant's quantity defined for each colony. The combination of several continuous nodes compounds a solution if they satisfy the failure criteria meaning that supports the specified tensions.

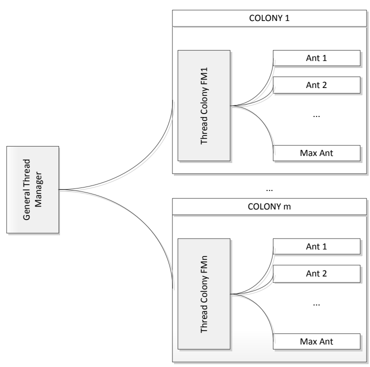


Figure 3. General algorithm´s structure.

The algorithm model proposal is as follow:

```
ACO_Calculation()
   Paremeters_Inicialization(Vf, Vm, Ef, Em, Gf, Gm, νf)
   FM_Space_Generation()
   for each FM_Space in pararel do
      repeat
         repeat in pararel from k=1 to ants_quantity
            New_Ant()
         end repeat
         repeat in pararel from k=1 to candidate_solutions
            Pheromone_Update()
         end repeat
      until objective found or max_cycle reach
   end for
   Find_Final_Solution(colonies, objective)
end function
```

Algorithm 2. ACO Algorithm for composite laminates.

3.2 New Ant

This function generates an individual ant, like the generic ACO algorithm, that will travel the solution space trying to find candidate solutions. In an iterative way, an ant put a pheromone trail in a forward path traveling each node generated, calculating the stiffness and striving matrixes for each laminate. In this case the solution is evaluated for not exceed the material volume fraction and respect the maximum number of adjacent sheets stack as a result of stacking of layers with the same fiber orientation. This limitation is due to avoid the residual tension problem (Barbero, 1999). Then, each branch is evaluated according to the failure criteria and to determine the candidate solutions. This function repeats until it obtains the composite laminate or the maximum cycle value is reach.

3.3 Pheromone Update

From an economic point of view is convenient that the laminate is compose with the least laminates number and the volume fraction be minimum, so lower laminates number and lower volume fraction means lower cost. While from a security point of view, is convenient that the tension be distributed on the fiber direction to avoid the negative effects of not provided tensions. Due to this criterion we have defined the pheromone update function (PU) in a way that the pheromone attenuation will be calculated as:

$$PU = F^{-1}\left(\delta, \frac{P_1^{\alpha}}{V_f.(n.e)^{\beta}.(R+1)^{\gamma}.(P_2+1).(P_{12}+1)}\right) \tag{8}$$

where: P_1 is the longitudinal coefficient along the direction of the fibers; P_2 is the longitudinal coefficient perpendicular to the direction of the fibers; P_{12} is the shear coefficient; V_f is the laminate volume fraction; R indicates the number of layers that break and α, β, γ are real values. Finally, delta is a parameter to speed up the convergence and enhance the algorithm. In our case, the solutions that were using for several ants will be receiving a bigger pheromone level and therefore these solutions will be the most use by the ants in future iterations.

The prototype was implemented in the programming language Python, and due to the heavy computational calculation, we used the NumPy library to speed up and to obtain accuracy results.

4 EXPERIMENTAL RESULTS

The problem is to determine the composition and thickness, e, of a cylindrical tank, where e is small compared to the average radius, r, and subjected to internal pressure p. Figure 4 shows the cylindrical tank along with the stresses on an element of it.

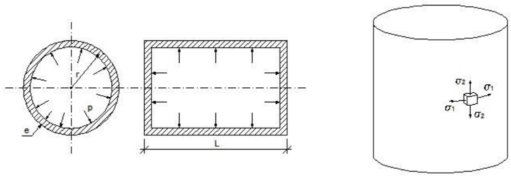


Figure 4. Thin-walled cylindrical tank subjected to interior pressure.

The forces resulting from the equilibrium equation are defined as:

$$\sigma_1 = \frac{p.r}{e} \qquad \sigma_2 = \frac{p.r}{2.e} \tag{9}$$

Let N_1 y N_2 the normal forces, resulting from the normal tension σ_1 y σ_2 by unity length, their values are:

$$N_1 = p.r \qquad N_2 = \frac{p.r}{2} \tag{10}$$

For example, if

$$p = 18\frac{Kg}{cm^2} = 1.7685\, MPa \;\; y \;\; r = 1m \tag{11}$$

Then

$$N_1 = p.r = 1.7685 \cdot 10^6 \cdot 1 = 1.7685.10^6 \,\frac{N}{m} \tag{12}$$

$$N_2 = \frac{p.r}{2} = \frac{1.7685 \cdot 10^6 \cdot 1}{2} = 0.8829.10^6 \,\frac{N}{m} \tag{13}$$

Table 1 shows the statistics corresponding to 300 tests completed with the proposal algorithm in this work. The results shown correspond to the least number of laminas of the material obtained, NL_{min}, the number of times this solution is obtained N_1 its average NL_{mean}, and its standard deviation σNL. The table 2 present the characteristics of the best laminate obtained in the series of tests. Finally, the Tsai-Wu coefficients are shows graphically corresponding to the laminas of the best laminate obtained. In this example, a large degree of uniformity of the coefficients P_k around 1 can be observed that indicates an excellent exploitation of all the laminas.

Algorithm	NL_{min}	N_1	NL_{mean}	σNL
ACO Model	20	13	22.84	1.1629

Table 1. Statistics corresponding to the series of tests.

All solutions obtained by Ant Colony based model have been validated by the ANSYS software package (Ansys Multiphysics, Release 11.0) offering values of $P_k < 1$, that is, the laminas do not break.

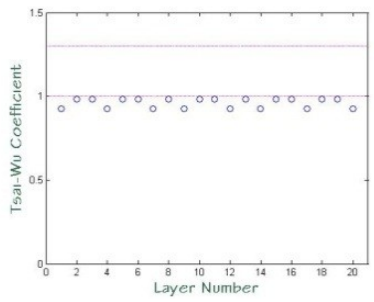
<table>
<tr><th>Characteristic</th><th>Values</th><th>P_k Coefficients (Tsai-Wu)</th></tr>
<tr><td>Fibre</td><td>P-100</td><td rowspan="5">
</td></tr>
<tr><td>Matrix</td><td>Peek</td></tr>
<tr><td>VF</td><td>0.50</td></tr>
<tr><td>Laminates</td><td>[-30° 40° 40° 30° 40° -40° -30° -40° 30° -40°]</td></tr>
<tr><td>Thickness (mm)</td><td>3.60</td></tr>
</table>

Table 2. Characteristics of the best laminate obtained in the series of tests.

5 CONCLUSIONS

In this work we propose an Ant Colony Optimization algorithm for the design of composite material geometry and composition. This model presents a function that generates the space solution based in the fiber and matrix combination. Besides, we propose a function to update the pheromone trail based in safety and economic criteria's.

Finally, the proposal algorithm has been applied to a real problem, obtained a promising result. These results have been showed that the adjustment coefficients obtained from P_k is close to 1, meaning that all laminas work near to their strength and consequently reduced the product cost, using in an efficient way the material. Based on these results we can conclude that we are opening a new research line in this field.

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