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USING GENETIC ALGORITHMS AND ARTIFICIAL NEURAL NETWORKS FOR RELIABILITY BASED OPTIMIZATION OF LAMINATED COMPOSITE STRUCTURES

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Abstract. The design of anisotropic laminated composite structures is very susceptible to changes in loading, angle of fiber orientation and ply thickness. Thus, optimization of such structures, using a reliability index as a constraint, is an important problem to be dealt. This paper addresses the problem of structural optimization of laminated composite materials with reliability constraint using a genetic algorithm and two types of neural networks. The reliability analysis is performed using one of the following methods: FORM, modified FORM (FORM with multiple checkpoints), the Standard or Direct Monte Carlo and Monte Carlo with Importance Sampling. The optimization process is performed using a genetic algorithm. To overcome high computational cost it is used Multilayer Perceptron or Radial Basis Artificial Neural Networks. It is shown, presenting two examples, that this methodology can be used without loss of accuracy and large computational timesavings, even when dealing with structures having geometrically non-linear behavior.

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1 INTRODUCTION

In the optimization of laminated composite structures, the design variables related to the optimal configurations may be the ply number, fiber orientation angles, thickness of each layer, number of materials and the sequence of lamination. The result of the optimization procedure consists of systems with anisotropic mechanical behavior that are also highly sensitive to the direction of applied loads. Any change in the applied load direction, fiber orientation or thickness of the layers may affect the stress state, leading to a reduction in the structural performance or in the reliability index. Thus, design of structures with anisotropic laminated composite materials should take into account such uncertainties in loads and material properties. Consequently, reliability of such optimized designs becomes especially important in the field of laminated composite structures (Miki *et al.*, 1980). This subject is not new and a critical appraisal and survey related to methodologies for reliability evaluation of fiber reinforced composite materials can be found in Shaw *et al.* (2010)

The main objective of this paper is to present a new methodology to determine the optimal configuration of laminated composite structures with reliability constraints. The main structural behavior aspects were modeled by finite elements using well-known meso-scale models based on elasticity theory and failure of composite materials simulated by the Tsai-Wu criterion.

The optimization process is performed using a genetic algorithm (GA). Genetic algorithms are optimization tools based on the concepts of natural selection and survival of the fittest individual with respect to some criterion. The design of the optimal sequence of layers in laminated composite materials (with their respective thickness and fiber orientation angles) is a minimization problem and due to its characteristics, genetic algorithms are more convenient than gradient methods, which often converge to solutions that represent local minima (Goldberg, 1989). Moreover, in commercial projects of this type of structure, fiber orientation angles, number and thickness of layers are discrete variables, a fact that encourages the use of genetic algorithms, because this tool is suitable for computational problems involving discrete variables and combinatorial optimization.

In this paper the reliability analysis is carried out using one of the following methods: First Order Reliability Method (FORM), modified FORM with multiple check points (FORM-MCP), Standard or Direct Monte Carlo (MC) and Monte Carlo with Importance Sampling (MCIS). These methods and concepts related to structural reliability are widely covered in texts such as Ang and Tang (1984), Haldar & Mahadevan (1999), Melchers (1999), among others, as well as a large number of articles published in several International Journals. The Tsai-Wu criterion is adopted as the limit state function used to evaluate the reliability index (Daniel & Ishai, 1994; Jones, 1999; Gurdal *et al.*, 1999).

The finite element analysis (FEA) was performed using the Discrete Kirchhoff Triangular element (DKT) for thin plates (Bathe & Batoz, 1980), coupled with the Constant Strain Triangular element (CST). The element was adapted to analyze laminated composite structures, following the classical theory of laminates (Jones, 1999, Daniel & Ishai, 1994).

In order to reduce the computational cost in the reliability-based optimization of laminated composite structures, two artificial neural networks were used: Multilayer Perceptron Neural Network (MPNN) and Radial Basis Neural Network (RBNN) (Haykin, 1994 and Gomes, 2004).

The main contribution of this paper relies on the computational improvements obtained when optimizing large structures with reliability constraints. The combination of a global optimizer for discrete/continuous variables associated to an approximated structural re-

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analysis may render unfeasible/unworthy problems to become tractable.

2 COMPOSITE MATERIALS FAILURE CRITERION AND RELIABILITY ANALYSIS

The evaluation of principal stress components is sufficient to indicate the failure in cases where isotropic materials are used. The failure can be assessed by traditional theories of failure analysis, such as maximum stress, maximum strain, Tresca, von Mises, among others. Regardless of the directions of principal stresses, their magnitudes are compared with experimental strength values in order to verify the occurrence of failure. The main difference between isotropic materials and laminated composite materials is the directional dependence of the strengths, which occurs in the latter case, where failure takes place in the direction of the fibers or the polymeric matrix. There are several failure criteria for composite laminates reinforced by fibers, such as maximum strain, Tsai-Hill, Hoffman and Tsai-Wu (Kaw, 2006). Among these methods, Tsai-Wu criterion is the most widely used by several authors because it represents quite well the actual behavior of this type of structure. This criterion takes into account the interactions between different stress components. The two coordinate systems used here are shown in Figure 1, where *1* and *2* represent the axes of reference and θ is the angle between the axes *x* and 1 and between axes *y* and *2*. Since the stress components in the direction of the reference axes (*1-2*) are rotated to the principal axes of the material (x-y), the Tsai-Wu criterion for plane stress state can be evaluated using the following equation:

$$F_x S_x + F_y S_y + F_{xx} S_x^2 + F_{yy} S_y^2 + F_{ss} S_{xy}^2 + 2F_{xy} S_x S_y = 1 \tag{1}$$

where $F_{xx} = 1/R_x R'_x$, $F_x = 1/R_x - 1/R'_x$, $F_{yy} = 1/R_y R'_y$, $F_y = 1/R_y - 1/R'_y$, $F_{ss} = 1/R_s^2$ and $F_{xy} = F_{xy}^* \sqrt{F_{xx} F_{yy}}$.

The factor F_{xy}^* is taken as being equal to *-1/2*. This value is only valid for a von Mises-Hencky stress criterion basis. The subscripts *x* and *y* indicates, fiber orientations, while *s* means shear. The symbols with apostrophe indicate compression strengths, whereas symbols without apostrophe indicate tensile strengths. R_x is the ultimate longitudinal tensile strength, R'_x is the ultimate longitudinal compressive strength, R_y is the ultimate transverse tensile strength, R'_y is the ultimate transverse compressive strength and R_s is the in-plane shear strength. S_x, S_x and S_{xy} are stress components referred to the system (*x*-*y*).

Assuming an elastic material behavior, a Tsai-Wu factor λ, which multiplies all stress tensor components, can be evaluated concerning the safety margin of the stress state. This is indicated by Daniel and Ishai (1994) by solving the following equation for λ :

$$\lambda^2 (F_{xx} S_x^2 + F_{yy} S_y^2 + F_{ss} S_{xy}^2 + 2F_{xy} S_x S_y) + \lambda (F_x S_x + F_y S_y) - 1 = 0 \tag{2}$$

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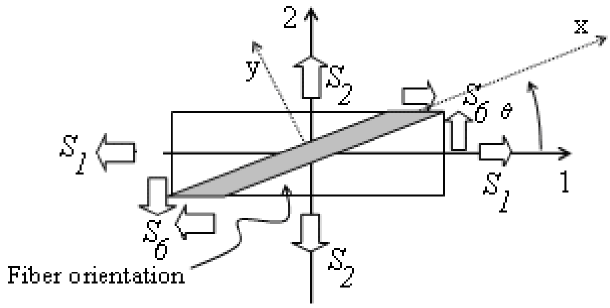

Figure 1: Coordinate system for unidirectional composite materials

A mathematical expression for unidirectional composite failure is written as follows:

$$g(\mathbf{X}) = g(x_1, x_2, ..., x_n) \leq 0 \tag{3}$$

where $g(\mathbf{X})$ represents the safety margin and $\mathbf{X}$ is the n-dimensional vector of random variables ($x_i, i = 1, 2, ..., n$) that affects the material strength or structural behavior. $g(\mathbf{X}) \leq 0$ means failure and $g(\mathbf{X}) > 0$ means that the material is in the safety domain. Sometimes, function $g(\mathbf{X})$ is referred as the Limit State Function (LSF). Generally speaking, the failure probability can be evaluated using the joint probability density function $f_X(x_1, x_2, ..., x_n)$ by the following expression:

$$P_f = \iint \cdots \int_D f_X(x_1, x_2, ..., x_n) dx_1 dx_2 ... dx_n \tag{4}$$

where D means the failure domain (where $g(\mathbf{X}) \leq 0$).)

Consider a thin plate of composite material subjected to a plane stress state, as indicated by Figure 1, where the random variables $\mathbf{X}$ are the stress components S_1, S_2 and S_6 the experimental material strengths along fiber and transversal directions $R_x, R'_x, R_y, R'_y, R_s$ and fiber orientation angles θ. Rotating these stress components to the fiber direction and distributing to the other layers of the composite accordingly to ply ratios, one may obtain the stress state acting on a lamina (S_x, S_y, S_{xy}). So, in this case, for instance, one may assume $\mathbf{X} = (S_1, S_2, S_6, R_x, R'_x, R_y, R'_x, R_s, \theta)$ as the vector of random variables. It should be noticed that the random stress components S_1, S_2, S_6 generate random stress components on the fiber and transversal directions, which requires a structural analysis to be evaluated.

Substituting equation (1) into (3), the limit state function $g(\mathbf{X})$, at a particular point in the composite material, becomes:

$$g(\mathbf{X}) = 1 - \left\{ F_x S_x + F_y S_y + F_{xx} S_x^2 + F_{yy} S_y^2 + F_{ss} S_{xy}^2 + 2 F_{xy} S_x S_y \right\} \tag{5}$$

It should be emphasized that this equation should be verified at the top, middle and bottom of each layer belonging to the composite material. The integration of equation (4) becomes hard if equation (5) is used as the limit state function, since the problem deals with several random variables and the stress state is a function of geometrical dimensions and external

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loads as well. Besides, function $f_X(\mathbf{X})$ is not known *a priori* because usually there are not enough available statistical data. In this paper, using a finite element analysis, a limit state function is built based on the Tsai-Wu factors $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ...)^T$ evaluated at each element integration point and at each layer (at the top, middle and bottom), as expressed by the following equation:

$$g(\mathbf{X}) = min(\boldsymbol{\lambda}) - 1 \tag{6}$$

This equation holds provided a first ply failure for the composite is assumed. Therefore, if the minimal Tsai-Wu factor at any point is less than a unit value, this will mean failure ($g(\mathbf{X}) < 0$), otherwise, not all the stress states in the composite provoke failures. It is true that a more realistic approach for the failure mode can be constructed using a stochastic 3D field related to the geometrical imperfections in the finite element mesh and materials strengths at each integration point (some relevant data can be found in Cryssanthopoulos and Poggi, 1995 and Sriramula and Cryssanthopoulos, 2009). The authors have some papers related to the modeling of stochastic fields on concrete structures (Gomes and Awruch, 2002, Gomes and Awruch, 2004 and Gomes and Awruch, 2005) where the spatial variability of such material properties and geometrical imperfections are important. In this paper an exponentially correlated stochastic filed for ply thickness was be considered in the last example since in this example buckling effects are important. The stochastic filed generation follows the methodology indicated in Gomes and Awruch (2005).

In order to determine the failure probability or the reliability index, Reliability analyses are preformed using standard methods such as the Direct Monte Carlo (MC), Monte Carlo with Importance Sampling (MCIS), First Order Reliability Method (FORM) and FORM with Multiple Check Points (FORM-MCP). Details about MC, MCIS and FORM can be found in Melchers (1999) and Ang *et al.*(1984).

FORM-MCP is a variant of the Multiple Check Points Importance Sampling method presented by Miki (1986). In this case, sample points are searched close to the boundary of the failure and safety regions. FORM method is used instead of Monte Carlo Simulations with Importance Sampling in order to evaluate multiple design points. The parameter used to distinguish among different design points is the same indicated by Miki (1986) and Shao *et al.* (1992): the angle between the vector of design variables for each new design point should be larger than a previously specified value θ (in this paper, θ must be larger than 10^{-3}). The search is performed in random directions; the number of searches is a multiple of the number of random variables. Figure 2 shows the multiple checkpoints criterion in the standard non-correlated space for three random directions and three limit state functions. $H_i(\mathbf{U})$ are the limit state functions obtained from $g(\mathbf{X})$, which is the limit state function in the real space. In this case, $\mathbf{U}$ is the vector of random variables at the standard non-correlated space. Then, the i resulting values for failure probability ($P_f^{\,i}$) and the reliability index are given by $P_f = \Sigma\, P_f^{\,i}$ and, $\beta = \Phi^{-1}(1 - P_f)$ respectively, where $\Phi^{-1}(.)$ is inverse of the cumulative standard probability function. Index i indicates the number of random directions. For the case of Figure 2, $i=1, 2, 3$. More details can be found in Miki (1986). It is important to point out that in this work the failure of one layer represents the failure of the whole system, criterion which is known as first ply failure.

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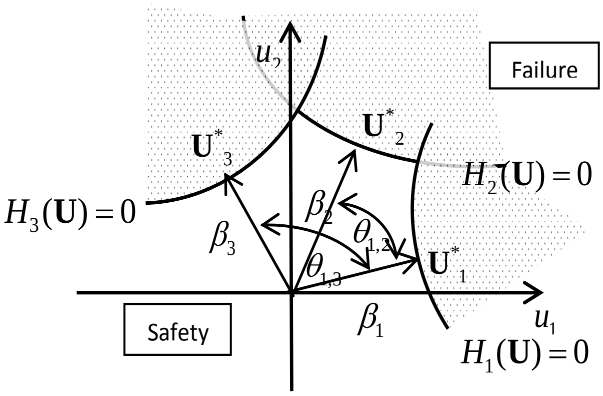

Figure 2: FORM with multiple check points in the standard non-correlated space.

3 GENETIC ALGORITHM (GA)

Genetic Algorithm (GA) is a computational search tool based on concepts of natural selection and survival of the fittest individual. One aspect of fundamental importance in the GA is the way the solutions are tracked. Instead of using derivatives or gradients, as in deterministic optimization algorithms, GA works with the objective function based on simple values of individuals. This feature makes the method suitable for problems involving discontinuous functions, and/or non-defined derivatives like in integer programming. Moreover, unlike deterministic optimization methods, which perform the search focusing on a single solution at a time, the GA works with a population of individuals in each generation. Thus, as several search points are maintained, the convergence or stagnation to local minima, if the starting point is poorly chosen, is prevented. All these aspects result in increased chances of finding the optimal solution or with other similar quality, even on problems that have hard search spaces with multiple local minimum (Goldberg, 1989).

The design of the optimal sequence of layers in laminated composite materials is a problem of global minimum. Due to the stochastic characteristics of Genetic Algorithms, they are more suitable than deterministic methods of optimization, which often converge to solutions representing a local minimum. Moreover, in commercial designs fiber orientation angles and the amount and thickness of layers are discrete variables, a fact that confirms the suitability of Genetic Algorithms for these kinds of problems.

More details related to the use of the method for weight optimization of composite structures can be found in Almeida and Awruch (2009), Muc and Gurba, (2001) and Naik *et al.* (2008).

4 ARTIFICIAL NEURAL NETWORKS

Artificial Neural Networks (ANN) may be characterized as computational models based on parallel distributed processing with particular properties such as the ability to learn, to generalize, to classify and to organize data. Several models have been developed for different specific computational tasks. These models may be divided into two groups: those with a supervised training and networks without a supervised training. In this paper, Multilayer Perceptron Neural Networks (MPNN) and Radial Basis Neural Networks (RBNN) are used. Both types of Networks have a supervised training, feed-forward architecture and they have been widely used as universal approximations for unknown functions of several variables

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with several outputs. More details can be found in Gomes and Awruch (2004).

4.1 Generation of sample data for artificial neural network training

To generate the sample data for artificial neural network training it is first carried out a search on random directions (in the non-correlated standard space) for points near the limit state function such as $H(\mathbf{U}) = 0$, in the standard non-correlated space. Once such points are found, the mean values of the distribution functions of the design variables are shifted in order to obtain samples (using Standard or Direct Monte Carlo) near the neighborhood of the safety/failure domain. Another set of random samples centered on mean values of the random variables are added to the original sample set in order to give a better behavior to the fitted limit state function that are located far from the failure domain. This is especially important if a gradient-based method, like FORM, is used, but not so important when Monte Carlo based methods are used. Figure 3 shows schematically how this sample data are generated in the non-correlated standard Gaussian space for a limit state function of two random variables. In this paper, the number of random directions is three times the number of random variables.

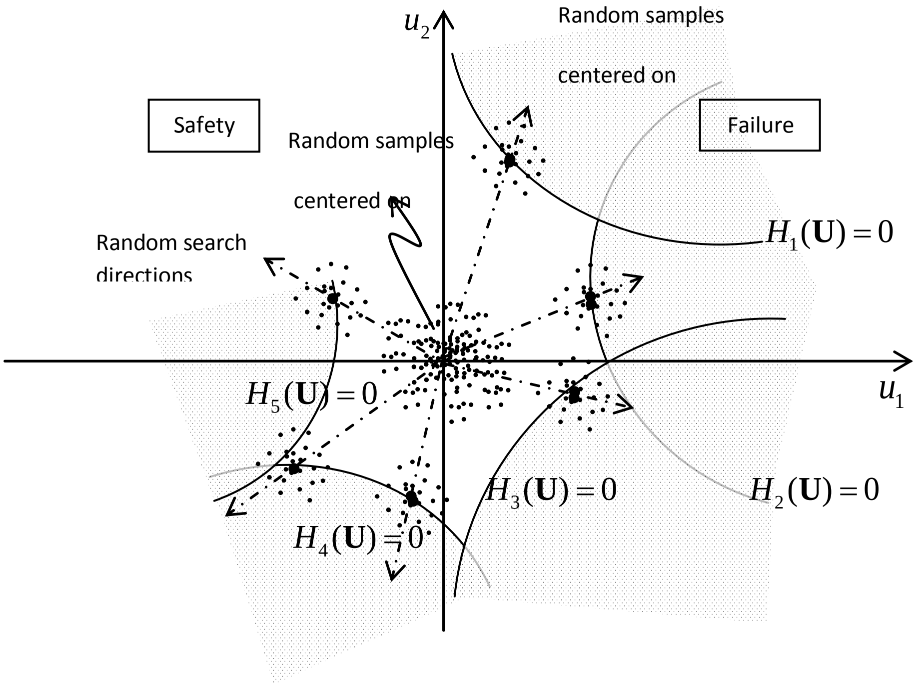

Figure 3: Generation of sample data set for neural network training.

5 NUMERICAL RESULTS

5.1 Example 1 – Optimization of a laminated composite plate with reliability constraint

This example deals with the minimization of the total thickness of a laminated composite plate with linear behavior. The total number of layers is N and the thickness of layer i is h_i ($i=1,2,...,N$). In all cases studied in this section, the cost function is the total thickness of the plate (h_t) and the constraint was the minimum reliability index required by the system (β_{req}), which is a value defined by the user. The optimization problem takes the following

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form:

$$\begin{aligned} &\text{Find} && h_i\,(i=1,2,...,N) \\ &\text{such that} && h_t=\sum_{i=1}^{N} h_i \quad \text{is a minimum} \\ &\text{subjected to} && \beta \geq \beta_{req} \end{aligned} \tag{7}$$

The fiber orientation angle of each ply of the laminated composite plate with four layers remains constant and according to the following distribution [0^0,45^0,45^0,0^0], as shown in Figure 4.

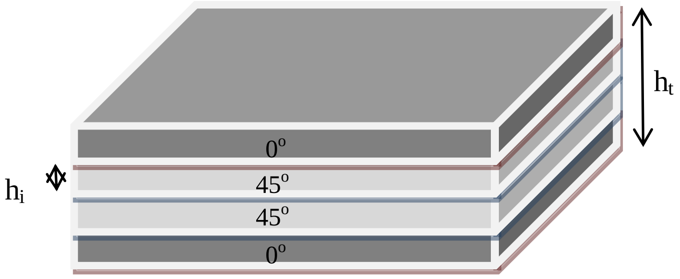

Figure 4: Laminated composite plate with four layers.

The material used here was Graphite / Epoxy (T300/5208). Table 1 presents the deterministic mechanical properties.

Material	E_1	E_2	E_{12}	ν_{12}
T300/5208 Grafite/Epóxi	181 GPa	10.30 GPa	40 GPa	0.28

Table 1: Deterministic mechanical properties.

In this example nine random variables were considered, where four variables are the applied loads N_1, N_2, N_{12} and M_1, arranged as shown in Figure 5, and five variables are strengths $R_x^T, R_x^C, R_y^T, R_y^C$ and R_{xy}, where indexes T and C mean, respectively, tension and compression whereas R_{xy} is the shear strength. In Figure 5, (*x*, *y*) is the fiber orientation system and (*1*, *2*) is the global system.

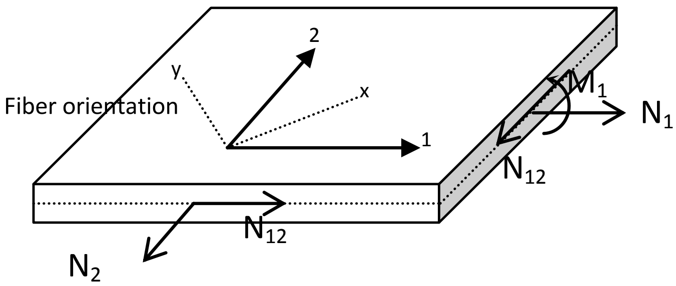

Figure 5: Loads acting on the laminated composite plate.

The statistical properties of the random variables are listed in Table 2. Although some authors prefers the use of Weibull distributions for the statistical analysis, this example was

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based on suggestions of Kan and Chang (1997), Frangopol and Recek (2003) and Murotsu *et al.*(1994) in order to allow comparisons. Indications for more accurate distribution type data can be found in Chrissantopoulos and Poggi (1995).

In all simulations, it was adopted a constraint target reliability index of $\beta_{req} = 3.0$. The optimization was performed using a GA, which input data are listed in Table 3. The reliability index was calculated using Monte Carlo, Monte Carlo with Importance Sampling, FORM and modified FORM (FORM-MCP).

No.	Symbol	Unit	Mean Value	Coeff. of Variation	Distribution type
1	N_1	KN/m	100.0	0.20	Lognormal
2	N_2	KN/m	200.0	0.20	Lognormal
3	N_{12}	KN/m	40.0	0.20	Lognormal
4	M_1	N.m/m	0.1	0.20	Lognormal
5	R_x^T	MPa	1500.0	0.20	Lognormal
6	R_x^C	MPa	1500.0	0.20	Lognormal
7	R_y^T	MPa	40.0	0.20	Lognormal
8	R_y^C	MPa	246.0	0.20	Lognormal
9	R_{xy}	MPa	68.0	0.20	Lognormal

Table 2: Statistical properties of random variables.

The limit state function considered here was the Tsai-Wu failure criterion and the stress state at the local axes of the laminated composite plate was carried out with two approaches: (a) using classical theory of composite plates using a closed form solution (Jones, 1999, Daniel & Ishai, 1994) and (b) using a finite element program (200 elements and 121 nodes), which uses the discrete Kirchhoff triangular element (DKT) (Bathe & Batoz, 1980), coupled with a constant stress triangular element (CST). The behavior of the element follows the classical theory of laminates (CTL) (Daniel & Ishai, 1994).

Number of individuals in the population	30
Maximum number of generations	30
Crossover probability	100%
Probability of mutation	1%
Stopping criterion (stand. dev. of individuals of the population)	1.0×10^{-5}
Number of design variables (thickness)	4
Lower limit of the design variables (m)	0.5×10^{-3}
Upper limit of design variables (m)	3.0×10^{-3}
Number of bits of each design variable	16

Table 3: Data input for the genetic algorithm program.

Since the used GA is based on a binary codification, each design variable will present discrete values that depend on the number of bits used for the codification. The resolution for

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each design variable can be calculated using the following expression (Goldberg, 1989):

$$R = \frac{U - L}{2^n - 1} \tag{8}$$

where n is the number of bits given to each design variable, while U and L are, respectively, upper and lower limits of the design variables. In this example the resolution for the minimization of the thickness is R=3.815x10^{-5} m. The search space, which corresponds to the number of thickness combinations, is $(2^{16})^4$=1.84x10^{19} which is unworthy for exhaustive search.

The cost function is the sum of the thickness of each ply and the penalty factor was set as 10^5. Equation (9) shows how the cost function, which depends on the reliability index of each individual, is evaluated.

$$\text{Minimize} \quad (\sum_{i=1}^{4} h_i)[1 + (\beta_{req} - \beta)^2 10^5] \tag{9}$$

where h_i , $i = 1,4$ represents the thickness of each layer.

Figures 6 to 9 show results for the optimal solution (where evolution of the thickness of the different layers and the total thickness of the plate along different generations are presented) for some methods to evaluate the reliability index. The local stress state was obtained using a closed solution for laminated composite rectangular plates, given by Kaw (2006).

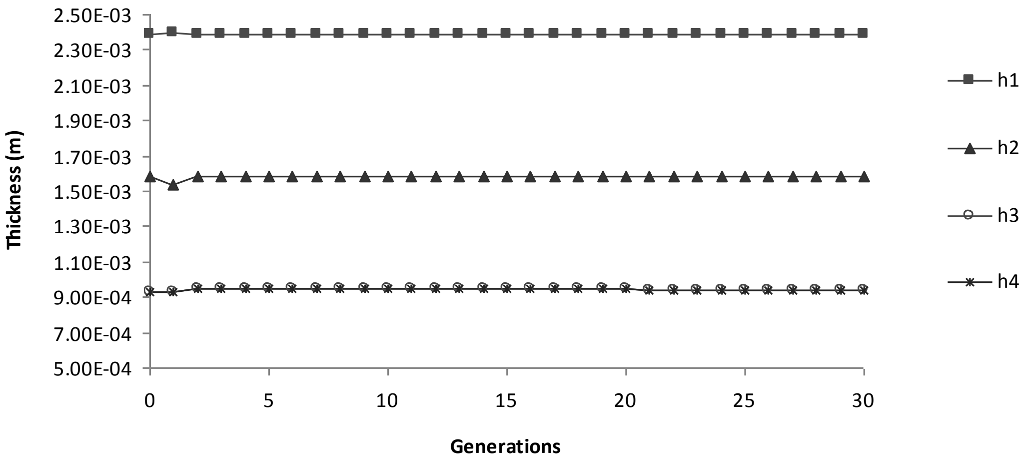

Figure 6: Layer thickness for the optimal solution of the laminated composite plate using Direct Monte Carlo Method for reliability index evaluation (the limit state function is determined analytically).

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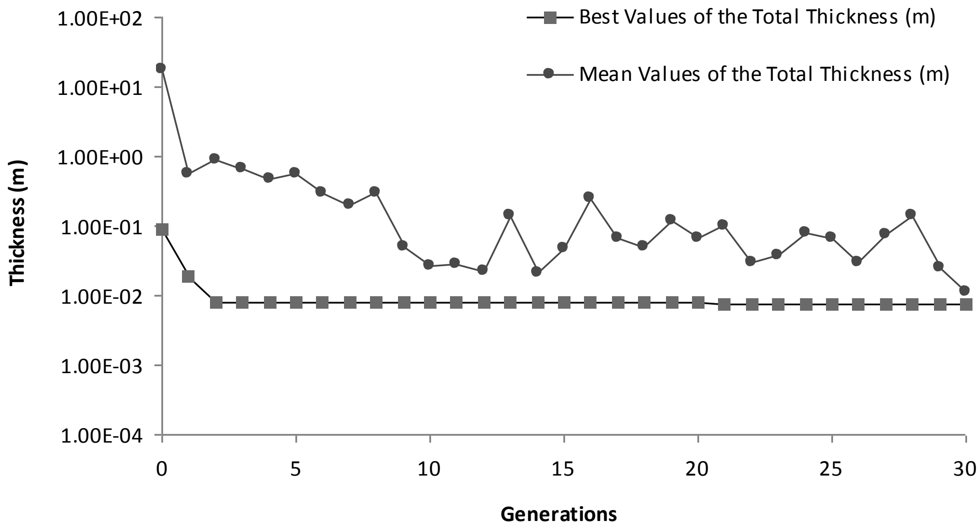

Figure 7: Total thickness of the best individual and mean values of the population's total thickness of the laminated composite plate using Monte Carlo Method for reliability index evaluation (the limit state function is determined analytically).

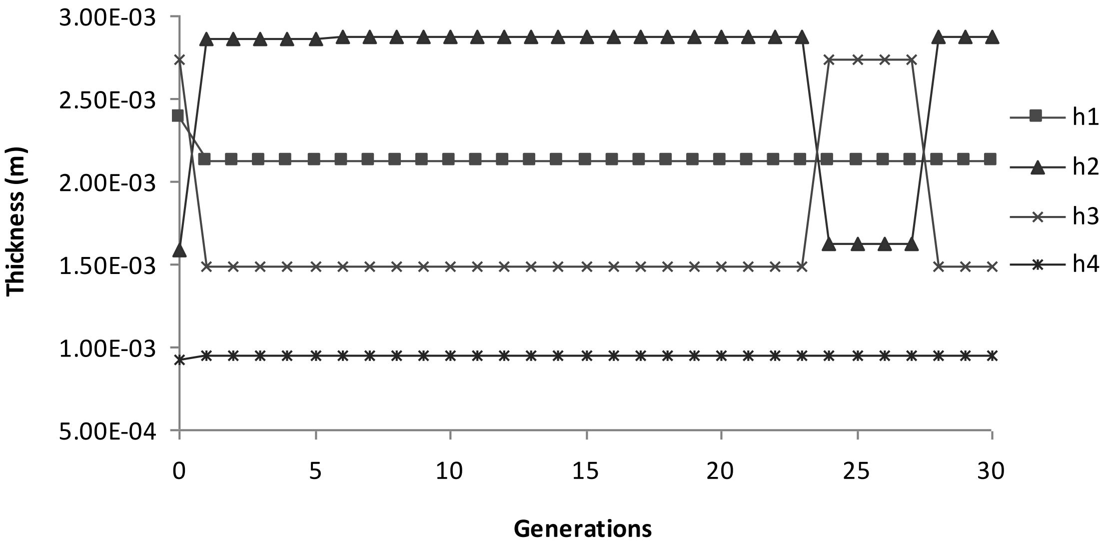

Figure 8: Layer thickness for the optimal solution of the laminated composite plate using FORM to calculate the reliability index (the limit state function is determined analytically).

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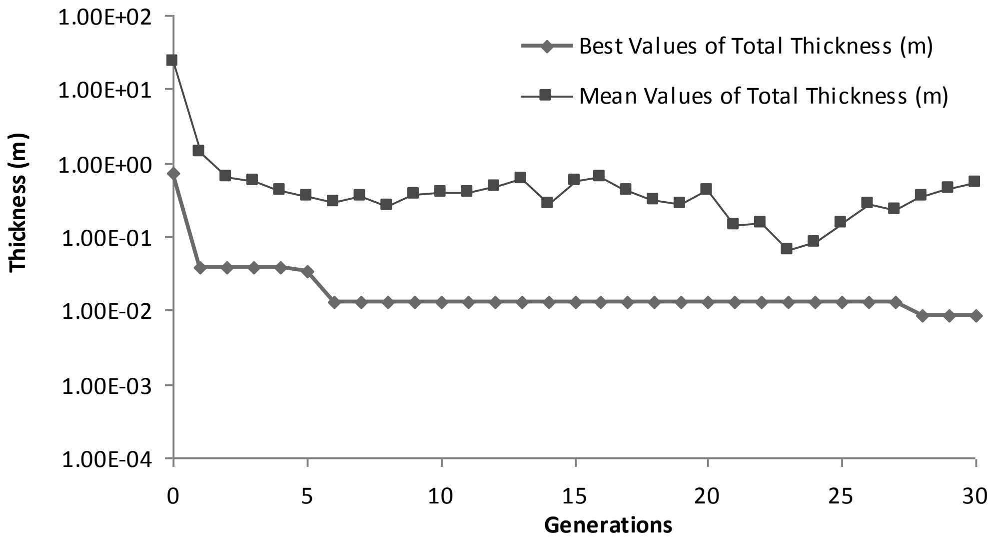

Figure 9: Total thickness of the best individual and mean values of the population's total thickness of the laminated composite plate using FORM to calculate the reliability index (the limit state function is determined analytically).

It can be noticed in the previous figures that the genetic based optimization process converged, in most cases, due to the criterion based on the maximum number of generations instead of diversity criterion, even when optimum value was early found. The diversity parameter (which is the coefficient of variation of the cost function, i.e., the ratio of standard deviation divided by the mean value) used as convergence criterion seems not to be a suitable parameter to indicate convergence. This shows that a more suitable convergence criterion reducing the number of generations and excessive number of simulations must be used. Perhaps the reduction of the heuristic parameter pm (probability of mutation) may reduce the diversity parameter. This is an issue to be investigated in future papers.

The results presented in Figures 7, 9 and 11 allows to observe that the total thickness for initial generations is weighted by the constraint violation (in this case the reliability constraint), which is the way as Penalization Techniques account for constraints (see Eq. 9) explaining values of the total thickness that are higher than the maximum physical total thickness.

Figure 8 and Fig. 10 suggest that there are symmetrical layer configurations (same total thickness and symmetrical inner layers) which have similar values for the reliability index. This may hinder the algorithm in order to reach the minimum cost function.

The other tests (Monte Carlo with Importance Sampling, FORM and FORM-MCP using finite element analysis) show similar results and behavior regarding evolution of design variable and cost function. In the examples where the stress state is obtained by a finite element analysis, the simulations are performed using FORM and modified FORM (FORM-MCP) to obtain the reliability index, since Monte Carlo methods would give a very expensive processing time.

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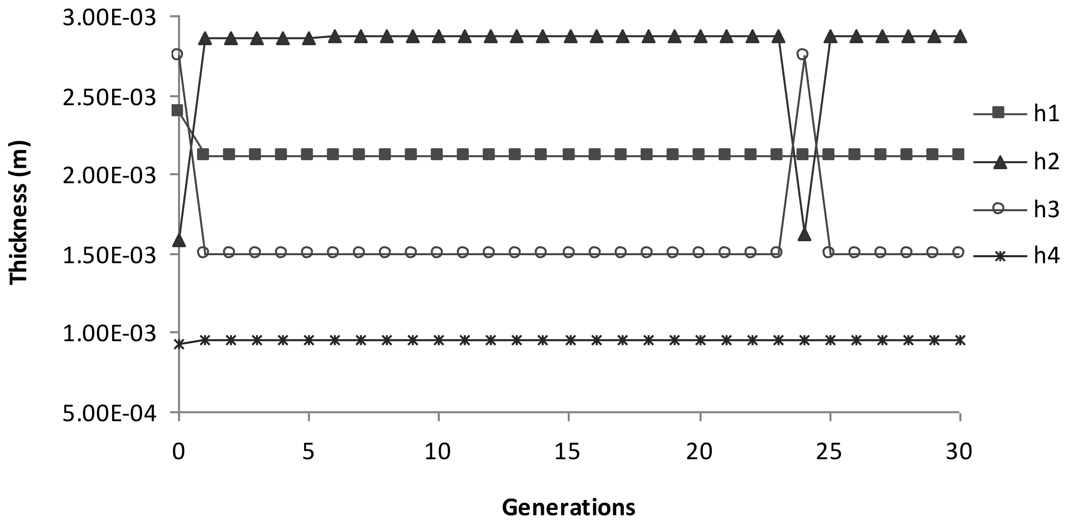

Figure 10: Total and layers thickness for the optimal solution of the laminated composite plate using modified FORM (FORM-MCP) to calculate the reliability index (the limit state function is determined analytically).

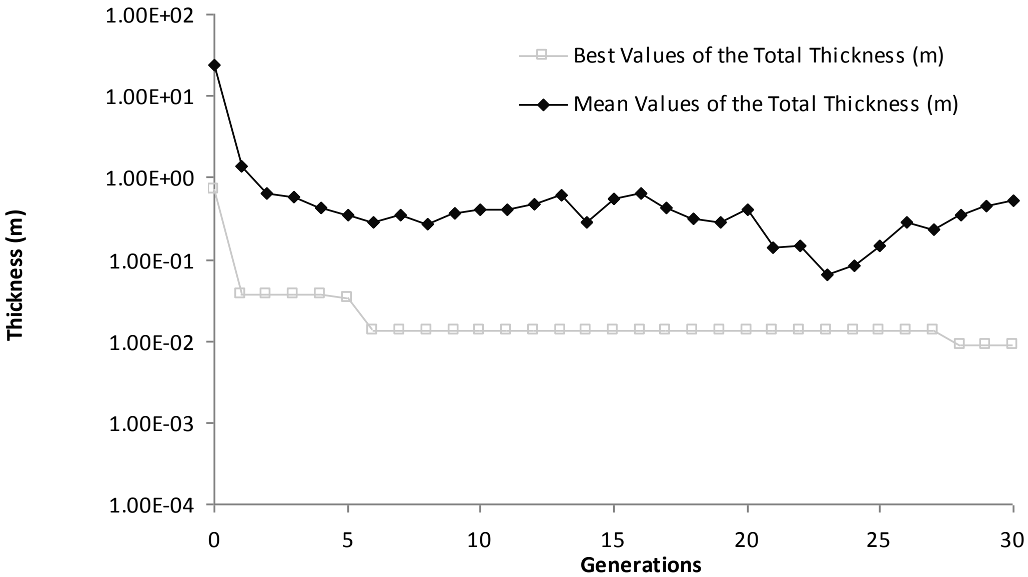

Figure 11: Total thickness of the best individual and mean values of the population's total thickness of the laminated composite plate using modified FORM (FORM-MCP) to calculate the reliability index (the limit state function is determined analytically).

In this example, RBNN and MPNN were also used in order to reduce the processing time spent in the optimization process. The training procedure used here was to train the networks so that they could provide directly the reliability index, from a particular configuration of the laminate (one specific individual of the GA population). The reliability index used for training the neural networks was calculated using FORM (while the value of the limit state function was obtained using a finite element program). A total number of 300 samples, collected according to section 4.3 were used. The network architectures were (4:300:1) for RBNN and (4:10:10:10:1) for MPNN.

Table 4 summarizes all the tests performed in this work and presents a comparison of the computational cost using Artificial Neural Networks and finite elements. The processing time for modified FORM (FORM-MCP) using finite element to evaluate the limit state function

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was considered as the reference for processing time comparisons.

Table 5 presents the relative errors using neural networks with reference to the solution with FORM and finite elements for the design variables.

The results show a drastic reduction in processing time when the optimization is performed using neural networks to simulate the calculation of reliability index. The relative errors are small and do not exceed 3.51% (thickness h_4 using RBNN).

Method	Relative Processing Time (s)	h_1 (m)	h_2 (m)	h_3 (m)	h_4 (m)	h_t (m)	β
Monte Carlo +closed form solution	2.677	2.39×10^{-3}	1.58×10^{-3}	2.74×10^{-3}	9.37×10^{-4}	7.66×10^{-3}	3.000
Monte Carlo with Importance Sampling +closed form solution	8.026×10^{-1}	2.12×10^{-3}	2.87×10^{-3}	1.50×10^{-3}	9.50×10^{-4}	7.30×10^{-3}	2.981
Modified FORM+closed form solution	1.466×10^{-1}	2.12×10^{-3}	2.87×10^{-3}	1.49×10^{-3}	9.50×10^{-4}	7.42×10^{-3}	2.978
FORM+closed form solution	1.796×10^{-3}	2.12×10^{-3}	2.87×10^{-3}	1.49×10^{-3}	9.50×10^{-4}	7.42×10^{-3}	2.978
Modified FORM+FEM	1.000	2.390×10^{-3}	1.580×10^{-3}	2.740×10^{-3}	9.40×10^{-4}	7.650×10^{-3}	2.999
FORM+FEM	9.475×10^{-2}	2.390×10^{-3}	1.580×10^{-3}	2.740×10^{-3}	9.40×10^{-4}	7.65×10^{-3}	2.995
RBNN-training	2.500×10^{-2}						
RBNN-simulation	3.381×10^{-5}	2.377×10^{-3}	1.576×10^{-3}	2.724×10^{-3}	9.075×10^{-4}	7.585×10^{-3}	3.000
MPNN-training	2.260×10^{-2}						
MPNN-simulation	2.305×10^{-5}	2.390×10^{-3}	1.541×10^{-3}	2.745×10^{-3}	9.319×10^{-4}	7.608×10^{-3}	3.000

Table 4: Comparison of processing time using neural networks and finite element for the optimization of the laminated composite plate thickness.

Method	Error h_1 (%)	Error in h_2(%)	Error in h_3(%)	Error in h_4(%)	Error in h_t(%)
RBNN	0.544	0.253	0.584	3.510	0.850
MPNN	0.000	2.468	0.182	0.862	0.549

Table 5: Relative errors of design variables (%) using neural networks and finite elements for thickness optimization of a laminated composite plate.

5.2 Example 2 - Reliability based optimization of the ply angles of the layers on a laminated composite shell with non-linear behavior subjected to an external pressure load

5.2.1. Problem description

In this example, the reliability index using the Finite Element Method (FEM) and Artificial Neural Networks (ANN) of a semi-cylindrical shell with geometric nonlinear behavior is calculated. An external pressure load P = 250000 Pa acts along the outer surface of the structure. The dimensions and boundary conditions, taken from Almeida and Awruch (2009) are shown in Figure 12. The finite element model is composed of 200 DKT elements modified to account for the composite layers with arbitrary ply angles.

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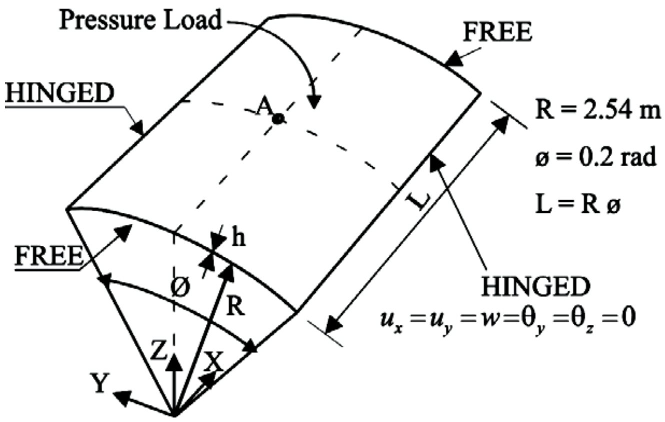

Figure 12: Composite shell under pressure load (Almeida and Awruch, 2009).

The total thickness of the laminated composite shell is 12.6mm, with 28 plies (*h*=0.45mm for each ply) and fiber orientation angle given by $\left[90^0_4, \mp 45^0, 90^0_4, \mp 45^0, 90^0_2\right]_s$ measured with respect to the longitudinal direction of the shell. The material considered is glass-epoxy, with elastic mechanical properties and strengths given by E_1 = 39GPa, E_2 = 8.6 GPa, E_{12} = 3.8 GPa and ν_{12} = 0.28 R^t_x = 1080 MPa, R^c_x = 620 MPa, R^t_y = 39 MPa R^c_y = 128 MPa, R_{xy} = 89MPa. The limit state function is defined by the Tsai-Wu criterion. A typical load-displacement curve for point A, as result of analysis using deterministic values, is shown in Figure 13.

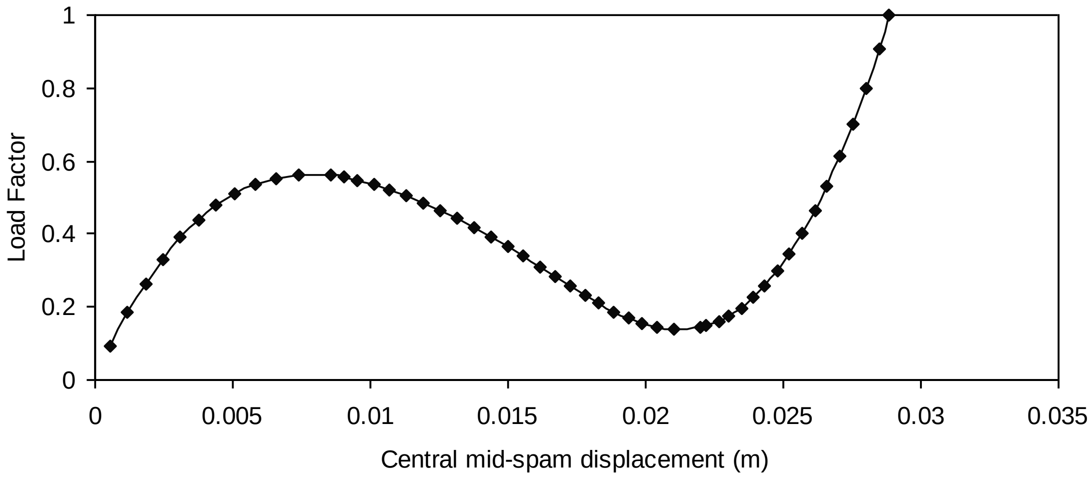

Figure 13: Non-linear load-deflection curve of point A of the hinged semi-cylindrical shell using a deterministic analysis.

In this example, five simple random variables were considered for material properties, as indicated in Table 6, and an isotropic stochastic field of ply thickness was considered using an exponential correlation function given by $R(\xi)=\exp[-(|\xi|/d)^2]$, where ξ means the distance between two finite element centers and *d* is the correlation length. A Normal probability distribution with COV=5% and a correlation length *d*=125mm were adopted as indicated by Sriramula and Chryssanthopoulos (2009) and Young and Ngah (2007).

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Random Variable	Unit	Mean Value	Coeff. of Variation	Distribution Function Type
R_x^t	Pa	1.08x10^9	0.2	Log-Normal
R_x^c	Pa	6.2x10^8	0.2	Log-Normal
R_y^t	Pa	3.9x10^7	0.2	Log-Normal
R_y^c	Pa	1.28x10^8	0.2	Log-Normal
R_{xy}	Pa	8.9x10^7	0.2	Log-Normal
h	mm	0.45	0.05	Normal

Table 6: Statistical parameters for random variables.

Using Monte Carlo numerical simulations, Shaw *et al.*(2010) found a slight correlation between R_x^t and R_y^t of about 0.19 and this was also imposed to the corresponding variables in this work. Correlations values of other strength parameters were not found in the literature, although they may easily be included, if available.

In the optimization of the fiber orientation angles, for practical reasons (Almeida and Awruch, 2009), symmetry of the laminate arrangement was assumed. The same random variables, i.e., the five strength parameters of the Tsai-Wu failure surface with some correlation and a spatial stochastic field for the ply thickness were also assumed. Thus, this example is an optimization problem with seven design variables $\left[\pm\theta_1,\pm\theta_2,\mp\theta_3,\pm\theta_4,\pm\theta_5,\mp\theta_6,\pm\theta_7\right]_s$. These design variables could assume, for constructive practical reasons, discrete values, and the following values were adopted: -45^o, 0^0, 45^o and 90^o. Thus, using the genetic algorithm described in this work two bits per design variable were defined, so that for the fiber orientation angles there are a binary encoding 00, 01, 10, 11, giving 4^7 = 16384 fiber orientation angles combinations.

The cylindrical shell of twenty-eight layers was previously analyzed and the failure probability using a Tsai-Wu criterion as ultimate limit state was evaluated. In this case, the reliability index of the structure against ultimate failure is β = 2.27 using Monte Carlo method with Importance Sampling and the Finite Element Method for the limit state function evaluation. This index was also confirmed by FORM. The configuration of the laminate, based in a deterministic design is $\left[90^0_2,90^0_2,\pm45^0,90^0_2,90^0_2,\pm45^0,90^0_2\right]_s$.

5.2.2 Optimization of ply angles using the Finite Element Method

Due to processing time required for the analysis, optimization of ply angles, imposing as constraint a target value of the reliability index equal to β = 5.0, was performed using FORM for reliability assessment. The finite element mesh as well as the parameters for the nonlinear analysis is the same used in the previous section, where a deterministic analysis was carried out. The parameters used by the genetic algorithm are shown in Table 7.

Results obtained using a Finite Element Analysis (FEA) to evaluate the limit state function are given in Figure 14 and Figure 15.

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No. of design variables (*n*)	7
Discrete values Fo design variables	-45°, 0°, 45° and 90°
Design variable's No. of bits (*b*)	2
Probability of Mutation (p_M)	1%
Probability of **C**rossover (p_c)	90%
Population size (*npop*)	300
Maximum number of grnerations (*ngen*)	100
Cost Function to be minimized (*f*)	$f = c\,\lvert\beta - 5\rvert$
Stopping criterion by diversity of individual's cost function (COV= σ_f / μ_f)	5%
Penalty coefficient *c*	100

Table 7: Genetic Algorithm (GA) parameters.

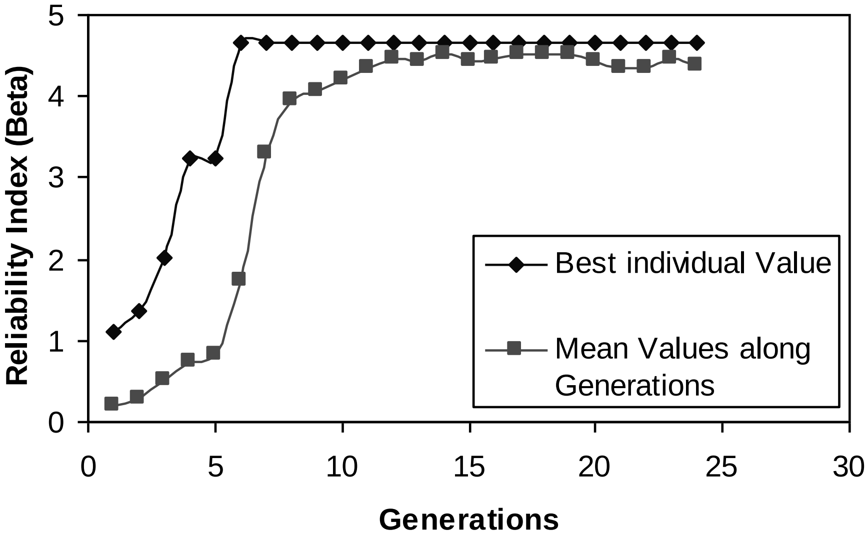

Figure 14: Reliability index for the best individual and mean values along generations using finite elements for limit state function evaluation.

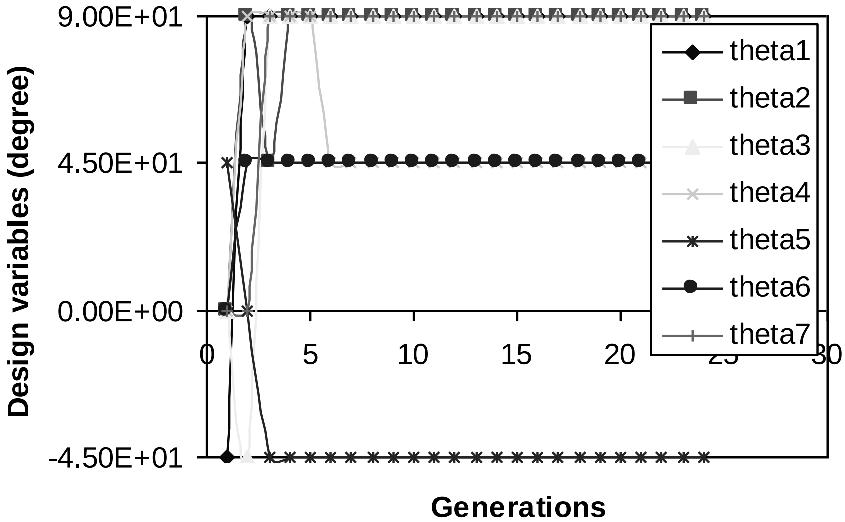

Figure 15: Reliability index of the best individual and mean values along generations using finite elements for limit state function evaluation.

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The best combination of fiber orientation angles that provides reliability index closer to the desired value is $\left[90^0_2, 90^0_2, 90^0_2, \pm 45^0, \pm 45^0, \pm 45^0, 90^0_2\right]_s$ and the reliability index value was β = 4.652.

Obviously, in this case, where design variables are discrete and cannot take any arbitrary value, the corresponding reliability index may not reach the required value, being the result of the optimization process the combination of angles that most closely approximates the constraint value of the reliability index. Therefore, it is possible, keeping the same number of layers of the laminated composite structure and changing only their fiber orientation angles, to increase the design reliability index from β=2.27 to β=4.652, which is a desirable situation, since there are not an additional production cost of the new laminated composite material.

5.2.3 Optimization of the ply angles using Artificial Neural Networks

In this section, Artificial Neural Networks (ANN) are trained to substitute the application of the finite element method only in the reliability analysis. Thus, for a given combination of fiber orientation angles, the ANN is trained to help in the evaluation of the corresponding reliability index. The architecture of the neural network used here has seven inputs (ply angles) and one output (reliability index). In the cases of Multilayer Perceptron Neural Network (MPNN) and Radial Basis Neural Network (RBNN) with architectures characterized by (7:15:15:15:1) and (7:120:1), respectively, were enough adopted for training process.

The parameters used in neural networks, such as learning rate, tolerance for convergence, types of activation function, momentum, etc. are the same used in the previous example, changing only the network architecture. The chosen training process consisted in the generation of 300 uniformly distributed samples where the search space is composed by 16384 combinations. Each of 300 samples (combinations of fiber orientation angles) the reliability index using FORM was evaluated. This stage is the most time consuming step in the analysis when ANN are employed, since several finite element analysis are necessary to generate samples. Thus, the samples were used for training and then the genetic algorithm was used to optimize the fiber orientation angles using the trained ANN. Optimization results by genetic algorithms using trained RBNN are presented in Figure 16.

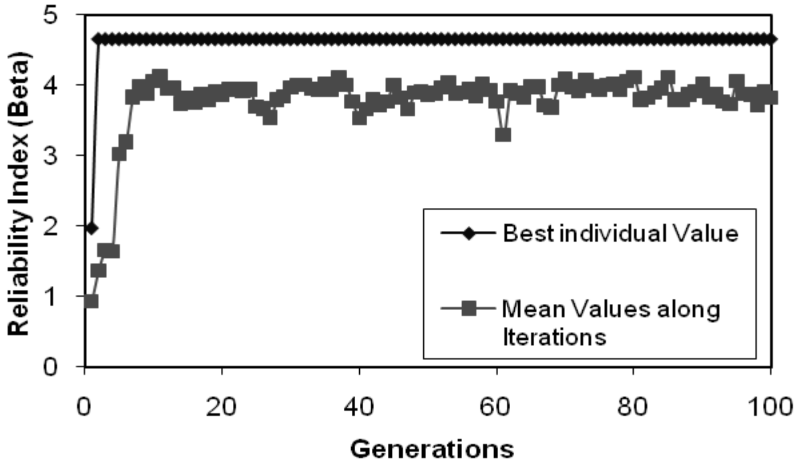

Figure 16: Reliability index history for the best individual and mean values along generations using Radial Basis Neural Network.

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In both cases, the optimum value of the combination of the fiber orientation angles was exactly the same which was found with the optimization using the finite element method, i.e. $\left[90^0_2, 90^0_2, 90^0_2, \pm45^0, \pm45^0, \pm45^0, 90^0_2\right]_s$ values of reliability index were β=4.65 for the RBNN and β=4.81 for the MPNN .

5.2.4 Comparisons regarding processing time

Table 8 shows the relative processing times, design variable values and reliability indexes obtained in the optimization process using FEA and ANN. It can be noticed that both types of ANN give considerable time savings on computer processing time. For both ANN, most of the processing time is spent in the training process, since the processing time spent by the ANN to calculate results is very small when compared with a complete FEA.
A small difference in the reliability index values using trained ANN with respect to those obtained using complete FEA (regardless the same optimum design variables) may be explained by a lack of fit of the ANN with the training data or by the small number of samples used in the training process.

Method	Relative Processing Time	θ_1 (º)	θ_2 (º)	θ_3 (º)	θ_4 (º)	θ_5 (º)	θ_6 (º)	θ_7 (º)	β
GA+FORM+FEM	1.00	90	90	90	45	45	45	90	4.65
RBNN –training (300 samples)	$2.56x10^{-1}$								
GA+FORM+ RBNN	$1.45x10^{-3}$	90	90	90	45	45	45	90	4.65
MPNN –training (300 samples)	$8.75X10^{-1}$								
GA+FORM+MPNN	$0.42x10^{-4}$	90	90	90	45	45	45	90	4.81

Table 8: Comparison of processing times for ply orientation optimization using the Finite Element Analysis and Artificial Neural Networks.

6 FINAL REMARKS

Some initial results were presented in this work dealing with reliability based optimization for structural problems involving laminated composite materials. A methodology to reduce the processing time using trained ANN when dealing with reliability based design optimizations was proposed.

For the structural optimization, a Genetic Algorithm (GA) was used. GA are very suitable tools to obtain global optimal solution in problems where laminated composite materials are employed, because these materials handle with discrete variables (such as fiber orientation angles and number of layers) and multiple local optima are probable when dealing with reliability constraints. In some examples it was noticed that the convergence criterion used by the algorithm to stop the optimization process needs to be investigated since maximum number of iterations prevailed with respect to the diversity criterion (based on the coefficient of variation) leading to excessive iterations.

To evaluate the reliability index four classical methods were used: Standard or Direct Monte Carlo Method (MC), Monte Carlo Method with Importance Sampling (MCIS), First Order Reliability Method (FORM) and FORM with Multiple Check Points (FORM-MCP). In order to assess accuracy in the analysis, the first example uses both the Finite Element Method (FEM) and closed solutions for laminated composite plates to evaluate the limit state

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function (the Tsai-Wu failure criterion was adopted) regarding the reliability index. In optimization problems, where the reliability index is used as a constraint, a complete finite element analysis (FEA) is very expensive in terms of computer processing time (especially if MC or even MCIS are employed). As an alternative to save computer-processing time, trained Artificial Neural Networks (ANN) were used to evaluate the reliability index for the examples presented. Two types of ANN were used: Multilayer Perceptron Neural Network (MPNN) and Radial Basis Neural Network (RBNN). Their efficiency depends mainly of the chosen architecture and training process. In this work, both ANN reduced de computer processing time and the corresponding errors with respect to a complete FEA were very small.

In the reliability-based optimization of the ply angles of the layers on a laminated composite shell with non-linear behavior, it can be noticed that the reliability index 5.0 was not attained since ply orientation has discrete values, resulting in a composite shell with reliability index about 4.65. Nevertheless it was possible, keeping the same number of layers of the laminated composite structure and changing only ply orientation angles, to increase the reliability index of the original design from β=2.27 to β=4.65. This is a highly desirable situation, since there are not additional production costs of the new laminated composite material.

In the last example, a small difference in the obtained reliability index value using trained ANN with respect to those obtained using complete FEA may be explained by a lack of fit of the ANN with the training data, indicating that the training process was not completed.

Future works would involve more complex problems with other limit state functions, such as delamination and hygrothermal effects. Improvements of the GA and the training process, as well as a parallel algorithm to solve large real problems could also be implemented.

REFERENCES

Ang, A. A-H., Tang, W. H., Probability Concepts in Engineering Planning and Design, Volume II: Decision, Risk and Reliability, 1984, Wiley & Sons.

Almeida, F. S., Awruch, A. M., Design optimization of composite laminated structures using genetic algorithms and finite element analysis, Composite Structures, 2009,88(3): 443-454. doi:10.1016/j.compstruct.2008.05.004

Bathe, K. J., Ho, L., Batoz, J., A study of Three-node Triangular Plate Bending Elements. Internacional Journal for Numerical Methods in Engineering, 1980,15: 1771 – 1812.

Cryssanthopoulos, M. K., Poggi, C., Probabilistic imperfection sensitivity analysis of axially compressed composite cylinders, Engineering Structures, 1995, 17(6): 398-406.

Daniel, I. M., Ishai. O., Engineering Mechanics of Composite Materials, Oxford University Press, 1994.

Frangopol, D. M., Recek, S., Reliability of fiber-reinforced composite laminate plates, Probabilistic Engineering Mechanics, 2003, 18: 119-137.

Goldberg, D. E., Genetic Algorithms in search, optimization and machine learning, R. Addison-Wesley Publishing, 1989, MA.

Gomes, H. M.; Awruch, A. M., Comparisons of Different Methods for Structural Reliability Analysis. Structural Safety, 2004, 26: 49-67. doi:10.1016/S0167-4730(03)00022-5.

Gomes, H. M., Awruch, A. M., Reliability of Reinforced Concrete Structures Using Stochastic Finite Elements, Engineering Computations, 2002, 19(7):764-786.

Gomes, H. M., Awruch, A. M., Reliability Analysis of Concrete Structures with Neural Networks and Response Surfaces, Engineering Computations, 2005, 22(1):110-128.

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Haldar, A., Mahadevan, S., Probability, Reliability and Statistical Methods in Engineering Design, 1999, Wiley & Sons.

Haykin, S., Artificial Neural Networks: a comprehensive foundation, New Jersey, USA: Prentice Hall, 1999.

Kaw, A. K. Mechanics of Composites Materials, 2nd Ed., CRC Taylor & Francis Group, Boca Raton, 2006, 457p.

Kam, T.Y., Chang, E.S., Reliability formulation for composite laminates subjected to fist-ply failure, Composite Structures, 1997, 38(1-4):447-452.

Melchers, R. E. Structural Reliability Analysis and Prediction, Wiley; 2nd edition, 1999, 456p.

Miki, M., 1986, Optimum Design of Fibrous Laminated Composite Plates Subject to Axial Composition, Composites ´86: Recent Advances in Japan and the United States, edited by K. Kawata, S. Umekawa e A. Kobayashi, Japan Society for Composite Materials, Tokio, 673 – 680.

Miki, M. Yoshiada, M. Tetsou, T. and Shaoewn, S., 1980, Reliability of Unidirecional Fibrous Composites, American Institute of Aeronautics and Astronautics Journal, 1980, 28(11):1980-1986.

Muc, A., Gurba, W., Genetic algorithms and finite element analysis in optimization of composite structures, Composite Structures, 2001. 54(2-3): 275-281.

Murotsu, Y., Miki, M., Shao, S., Reliability design of fiber reinforced composites. Structural Safety, 1994, 15: 35-49.

Naik, G. N., Gopalakrishnan, S., Ganguli, R., Design optimization of composites using genetic algorithms and failure mechanism based failure criterion, Composite Structures, 2008, 83(4):354-367.

Ngah, M. F., Young, A., Application of the spectral stochastic finite element method for performance prediction of composite structures. Composite Structures, 2007, 78(3):447-56.

Shao, S., Murotsu, Y., 1992, Reliability evaluation methods for systems with complex limit states. In: R. Rackwitz and P. Thoft-Christensen (Eds.), Reliability and Optimization of Strctural Systems ’91, Springer , Berlin, pp.325-338.

Sriramula, S., Cryssanthopoulos, M. K., Quantification of uncertainty modeling in stochastic analysis of FRP composites, Composites: Part A, 2009, 40:1673-1684.

Shaw, A., Sriramula, S., Gosling, P. D., Cryssanthopoulos, M. K., A critical reliability evaluation of fibre reinforced composite materials based on probabilistic micro and macro-mechanical analysis, Composites: Part B, 2010, 41:446-453.

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