Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXXII, págs. 3563-3574 (artículo completo)
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Mendoza, Argentina, 19-22 Noviembre 2013

VIBRATIONS OF A FRAME STRUCTURE WITH A CRACK

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Keywords: Frame structure, crack, vibrations, elastic hinge, damage.

Abstract. The study of the dynamic properties of frames is very important in structural design, as they are the cornerstone for many resistant structures. A crack on a structural member introduces a local flexibility which is a function of the crack depth. This new flexibility condition changes the dynamic behavior of the structure. The knowledge of the influence of the crack on the characteristic dynamic parameters of the frame makes it possible to determine both the position of the crack and its magnitude in damaged frames. In this paper, the crack is modeled by means of an elastically restrained hinge using Chondros' formulation while the analytical analysis is based on variational calculus and the Euler-Bernoulli beam theory to describe the transversal displacements of the frame members. In the laboratory a device was built to measure experimentally the natural frequencies of steel frames. The first ten natural frequencies of in plane vibration of cracked L-frames are obtained, different locations and depths of the crack are considered. The experimentally measured frequencies are compared with the frequencies obtained from the proposed analytical solution, the finite element method and with values reported in previous studies published on the subject by other authors.

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1 INTRODUCTION

The problem of the influence of a crack in a welded joint on the dynamic behavior of a structural member has been studied in a thorough paper by Chondros and Dimarogonas, 1980.

The static and dynamic analysis of beams with single or multiple concentrated damages, produced by cracks, has received an increasing interest in recent years.

A very complete description of the state of the art in the field with the mention and description of the most important work was done by Caddemi and Morassi, 2013.

There it is explained that generally, it is assumed that the amplitude of the deformation is enough to maintain the crack always open, this model offers the great advantage to be linear and, therefore, it leads to efficient formulations for solving both static and dynamic problems.

From the first studies, it became clear that localized damage produces a local reduction in the stiffness of the beam (Thomson, 1943). Many models have been proposed in the literature to describe open cracks on beams, the flexibility modeling of cracks is quite common (Adams et al., 1978). In the case of beams under plane flexural deformation, a crack is modeled by inserting a massless, rotational elastic spring at the damaged cross section (Gudmnundson, 1983; Sinha et al., 2002).

The aim of this study is to verify, through an experimental device, the accuracy of the representation of the crack by a rotational spring.

2 STRUCTURAL MODEL AND ANALYTICAL SOLUTION

We deal with the vibration of L-frames assuming an internal crack in different positions of the horizontal part of the frame.

The two parts of the L-shaped geometry are joined at right angle, with the end of one of them clamped and the end of the other elastically restrained. Figure 1 depicts the structure under study.

The position of the crack is defined by the coordinate l_2 and locally affects the flexural stiffness of the L-frame. It is modeled as a massless, rotational elastic spring at the damaged cross section connecting the two adjacent segments of the beam (Gudmnundson, 1983; Sinha et al., 2002)

The magnitude adopted for the flexibility constant of the equivalent spring (α_C), is obtained by means of the expression proposed by Chondros et al., 1998 which was found with fracture mechanics methods.

$$\beta_C = \frac{6\pi(1-\nu^2)h}{EI}\phi_C(z); \tag{1}$$

with

$$\phi_C(z) = 0.6272\, z^2 - 1.04533 z^3 + 4.5948 z^4 - 9.973 z^5 + 20.2948 z^6 - 33.0351 z^7 + 47.1063 z^8 - $$
$$- 40.7556 z^9 + 19.6 z^{10};$$

where $z = h_C / h$, h is the height of the cross section and h_C is the depth of the crack.

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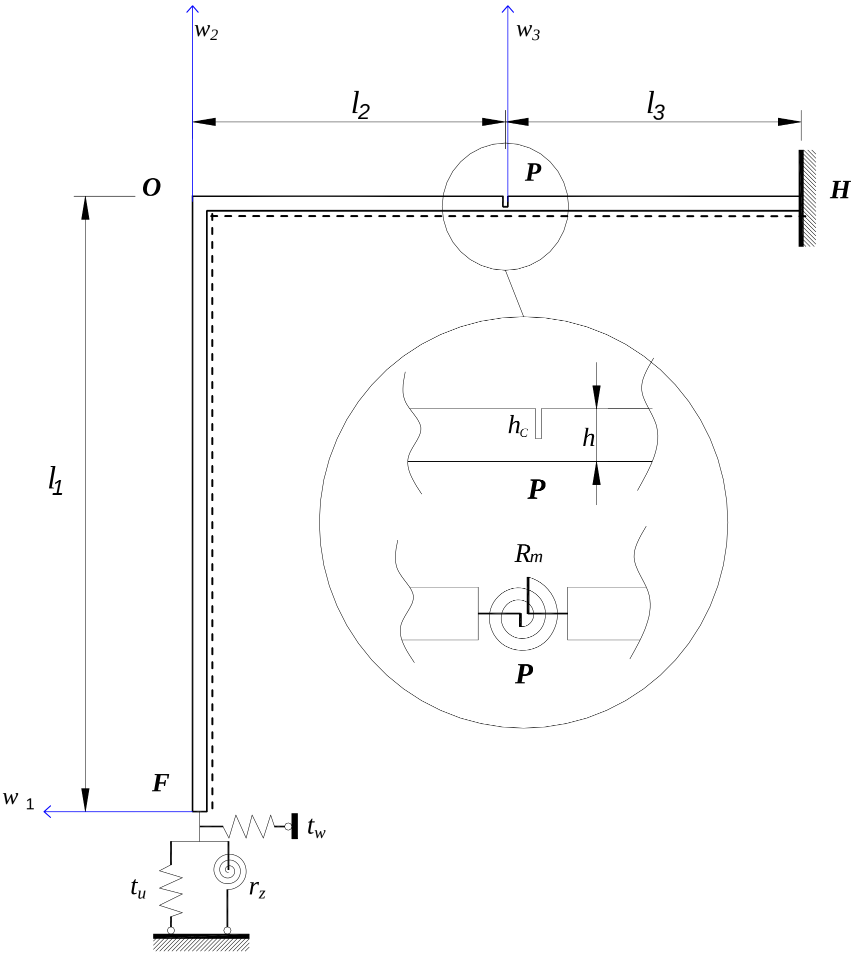

Figure 1: L-frame structure

The frame-structure has three beam members: the beam *FO* of length l_1, the beam *OP* of length l_2 and the beam *PH* of length l_3; each of them having uniform properties. The beams are modeled using the Euler-Bernoulli beam theory.

The external end *H* is a classical clamped support and the external end *F* is supported by two translational springs of stiffness t_w and t_u and a rotational spring of stiffness r_z. At point *P*, and modeling the crack, there is an internal hinge elastically restrained against rotation between beams 2, *OP*, and 3, *PH*, this semi-rigid connection is materialized by a rotational spring of stiffness:

$$r_m = {}^{1}\!/_{\beta_C} \tag{2}$$

The flexural rigidity, the mass density, the length and the area of the cross section of each beam are E_iI_i, ρ_i, l_i and A_i, with i=1, 2, 3.

Three co-ordinate systems are located as they are shown in Fig. 1, and its origins are taken to be at points *F*, *O* and *P* of each beam. At abscissa x_i ($0 \leq x_i \leq l_i$), w_i is the transverse displacement of the beam *i*, $\theta_i = \partial w_i / \partial x_i$ is the section rotation and u_i is the axial displacement at any time *t*. The deformation of a beam in *x* direction is not taken into account, since the beams are considered infinitely rigid in the axial direction.

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The sign convention used for the positive shear force spins an element clockwise (up on the left, and down on the right). Likewise the normal convention for a positive bending moment elongates the reference fiber of the beam indicated by the dotted line. Fig. 2 shows the sign convention to be employed.

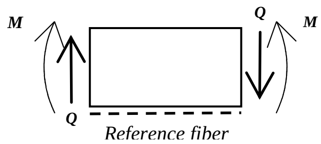

Figure 2: Sign convention for positive shear force (Q) and bending moment (M)

For free vibration, the bending moment and the shear force expressions are:

$$Q_i\left(x_i,t\right)=E_iI_i\frac{\partial^3 w_i\left(x_i,t\right)}{\partial x_i^3};\; M_i\left(x_i,t\right)=E_iI_i\frac{\partial^2 w_i\left(x_i,t\right)}{\partial x_i^2} \quad (3)$$

To express equations in dimensionless form, the non-dimensional parameter is introduced:

$$X_i=x_i/l_i;\text{ with } X_i\in[0,1]\;\forall i=1,2,3. \quad (4)$$

The displacements u_i and w_i, and θ_i may be expressed in terms of the dimensionless coordinates as follows

$$W_i\left(X_i,t\right)=w_i\left(x_i,t\right)/l_i;\theta_i\left(X_i,t\right)=\frac{\partial W_i\left(X_i,t\right)}{\partial X_i}=\frac{\partial w_i\left(x_i,t\right)}{\partial x_i};U_i\left(X_i,t\right)=u_i\left(x_i,t\right)/l_i. \quad (5)$$

The characteristics of beam 1 are used as ´reference´:

$$EI=E_1I_1,\; \rho A=\rho_1A_1,\; l=l_1, \quad (6)$$

to define the ratios:

$$\nu_{EI_i}=\frac{E_iI_i}{EI},\; \nu_{\rho A_i}=\frac{\rho_iA_i}{\rho A}; \quad (7)$$

and the dimensionless spring stiffness:

$$T_w=t_w\frac{l^3}{EI},\; T_u=t_u\frac{l^3}{EI},\; R_z=r_z\frac{l}{EI},\; R_m=r_m\frac{l}{EI}. \quad (8)$$

Finally the dimensionless frequency coefficient is expressed as:

$$\lambda^4=l^4\omega^2\,\rho A/EI, \quad (9)$$

where ω is the circular natural frequency of the vibrating system in radians per second.

Under the described conditions and applying the technique of variational calculus (Grossi,

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2010), (Ratazzi et al., 2012) the governing differential equations of the problem and the boundary and continuity conditions are:

$$\frac{\partial^4 W_1}{\partial X_1{}^4}(X_1,t)+k_1^4\frac{\partial^2 W_1}{\partial t^2}(X_1,t)=0\,; \tag{9}$$

$$\frac{\partial^4 W_2}{\partial X_2{}^4}(X_2,t)+k_2^4\frac{\partial^2 W_2}{\partial t^2}(X_2,t)=0\,; \tag{10}$$

$$\frac{\partial^4 W_3}{\partial X_3{}^4}(X_3,t)+k_3^4\frac{\partial^2 W_3}{\partial t^2}(X_3,t)=0\,; \tag{11}$$

with $k_i^4=\dfrac{\rho_i A_i}{E_i I_i}l_i^4\,, i=1,2,3$.

$$v_{l_1}W_1(1,t)=0; \tag{12}$$

$$v_{l_2}W_2(1,t)=v_{l_3}W_3(0,t); \tag{13}$$

$$\frac{\partial W_1}{\partial X_1}(1,t)=\frac{\partial W_2}{\partial X_2}(0,t); \tag{14}$$

$$\frac{v_{EI_1}}{v_{l_1}}\frac{\partial^2 W_1}{\partial X_1^2}(1,t)=\frac{v_{EI_2}}{v_{l_2}}\frac{\partial^2 W_2}{\partial X_2^2}(0,t); \tag{15}$$

$$\frac{v_{EI_2}}{v_{l_2}}\frac{\partial^2 W_2}{\partial X_2^2}(1,t)-R_m\left(\frac{\partial W_3(0,t)}{\partial X_3}-\frac{\partial W_2(1,t)}{\partial X_2}\right)=0; \tag{16}$$

$$\frac{v_{EI_3}}{v_{l_3}}\frac{\partial^2 W_3}{\partial X_3^2}(0,t)-R_m\left(\frac{\partial W_3(0,t)}{\partial X_3}-\frac{\partial W_2(1,t)}{\partial X_2}\right)=0; \tag{17}$$

$$\frac{v_{EI_1}}{\left(v_{l_1}\right)^2}\frac{\partial^3 W_1}{\partial X_1^3}(0,t)+T_w\,v_{l_1}\,W_1(0,t)=0; \tag{18}$$

$$\frac{v_{EI_2}}{v_{l_2}^2}\frac{\partial^3 W_2}{\partial X_2^3}(1,t)-\frac{v_{EI_3}}{v^2_{l_3}}\frac{\partial^3 W_3}{\partial X_3^3}(0,t)=0 \tag{19}$$

$$\frac{v_{EI_2}}{\left(v_{l_2}\right)^2}\frac{\partial^3 W_2}{\partial X_2^3}(0,t)+T_u\,v_{l_2}W_2(0,t)-\frac{l^4}{EI}\rho_1 A_1 v_{l_1}v_{l_2}\frac{\partial^2 W_2}{\partial t^2}(0,t)=0; \tag{20}$$

$$\frac{v_{EI_1}}{v_{l_1}}\frac{\partial^2 W_1}{\partial X_1^2}(0,t)-R_z\left(\frac{\partial W_1}{\partial X_1}(0,t)\right)=0; \tag{21}$$

$$v_{l_3}W_3(1,t)=0\,; \tag{22}$$

$$\frac{\partial W_3}{\partial X_3}(1,t)=0. \tag{23}$$

The last term in expression (20) is due to the rigid body vertical translation of beam 1 of length l_1, which is a consequence of assuming infinity axial rigidity.

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Using the well-known separation of variables method (Grossi, 2010), solution of Eqs. (9) to (11), free vibrations of the system can be expressed in the form.

$$W_1(X_1,t)=\sum_{n=1}^{\infty}W_{1n}(X_1)\,e^{i\omega t}; \tag{24}$$

$$W_2(X_2,t)=\sum_{n=1}^{\infty}W_{2n}(X_2)\,e^{i\omega t}; \tag{25}$$

$$W_3(X_3,t)=\sum_{n=1}^{\infty}W_{3n}(X_3)\,e^{i\omega t}. \tag{26}$$

The functions W_{1n}, W_{2n} and W_{3n} represent the corresponding transverse modes of natural vibration of each beam member and are given by

$$W_{1n}(X_1)=C_1\cosh(\lambda_n\alpha_1X_1)+C_2 senh(\lambda_n\alpha_1X_1)+C_3\cos(\lambda_n\alpha_1X_1)+C_4 sen(\lambda_n\alpha_1X_1); \tag{27}$$

$$W_{2n}(X_2)=C_5\cosh(\lambda_n\alpha_2X_2)+C_6 senh(\lambda_n\alpha_2X_2)+C_7\cos(\lambda_n\alpha_2X_2)+C_8 sen(\lambda_n\alpha_2X_2); \tag{28}$$

$$W_{3n}(X_3)=C_9\cosh(\lambda_n\alpha_3X_3)+C_{10} senh(\lambda_n\alpha_3X_3)+C_{11}\cos(\lambda_n\alpha_3X_3)+C_{12} sen(\lambda_n\alpha_3X_3); \tag{29}$$

where $\alpha_i=\nu_{l_i}\sqrt[4]{\nu_{\rho A_i}/\nu_{EI_i}}$ is a mechanical and geometrical parameter, with $i=1,2,3$; $\lambda_n=\sqrt[4]{l^4\omega_n^2\,\rho A/(EI)}$ is the dimensionless frequency coefficient of mode of vibration n and C_1, C_2,…, C_{12} are arbitrary constants to be determined.

Replacing Eq. (27), (28) and (29) in Eq. (24), (25) and (26); and these ones in Eq. (12) to (23) a linear system of equations in the unknown constants C_1, C_2,…, C_{12} is obtained.

For a non-trivial solution to exist the determinant of the coefficient matrix in the linear system of equations should be equal to zero and the roots of the transcendental frequency equation are the dimensionless frequency coefficients λ_n of the mechanical system in Figure 1.

The results were determined by using the Mathematica software (Mathematica, 2012) with at least five significant figures. (They are indicated as "*Analytical*" in Table 1 and "*Analytical (Eigenvalue)*" in Tables 2 to 5).

3 EXPERIMENTAL DEVICE

The frame was modeled with a rod of steel 5/8 "× 1/8" (b=15.875mm, h= 3.175mm).

Both members of the frame have the same length: 446 mm, cross sectional area and material properties:

$$\nu_{\rho A_i}=1;\ \nu_{EI_i}=1;\ \ \forall\ \ i=1,2,3;\ \nu_{l_1}=2\nu_{l_2}=2\nu_{l_3}=1.$$

The constructed model was tested on a steel base composed of UPN 100 and linked to it by two welded presses (Fig. 3). According to the mass and inertia differences between the structure and the base, the outer connections can be assumed as two clamped edges (Fig. 4).

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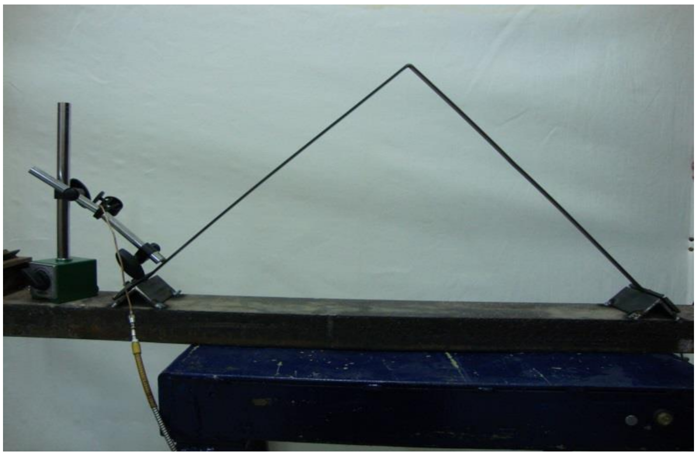

Figure 3: Experimental device: Clamped frame and proximiter

Figure 4: Clamped edge

The crack was modeled with a thickness of 1mm using a saw (Fig. 5). All precautions were taken so that the cut is made smoothly and its depth is uniform: A piece of hard steel was employed, with a gap where the beam is embedded up to the desired depth. (Fig. 6 a, b)

Figure 5: Crack in the strip

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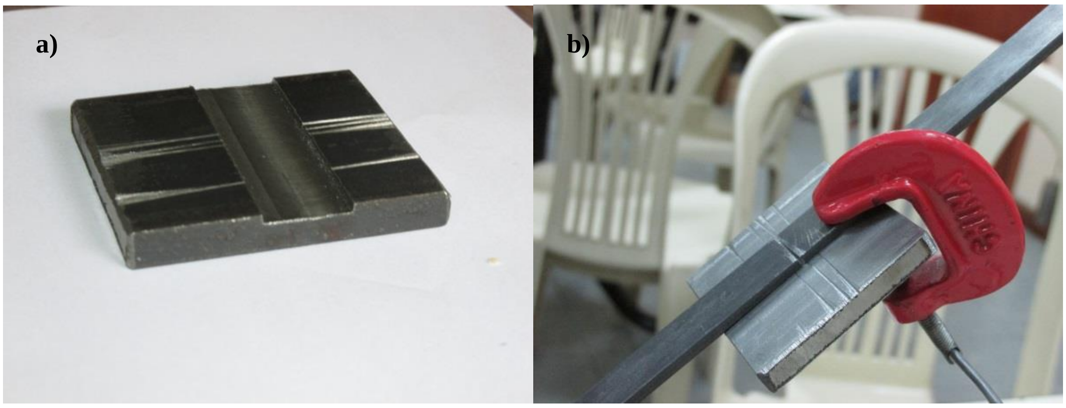

Figure 6: Auxiliary device to model the crack

In order to determine the mechanical parameters of the material of the strip was taken as reference a value widely verified in solid mechanics: the fundamental frequency of a cantilever beam of length 417 mm. built with the strap. It is known that the corresponding eigenvalue is 1.8751

The measured frequency: 14.85 *Hz*, then:

$$f=\frac{1}{2\pi}\frac{\lambda^2}{l^2}\sqrt{\frac{EI}{\rho A}};\ 14.85Hz=\frac{1}{2\pi}\frac{(1.8751)^2}{(0.417m)^2}\sqrt{\frac{Eh^2}{12\rho}}\ \Rightarrow\ \sqrt{\frac{E}{\rho}}=5034.7\frac{m}{seg}. \quad (30)$$

This value is employed in the numerical determinations.

4 FINITE ELEMENT METHOD

Numerical examples are solved by means of the finite element method, using the software ALGOR 23.1 (ALGOR software, 2009). The column and the beam are divided into 100 beams elements respectively, each beam element with three degrees of freedom.

The crack was modeled by a very small beam element, its length is equal to the width of the cut (1 mm) and height equal to the portion of the material that is not affected by cutting.

5 NUMERICAL RESULTS

Table 1 presents the first ten coefficients of natural frequency of vibration of a clamped-clamped frame, without internal hinge. The values obtained by means of the analytical approach are in very good concordance from an engineering viewpoint with those obtained using the finite element method and with particular cases available in the literature. Experimental data show a striking agreement with aforementioned values.

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i	1	2	3	4	5	
λ_i	3.9331	4.7235	7.0540	7.8255	10.149	Analytical
λ_i	3.9222	4.7145	7.0376	7.7588	10.007	Albarracín et al.(2005)
(f_i)	56.62	82.08	183.07	225.29	378.99	Analytical (*Hz*)
(f_i)	56.76	83.01	182.50	228.88	382.08	Experimental (*Hz*)
(f_i)	56.70	82.71	183.69	227.86	383.01	FEM (*Hz*)
i	6	7	8	9	10	
λ_i	10.839	13.310	14.137	16.345	17.179	Analytical
(f_i)	432.19	651.784	735.28	982.92	1085.7	Analytical (*Hz*)
(f_i)	448.00	653.080	737.92	992.43	1105.3	Experimental (*Hz*)
(f_i)	446.31	654.400	736.86	997.42	1098.8	FEM (*Hz*)

Table 1: First ten natural frequencies for C-C frames.

Tables 2 to 5 exhibit the results for the same frame with a crack artificially produced which varies in depth and location as indicated in each case.

The crack is located in the first third of the horizontal beam (Tables 2 and 3), in the middle (Table 4) and in the second third (Table 5)

Applying Eqs. (1), (2) and (8) one obtains R_m=8.40 if the crack depth h_c involves 75% of the height of the section (Tables 3, 4 and 5), and R_m=44 for h_c=0.5 (Table 2).

l_2/l_1	h_c/h	R_m		i=1	2	3	4	5	
0.33	0.50	44	λ_i	3.9195	4.7144	7.0155	7.7862	10.1331	Analytical (Eigenvalue)
			(f_i)	56.45	81.67	180.86	217.40	377.32	Analytical (*Hz*)
			(f_i)	56.45	83.01	181.88	225.22	382.26	Experimental (*Hz*)
			(f_i)	56.65	82.52	182.48	226.46	380.86	FEM (*Hz*)
				i=6	7	8	9	10	
			λ_i	10.8305	13.2873	14.0833	16.2749	17.0768	Analytical (Eigenvalue)
			(f_i)	431.05	648.79	728.86	975.50	1071.64	Analytical (*Hz*)
			(f_i)	447.39	650.63	731.20	979.78	1093.75	Experimental (*Hz*)
			(f_i)	445.93	652.85	733.77	989.88	1091.85	FEM (*Hz*)

Table 2: First ten natural frequencies for C-C frames with a crack ($h_c/h = 0.5$) in the third of the horizontal beam

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$.l_2/l_1$	h_c/h	R_m		i=1	2	3	4	5	
0.33	0.75	8.40	λ_i	3.9117	4.6886	6.9269	7.7234	10.093	Analytical (Eigenvalue)
			(f_i)	56.22	80.78	174.25	217.64	374.35	Analytical (*Hz*)
			(f_i)	56.15	80.57	167.24	217.9	377.81	Experimental (*Hz*)
			(f_i)	56.36	81.35	174.06	219.89	378.82	FEM (*Hz*)
				i=6	7	8	9	10	
			λ_i	10.831	13.238	13.939	16.056	16.883	Analytical (Eigenvalue)
			(f_i)	430.26	643.97	714.02	947.31	1047.4	Analytical (*Hz*)
			(f_i)	446.78	626.22	705.57	927.12	1080.3	Experimental (*Hz*)
			(f_i)	445.00	643.86	718.84	947.03	1071.7	FEM (*Hz*)

Table 3: First ten natural frequencies for C-C frames with a crack ($h_c/h = 0.75$) in the third of the horizontal beam

l_2/l_1	h_c/h	R_m		i=1	2	3	4	5	
0.50	0.75	8.40	λ_i	3.8482	4.6617	7.03645	7.8254	9.9059	Analytical (Eigenvalue)
			(f_i)	54.41	79.18	181.44	224.73	360.59	Analytical (*Hz*)
			(f_i)	52.49	78.13	183.72	227.05	348.51	Experimental (*Hz*)
			(f_i)	54.27	79.61	182.32	227.74	362.42	FEM (*Hz*)
				i=6	7	8	9	10	
			λ_i	10.698	13.270	14.1316	16.084	16.847	Analytical (Eigenvalue)
			(f_i)	420.66	647.582	733.76	950.08	1043.3	Analytical (*Hz*)
			(f_i)	429.69	656.74	741.58	925.29	1067.50	Experimental (*Hz*)
			(f_i)	430.01	649.88	738.44	950.24	1068.08	FEM (*Hz*)

Table 4: First ten natural frequencies for C-C frames with a crack ($h_c/h = 0.75$) in the middle of the horizontal beam

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l_2/l_1	h_c/h	R_m		i=1	2	3	4	5	
			λ_i	3.842	4.7007	6.9814	7.7194	10.136	Analytical (Eigenvalue)
			(f_i)	54.02	80.86	177.82	217.27	376.77	Analytical (*Hz*)
			(f_i)	52.49	81.18	173.95	217.29	380.86	Experimental (*Hz*)
			(f_i)	53.84	81.55	177.94	219.26	381.56	FEM (*Hz*)
0.66	0.75	8.40		i=6	7	8	9	10	
			λ_i	10.8319	13.088	13.978	16.223	16.857	Analytical (Eigenvalue)
			(f_i)	432.21	629.46	717.98	967.16	1044.3	Analytical (*Hz*)
			(f_i)	442.12	610.35	728.15	949.71	1073.6	Experimental (*Hz*)
			(f_i)	444.62	621.84	724.18	971.67	1067.0	FEM (*Hz*)

Table 5: First ten natural frequencies for C-C frames with a crack ($h_c/h = 0.75$) in the second third of the horizontal beam

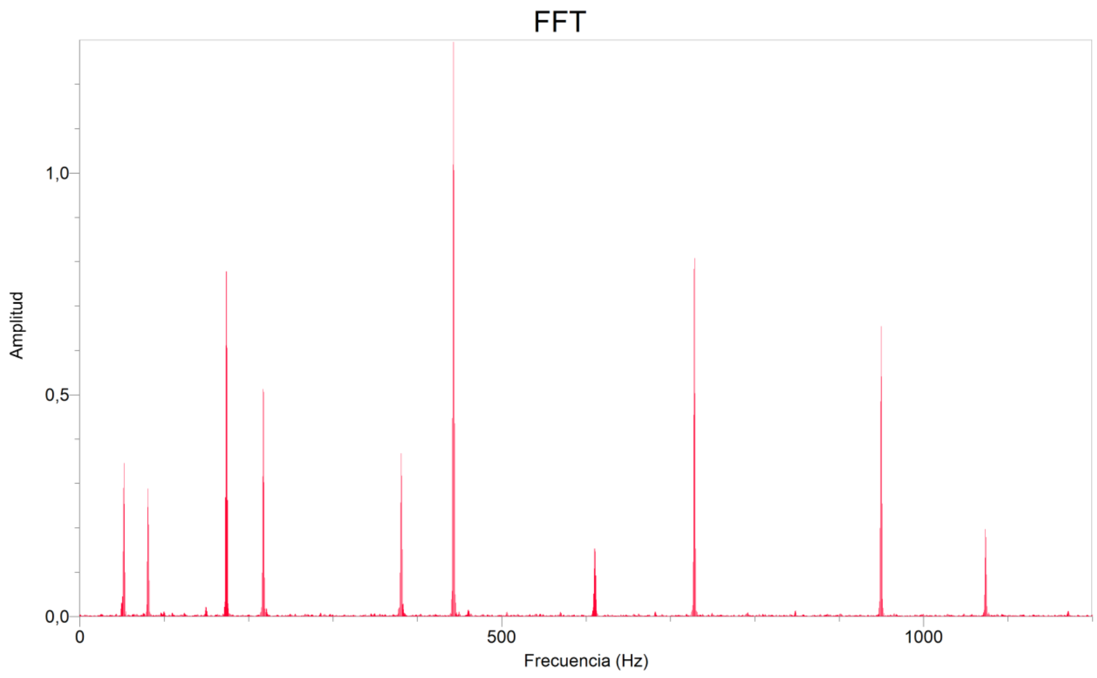

Figure 7: Dynamic response of the frame with a crack ($h_c/h = 0.75$) in the second third of the horizontal beam

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As it can be seen, the experimental values are again in surprisingly agreement with those obtained by means of the analytical procedure and the finite element method. There´s no case where the difference exceeds than 5 %, and generally it is less than 2%.

6 CONCLUSIONS

Through the study of the presented simplified model, it has been analyzed the effect of a crack on the natural frequencies of vibration of a clamped L-frame.

It has been demonstrated that the expressions proposed by Chondros et al., 1998 constitute a simple and reliable tool for modeling a crack in a plane flexural problem.

ACKNOWLEDGMENTS

The present work has been sponsored by Secretaría General de Ciencia y Tecnología of Universidad Nacional del Sur at the Department of Engineering and by Consejo Nacional de Investigaciones Científicas y Técnicas

The authors are indebted to the Chief of Mechanical Vibration Laboratory at the Department of Engineering, Ing. Santiago Maiz for his cooperation and useful suggestions and to Mr. Gabriel Leguizamón for his careful preparation of the experimental device.

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