Asociación Argentina de Mecánica Computacional

Mecánica Computacional Vol XXXIII, págs. 2277-2285 (artículo completo)
Graciela Bertolino, Mariano Cantero, Mario Storti y Federico Teruel (Eds.)
San Carlos de Bariloche, 23-26 Setiembre 2014

A NEW APPROACH TO MODEL A PNEUMATIC POSITIONING SYSTEM

Felipe Barreto Campelo Cruz[a], Kevin Maurício Menon Ribeiro[a], Alexandre Maier Eurich[a] and Frederic Conrad Janzen[b]

[a]*Departamento Acadêmico de Mecânica, Universidade Tecnológica Federal do Paraná, Avenida Monteiro Lobato, s/n – Km 04, Ponta Grossa, Paraná, Brasil, fbcruz@utfpr.edu.br, kevin_pg108@hotmail.com, alexandre_me91@hotmail.com.*

[b]*Departamento Acadêmico de Eletrônica, Universidade Tecnológica Federal do Paraná, Avenida Monteiro Lobato, s/n – Km 04, Ponta Grossa, Paraná, Brasil, fcjanzen@utfpr.edu.br.*

Keywords: Pneumatic Positioning System, Proportional Directional Control Valve, Dynamic Behavior.

Abstract. Pneumatic positioning systems are relatively light, clean and of low cost presenting a good relationship between weight and power. These features make this kind of system a good choice of actuating for most applications. It appears, however, that the nonlinearities inherent to these systems are factors that complicate there application, turning them into subject of research for different authors. In this context, it is observed that there is a point less explored then other ones, related to the mathematical modeling of the dynamics of the adopted control valve. Several authors neglect this dynamic behavior by considering it is very high compared to the rest of the systems dynamics. Related to this claim, it is intended, in this paper, to develop a mathematical model representative of a pneumatic positioning system including the dynamic behavior of a proportional directional control valve to establish its real relevance in the global model of the pneumatic positioning system. The theoretical analysis that allows the formulation of the mathematical model is based on the concepts of fluid mechanics and on the laws of conservation of mass and energy. The results for the validation of the model are obtained by computer simulation, which are then compared to experimental results.

1 INTRODUCTION

It is understood by positioning systems, those that enable to position a mechanical load at a desired location. This location is usually denoted by a set of cartesian or polar coordinates which may be fixed or variable in time.

Positioning systems are used in several applications, among which may be mentioned: rolling mills, agricultural machinery, aircraft rudders, active suspension systems, robotic manipulators and active tools.

It is described in this paper the main aspects related to a specific pneumatic positioning system and its respective mathematical modeling, aiming to continue and to contribute to an existing research line which has already led to several works developed by other researchers, such as: Andersen (2001), Cruz (2003), Ritter (2010), Ribeiro and Cruz (2012), Valdiero et al. (2012).

The choice of pneumatic systems for actuation of positioning systems is due to a number of factors that make them very attractive when compared to other actuation systems: these systems are often of low cost, clean, light and easy to assemble, presenting a good relationship between weight and power. In contrast, the application of these systems have some disadvantages among which may be mentioned their nonlinearities imposed mainly by the compressibility of the air, the system leaks and the friction between its moving parts.

Intending to validate the obtained model, results of computer simulation are presented, which are then compared to experimental results.

2 DESCRIPTION OF THE EXPERIMENTAL SET

For conducting experiments relevant to this work, it was used an experimental set (Figure 1) that consists of the following components: a proportional directional control valve model MPYE-5-1/8-HF-010B, a linear drive model DGPL-25-450-PPV-450-TLF, three manometric pressure sensors, two of the model SDE-D10-G2-H18-C-PU-M8 and one of the model SDE-10-10V. All components listed above are from the manufacturer FESTO. Completing the experimental set, it was also used a universal thermocouple type K, model MTR-01 from the manufacturer Minipa, with the purpose to measure and control the temperature.

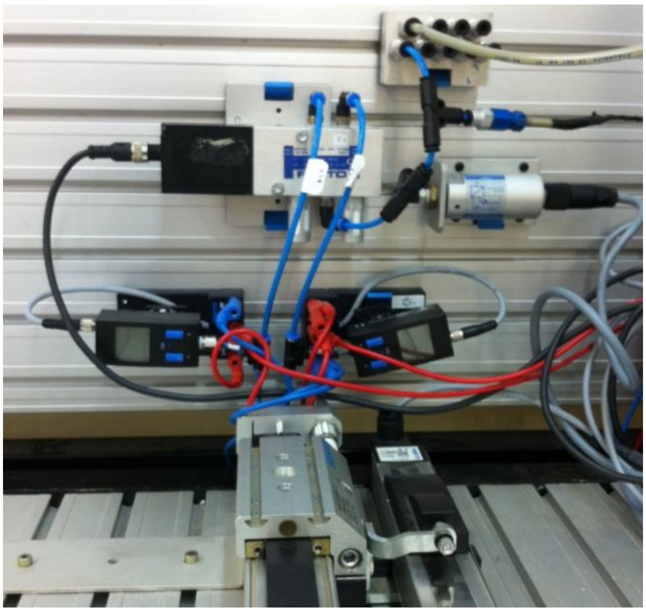

Figure 1: Experimental Set.

The functioning of this system occurs initially with a displacement of the spool of the employed valve, proportional to the value of the voltage signal applied to its solenoid. As the valve's spool moves, it causes to open or close some of the air passage orifices (control orifices), thus allowing the air to flow into one of the linear drive chambers. As a result of this air flow, it is generated a pressure difference between the chambers of the linear drive, which causes the displacement of its piston.

3 MATHEMATICAL MODELING

The presented mathematical modeling is based on the concepts of fluid mechanics and on the laws of conservation of mass and energy (Fox and McDonald, 2010; Streeter, 2000). As will be seen during this paper, some considerations will be needed to obtain the model, such as: an adiabatic and reversible process for the compression of the air volumes, which will occur at high speeds, what features an isentropic process, unidirectional flows, uniform speeds and the condition of static pressure upstream of the control orifices. Finally, the air is considered as an ideal gas.

For a better description of the developed mathematical model, the modeling is presented in stages according to the following sequence: modeling of the proportional directional control valve's spool dynamics, modeling of the mass flow rates and the continuity equations for the two control volumes considered in the system (volumes A and B) and finally, modeling of the linear drive's piston dynamics. Figure 2 illustrates each of the devices present in the system and how they are interconnected, and further evidences the control volumes mentioned:

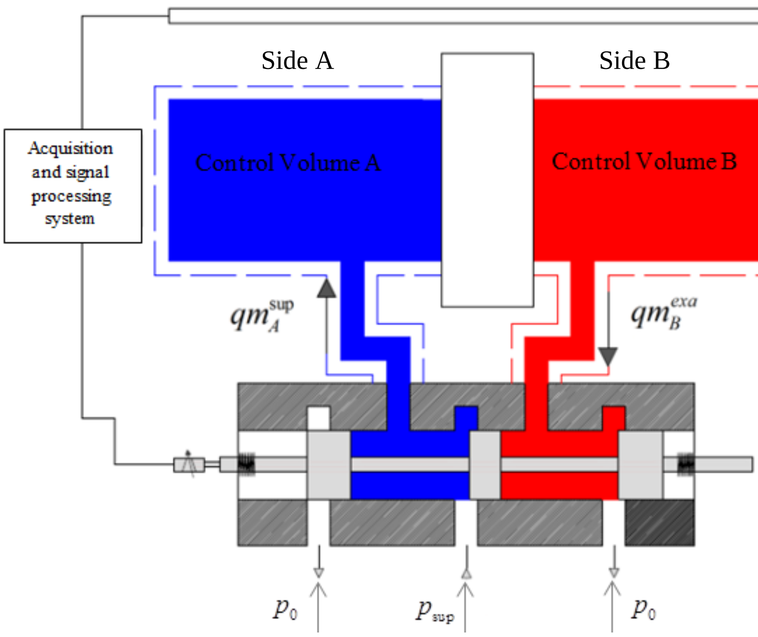

Figure 2: Schematic Drawing of the Pneumatic Positioning System.

where qm_A^{sup} is the mass flow of air in the supply of the control volume A, qm_B^{exa} is the mass flow of air in the exhaustion of the control volume B, p_0 is the local atmospheric pressure and p_{sup} is the manometric supply pressure of the system.

3.1 Proportional directional control valve's spool dynamics

The balance of forces acting on the valve's spool is responsible for its dynamic behavior ($\sum F = m_c.\frac{d^2x_c}{dt^2}$). The first force component that composes this sum is performed by the solenoid (F_s), the second one is related to the viscous damping ($B.\frac{dx_c}{dt}$) and the last one is related to the elastic behavior ($K_m.x_c$). Thus, and based on the 2nd Newton's law, considering the expression already written in the Laplace domain, one obtains:

$$x_c = \frac{K_s.|U - U_0|}{m_c.s^2 + B.s + K_m} \tag{1}$$

where x_c is the spool position, m_c is the spool mass and B and K_m are respectively the viscous damping and the elastic rigidity coefficients of the employed valve. Once the pressures act in a perpendicular direction to the spool's movement, its force components are neglected.

Additionally it is known that the force exerted by the solenoid is proportional to the applied voltage signal, that is:

$$F_s = K_s|U - U_0| \tag{2}$$

where K_s is the solenoid gain, U is the applied voltage signal (control signal) and U_0 is a voltage set point.

3.2 Mass flow through the proportional directional control valve

To describe the behavior of the mass flow through the control orifices of the proportional directional control valve, it is used a model already presented in other works such as Andersen (2001) and Cruz (2003).

According to Andersen (2001), the mass flow in the subsonic regime is given by:

$$qm = \frac{A_0.p_e}{\sqrt{T_e}}.\left\{\frac{2.\gamma}{(\gamma-1).R}.\left[\left(\frac{p_s}{p_e}\right)^{\frac{2}{\gamma}} - \left(\frac{p_s}{p_e}\right)^{\frac{(\gamma+1)}{\gamma}}\right]\right\}^{\frac{1}{2}} \tag{3}$$

where p_e and p_s are, respectively, the manometric pressures taken upstream and downstream of the considered control orifice. T_e is the absolute temperature of the air, A_0 is the cross sectional area of the considered control orifice, γ is the ratio of specific heats and R is the gas constant.

According to Andersen (2001), there is a maximum mass flow rate at which air can flow through a given area, once known the values of pressure and temperature. To obtain the pressure ratio corresponding to this maximum point, Eq. (3) has to be derived in relation to the pressure ratio and the resulting expression has to be set equal to zero:

$$\left(\frac{p_s}{p_e}\right)_{cr} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{4}$$

Taking a ratio of specific heats equal to 1.4 (value corresponding to the air), the pressure ratio described by Eq. (4) assumes a value of 0.528.

A more detailed analysis with respect to Eq. (3) can be found in Andersen (2001).

In the following sections, it will be discussed the behaviors assumed by the mass flow in the supply and in the exhaust of each one of the control volumes being considered (volumes A and B).

3.2.1 *Mass flow in the supply of the control volume being considered*

To represent the mass flow in the supply of the control volume being considered, it was adopted the following equation:

$$qm_i^{\sup} = \frac{A_0.p_{\sup}}{\sqrt{T_{\sup}}}.\left\{\frac{2.\gamma_i}{(\gamma_i-1).R}.\left[\left(\frac{p_i}{p_{\sup}}\right)^{\frac{2}{\gamma_i}}-\left(\frac{p_i}{p_{\sup}}\right)^{\frac{(\gamma_i+1)}{\gamma_i}}\right]\right\}^{\frac{1}{2}} \tag{5}$$

where i makes reference to the control volume being considered (volumes A or B), p_i is the manometric pressure related to the control volume being considered and $p_{\sup}$ is the manometric supply pressure of the system.

In order to obtain a simplified version, but still representative of the mass flow, Eq. (5) had its terms that appear in brackets expanded in terms of two binomial series until potency of order two, resulting in:

$$qm_i^{\sup} = K_0.\left(\frac{K_s.|U-U_0|}{m_c.s^2+B.s+K_m}\right).\left\{\frac{2.p_{\sup.}(p_{\sup}-p_i)}{T_{\sup}.R}\left[1-\frac{3}{2.\gamma_i}.\frac{(p_{\sup}-p_i)}{p_{\sup}}\right]\right\}^{\frac{1}{2}} \tag{6}$$

where:

$$A_0 = K_0.x_c = K_0.\left(\frac{K_s.|U-U_0|}{m_c.s^2+B.s+K_m}\right) \tag{7}$$

with K_0 representing the proportionality constant existing between the cross-sectional area of the considered control orifice and the position assumed by the proportional directional control valve's spool.

Considering an isentropic behavior for the system and also, assuming that the air behaves as an ideal gas, one obtains as the final expression for the mass flow in the supply, the following equation:

$$qm_i^{\sup} = \frac{\varepsilon_i.K_i.|U-U_0|.\sqrt{2.(p_{\sup}-p_i)}}{\left(\frac{1}{\omega_n^2}.s^2+\tau_i.s+1\right)} \tag{8}$$

where:

$$\varepsilon_i = \left[\frac{1 - \frac{3}{2.\gamma_i}.\frac{\Delta p_i}{p_{\sup}}}{1 - \frac{1}{\gamma_i}.\frac{\Delta p_i}{p_{\sup}} + \frac{(1-\gamma_i)}{2.\gamma_i^2}.\left(\frac{\Delta p_i}{p_{\sup}}\right)^2} \right]^{1/2} \tag{9}$$

$$K_i = \frac{K_0.K_s.\sqrt{2.\rho_i}}{K_m} \tag{10}$$

$$\tau_i = \frac{B}{K_m} \tag{11}$$

$$\omega_n = \sqrt{\frac{K_m}{m_c}} \tag{12}$$

with τ_i representing the time constant and ω_n the undamped natural angular frequency of the proportional directional control valve's spool.

3.2.2 *Mass flow in the exhaustion of the control volume being considered*

To represent the mass flow in the exhaustion of the control volume being considered, it was admitted that the pressure ratio, given by the pressures taken downstream and upstream of the considered control orifice, always assumes a value lower or equal to its critical value, given by:

$$\left(\frac{p_o}{p_i}\right)_{cr} = \left(\frac{2}{\gamma_i + 1}\right)^{\frac{\gamma_i}{\gamma_i - 1}} = 0.528 \tag{13}$$

which, in turn, provides a constant value for the mass flow, regardless of the pressure values involved. p_0 represents the local atmospheric pressure which, in this case, coincides with the pressure taken downstream of the considered control orifice.

Substituting the result of Eq. (13) in Eq. (3), considering also Eq. (7), Eq. (10), Eq. (11) and Eq. (12) and recalling the fact that the air has a ratio of specific heats equal to 1.4 and that it behaves in an isentropic manner and as an ideal gas, one obtains as the final expression representative of the mass flow in the exhaustion, the following equation:

$$qm_i^{exa} = \left(\frac{K_i.|U - U_0|}{\frac{1}{\omega_n^2}.s^2 + \tau_i.s + 1} \right).\sqrt{p_i}.0.484 \tag{14}$$

3.3 Continuity equation for a compressible flow in the control volume being considered

This equation makes reference to the conservation of mass in a specific control volume and through the surface area of passage that delimits it:

$$\int_{SC}\rho_i.v_i.dA_i+\frac{\partial}{\partial t}\int_{VC}\rho_i.dV_i=0 \tag{15}$$

where A_i is the surface area of air passage, V_i is the control volume being considered and ρ_i and v_i are, respectively, the density and the velocity of the air in V_i.

Considering that the air behaves as an ideal gas and assuming that the process has an isentropic behavior, one obtains as final expression for the continuity equation the following relationship:

$$q_{mi1}=q_{mi2}+\frac{p_i}{R.T_i}.\frac{dV_i}{dt}+\frac{V_i}{R.T_i.\gamma_i}.\frac{dp_i}{dt} \tag{16}$$

where q_{mi1} and q_{mi2} represent, respectively, the mass flow measurements upstream and downstream of the control volume V_i.

In the particular case of the control volume A, given by $V_A=A_c.X+V_{A0}$, it is considered that q_{mA2} does not exist and that q_{mA1}, for being the unique mass flow present in this case, was renamed only as q_{mA}. In view of these considerations, and in order to obtain an expression that describes the behavior of the pressure being regulated in the control volume A, Eq. (16) can be reduced to:

$$\frac{dp_A}{dt}=\frac{-A_c.\gamma_A.\dot{X}}{A_c.X+V_{AO}}.p_A+\frac{R.T_A.\gamma_A}{A_c.X+V_{AO}}.q_{mA} \tag{17}$$

where A_c and X represent, respectively, the useful cross-sectional area and the position of the linear drive's piston and V_{AO} represents a volume that includes the dead volume of the linear drive's chamber being considered, beyond the pipe connecting this chamber to one of the output control orifices of the valve.

Analogously to the control volume A, one obtains the following equation for the control volume B:

$$\frac{dp_B}{dt}=\frac{A_c.\gamma_B.\dot{X}}{A_c.(L-X)+V_{BO}}\cdot p_B-\frac{R.T_B.\gamma_B}{A_c.(L-X)+V_{BO}}\cdot q_{mB} \tag{18}$$

In this case, the control volume considered is given by $V_B=A_c(L-X)+V_{BO}$, q_{mB1} is the mass flow that becomes non-existent and q_{mB2}, for being the unique mass flow present in this case, was renamed only as q_{mB}. Additionally it follows that L and V_{BO} represent, respectively, the useful length of the linear drive and a volume that includes the dead volume of the linear drive's chamber being considered, beyond the pipe connecting this chamber to one of the output control orifices of the valve.

3.4 Linear drive's piston dynamics

$$M.\ddot{X}=(p_A-p_B).A_c \tag{19}$$

where M is the mass of the linear drive's piston.

It can be seen from Eq. (19), that the dynamic behavior of the linear drive was considered as being an exclusive result of the force imposed by the pressure gradient [$(p_A-p_B).A_c$]. The friction and the dead zone, although of extreme relevance for an accurate description of the linear drive's piston dynamics, are not considered in this paper, but are already being contemplated in the new work plans suggested by the authors of this paper.

4 RESULTS AND CONCLUSIONS

Figure 3 shows the dynamic behavior of the pressures that supply the two control volumes being considered.

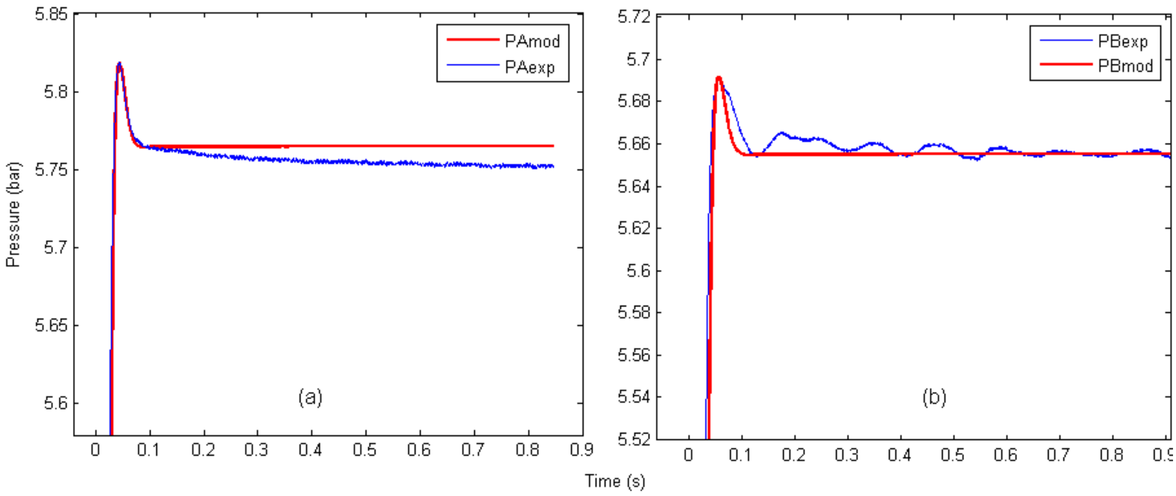

Figure 3: Dynamic Behavior of the Pressures Downstream the Control Valve: (a) Side *A;* (b) Side *B*.

It is worth noting that the curves obtained by computer simulation and exposed in Figure 3, are a direct result of the linearization process of Eqs. (1), (5) and (15). Comments regarding linearization and parametric survey can be found, respectively, at Ribeiro (2014) and Eurich (2014).

Analyzing the pressure behaviors in Figure 3, it can be observed that there is a great similarity between the dynamic behaviors obtained experimentally and by computer simulation. Based on these curves it can be concluded that the modeling process being performed tends to a satisfactory and representative result of the dynamic behavior of the real system.

Other experiments to comprove the validity of the proposed model are already being developed by the authors of this paper. Soon this model will be completely validated and will be ready for use.

5 ACKNOWLEDGEMENTS

We thank the Federal Technological University of Paraná (UTFPR) for the financial support given to the authors of this work, enabling its dissemination in the present event (ENIEF 2014).

REFERENCES

Andersen, B. W., *The analysis and design of pneumatic systems*, New York, USA, John Wiley & Sons, 2001.

Cruz, F. B. C., *Modelagem e controle não lineares de um posicionador servopneumático industrial*, 128 f., Dissertação (Mestrado) – Departamento de Engenharia Mecânica, Universidade Federal de Santa Catarina, Florianópolis, 2003.

Eurich, A. M., *Validação experimental de um modelo matemático representativo de um* *sistema pneumático de posicionamento,* Trabalho de Conclusão de Curso (Graduação) – Departamento Acadêmico de Mecânica, Universidade Tecnológica Federal do Paraná, Ponta Grossa, 2014. [Trabalho não publicado].

Fox, R. W., Pritchard, P. J., and McDonald, A. T., *Introdução à mecânica dos fluídos,* 7 ed., John Wiley & Sons, 2010.

Ribeiro, K. M. M., *Modelagem matemática de um sistema pneumático de posicionamento,* Trabalho de Conclusão de Curso (Graduação) – Departamento Acadêmico de Mecânica, Universidade Tecnológica Federal do Paraná, Ponta Grossa, 2014.

Ribeiro, K. M. M., and Cruz, F. B. C., *Modelagem matemática de um sistema pneumático de posicionamento*. In: 19° Congresso Nacional Dos Estudantes De Engenharia Mecânica (Creem), São Carlos, Agosto 2012.

Ritter, C. S., *Modelagem matemática das características não lineares de atuadores pneumáticos,* Dissertação (Mestrado) – Departamento de Física, Estatística e Matemática, Universidade Regional do Noroeste do Estado do Rio Grande do Sul, Ijuí, 2010.

Valdiero, A. C., Ritter, C. S., and Rasia, L. A., *Modelagem matemática e simulação computacional de um atuador pneumático considerando o efeito do atrito dinâmico,* In: Congresso Nacional De Matemática Aplicada E Computacional, XXXIV, Águas de Lindóia, 2012, p. 465 - 471.

Streeter, V. L., *Mecânica de fluídos,* 9 ed., McGraw-Hill, 2000.