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APPLICATIONS OF THE VIRTUAL ELEMENT METHOD FOR CRACKING ANALYSIS OF CEMENT-BASED COMPOSITES USING INTERFACE ELEMENTS

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Abstract. The very recently introduced Virtual Element Method (VEM) is a numerical method for solving partial differential equations that was created out of the mimetic difference method, but was later reformulated into the Galerkin framework. It is a generalization of the standard Finite Element Method (FEM) to general meshes made up by arbitrary polyhedra. The greatest advantage of VEM is to be able to deal with very complex geometries, i.e., made up by elements of any number of edges not necessarily convex, hanging nodes, flat angles, collapsing nodes, etc, while retaining the same approximation properties of FEM. In this paper we propose the simulating of the mechanical response of cement-based composites by means of a Virtual Element approach for discretization of the domain, along with a novel interaction between zero-thickness interface element for meso-scale analysis within the framework of the so-called discontinuous-based approach. Following a similar modeling approach already available in literature for FEM and iso-parametric interface elements, the formulation of the VEM for elasticity problems is a novelty that can be used to capture the key mechanical phenomena controlling concrete heterogeneity by taking advantage of the versatility of polyhedral meshes.

1 INTRODUCTION

In the last decades, meso-scale analysis of concrete material emerged as promising approach for concrete modeling and its failure prediction. Among other issues, the soundness and capabilities of meso-scale approaches deeply depend on the accurate description of the representative meso-geometry and the proper description of the non-linear numerical processes of the specimen when submitted to external actions (Häfner et al., 2006; Wriggers and Moftah, 2006).

A large amount of theoretical models and numerical tools have been proposed with the aim to realistically predict the pre- and post-cracking behavior of concrete at meso-scale levels. As a reference, we recall the work by Carol et al. (1997) where a fracture-based interface model for meso-scale analysis of concrete specimens subjected to several loading conditions such as tension, compression, biaxial and Brazilian tests (López et al., 2008) is described. The aforementioned formulation has been recently extended for modeling failure behavior of fiber-reinforced cementitious composites (Caggiano et al., 2012) and for coupled thermo-mechanical analysis of concrete failure under high temperature (Caggiano and Etse, 2015).

In this work, and in the framework of the so-called discontinuous-based approach, the simulating of the mechanical response of concrete composites is approached through the VEM for discretization of the domain, along with a novel interaction between zero-thickness interfaces for meso-scale analysis.

VEM is a new addition to the use of polytopal meshes. It was inspired by techniques used in the Mimetic Differences Method (Brezzi et al., 2005) but later was given a framework as a generalization of the standard FEM.The use of polyhedral meshes has many interesting possibilities such as allowing hanging nodes, improving adaptivity, introducing cracks and fractures as well as facilitating the meshing of complex geometries. Many approaches for polygonal finite elements already exist in the literature. As with all new developments, the VEM literature is still in its early stages, specially in experimental results, although it has been steadily growing since its introduction. On the other hand, the method is free from many of the most common disadvantages of other approaches for polygonal meshes and it has been shown to be robust when dealing with highly irregular meshes as well as relatively easy to implement.

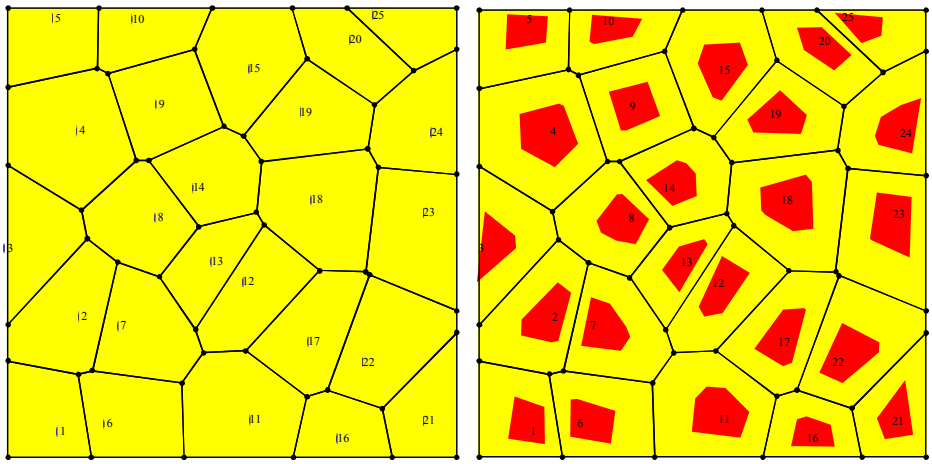
Zero-thickness interface elements, formulated in terms of contact stresses versus opening relationships, have been historically employed for modeling material discontinuities, i.e. mechanical contacts (Lei, 2001), bond phenomena (Chen et al., 1999; Baky et al., 2012) and crack evolutions in quasi-brittle materials like concrete (Xie and Jr., 2006; Ciancio et al., 2013). Several plasticity-based interface formulations have been proposed to predict failure behaviors of discontinuities in soil/rock mechanisms (Costa Daguiar et al., 2011). One of the most frequent use of interface elements in computational concrete mechanics is related to mesoscopic failure simulations (Stankowski et al., 1993; Lorefice et al., 2008).

After a brief literature review, Section 2 summarizes the meso-scale procedure for describing the composite nature of cementitious composites like concrete. The mathematical formulation of VEM as a generalization of standard FEM for (even non-convex) polyhedra is described in Section 3. Then, the basic equations behind the use of zero-thickness interface elements is given in Section 4. Section 5 introduces the potential applications and exemplifies some of the uses of the proposed model. Finally, some concluding remarks are reported in Section 6.

2 MESO-SCALE APPROACHES

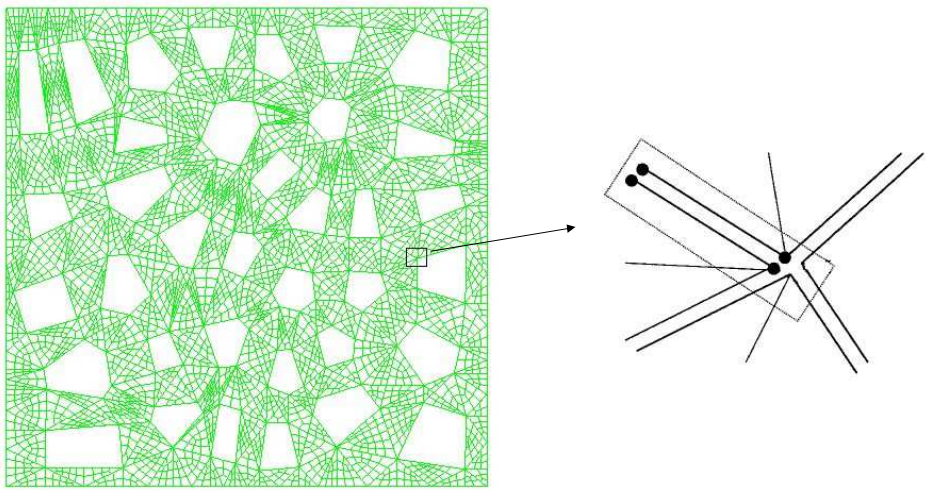
A meso-mechanical approach for the numerical analysis of concrete specimens subject to external actions is presented in this section. Cement-based composite will be represented as a

composite material characterized by larger aggregates embedded in a surrounding mortar plus fine aggregates matrix as schematically indicated in the meso-structure of Figure 1.



(a) (b)

Figure 1: Concrete meso-scale specimen: (a) random pertubed points (started from a regular array) and (b) Voronoi/Delaunay tessellation.



(a) (b)

Figure 2: Finite Element mesh: (a) meso-scale concrete specimen and (d) detail of zero-thickness interfaces.

Convex polygonal representation, adopted for representing both large aggregates and the surrounding mortar matrix, is numerically generated though standard Voronoi/Delaunay tessellation (Klein, 1989) from a regular array of points which are slightly perturbed as shown in Figure 1a. Both the polygonal particles and the areas between them (surrounding matrix of Figure 1b) are meshed with finite elements for generating the model analysis (Figure 2a). The continuum elements obtained by means of the above procedure are assumed as linear elastic. The idea is to simplify this mesh by replacing Finite Elements with Virtual Elements.

Once the specimen discretization in continuum elements is completed, the interface elements need to be introduced along all potential discontinuous lines. This process consists of a proper duplication of the nodes and subsequent changes in element nodal connectivities (Figure 2b). Thus, all (thermo-chemical-mechanical)-nonlinearities will be concentrated within zero-thickness interface elements defined throughout the adjacent edges of the elements of the mesh.

3 THE VIRTUAL ELEMENT METHOD

3.1 General formulation

In short, the VEM is a generalization of standard FEM to meshes made up by arbitrary, possibly non-convex polyhedra. It was first introduced in Beirão da Veiga et al. (2013) and the basic ideas are recalled here for the case of the general second order elliptic equation (described in Beirão da Veiga et al. (2016)). We choose to begin with this equation because it makes the presentation of the basic concepts clearer. This equation is used to model the concentration of some chemical substance under the processes of diffusion, transportation of mass due to the movement of the underlying medium and undergoing reaction. In its most general form, the time dependent equation is written as

$$\frac{\partial u}{\partial t} + \nabla \cdot (\beta u - \nu \nabla u) + \gamma u = f, \tag{1}$$

where u represents the concentration, ν is the diffusion coefficient, β is the flow velocity, γ is a source parameter and f is the source function that models the changes in concentration due to reactions. This equation is obtained from the conservation of mass in a continuum model and there may be nonlinearities present in the coefficients. When some of the coefficients are null, the equation reduces to special cases. In particular, when no reaction term is present ($\gamma = 0$) it simplifies to the advection diffusion (also known as convection-diffusion) equation, which is of particular interest in transport problems. The steady state equation is obtained by putting $\frac{\partial u}{\partial t} = 0$.

We present the main ideas for the construction of the local Virtual Element space $V_{k,h}^E$, the choice in degrees of freedom as well as the global space: given a domain F covered with a triangulation τ_h, for a desired order of accuracy k and with the space $\mathbb{P}_k$ of the polynomials of maximum degree k, let us define the local space $V_{k,h}^E$ as

$$V_{k,h}^E = \left\{ v_h \in \mathrm{H}^1(E) : v_{h|\partial E} \in C^0(\partial E),\ v_{h|e} \in \mathbb{P}_k(e)\ \forall e \subset \partial E,\ \Delta v_h \in \mathbb{P}_{k-2}(E) \right\},$$

where h is a mesh parameter, E is an element of the triangulation, ∂E is its border and e an edge. This definition already exhibits the main novelty of the VEM: the base functions in the local space are not necessarily explicitly known. In fact, they are only known at the boundary of the element, where they are piecewise-continuous polynomials, and they satisfy a certain property of their Laplacian in the interior. The global virtual element space is simply

$$V_{k,h} = \left\{ v_h \in \mathrm{H}^1(F) : v_h|_E \in V_{k,h}^E\ \forall E \in \tau_h \right\}.$$

Let us define the local terms of the weak formulation of the problem as

$$a^E(u,v) = (\nu \nabla u, \nabla v)_E, \quad b^E(u,v) = (\beta \cdot \nabla u, v)_E, \text{ and } c^E(u,v) = (\gamma u, v)_E. \tag{2}$$

The main idea in the VEM is to approximate these bilinear forms without explicitly knowing the shape functions in $V_{k,h}^E$ so as to retain optimal convergence of the error. In order to do so, let

us consider a basis for $\mathbb{P}_k$ given by the polynomials p_α with $\alpha = 1, ..., dim(\mathbb{P}_k)$. The notation ϕ_i will be use for the shape functions in $V_{k,h}^E$. The following projection operators are introduced: $\Pi_k^0 : V_{k,h}^E \to \mathbb{P}_k$, that is the solution of an L^2 projection problem given by

$$(p_\alpha, \Pi_k^0 \phi_i)_E = (p_\alpha, \phi_i)_E \quad \forall \alpha = 1, ..., dim(\mathbb{P}_k), \tag{3}$$

and the second projector $\Pi_{k-1}^0 : V_{k,h}^E \to (\mathbb{P}_{k-1}(E))^2$ also given by a L^2 projection of $\nabla\phi_i$ satisfying

$$(p_\alpha, \Pi_{k-1}^0 \nabla\phi_i)_E = (p_\alpha, \nabla\phi_i)_E \quad \forall \alpha = 1, ..., dim(\mathbb{P}_{k-1}). \tag{4}$$

Note that only the basis for the space $\mathbb{P}_{k-1}$ is needed in this case, since the projection of the gradient is a polynomial of degree $k-1$. Another crucial point in the description of the method is that these projectors can be exactly computed for functions in the local VEM space using their Degrees Of Freedom (DOF) and integration by parts (we refer the reader to the references provided for further details). The DOF are usually k vertex values, $k-1$ values for each edge interior and the rest are moment DOFs in the interior of the element. The total number of DOF is $nk + k(k-1)/2$ where n is the number of edges of the element.

If we introduce the following decomposition of the shape functions ϕ_i we obtain

$$\begin{aligned} \phi_i &= \Pi_k^0 \phi_i + (I - \Pi_k^0)\phi_i, \\ \nabla\phi_i &= \Pi_{k-1}^0 \nabla\phi_i + (I - \Pi_{k-1}^0)\nabla\phi_i, \end{aligned}$$

where I is the identity operator.

Finally, given v_h and $u_h \in V_{k,h}^E$ it is possible to obtain the discrete bilinear forms as

$$\begin{aligned} a_h^E(u_h, v_h) &= (\nu \Pi_{k-1}^0 \nabla u_h, \Pi_{k-1}^0 \nabla v_h)_E + S_a^E((I - \Pi_{k-1}^0)\nabla u_h, (I - \Pi_{k-1}^0)\nabla v_h), \\ b_h^E(u_h, v_h) &= (\beta \cdot \Pi_{k-1}^0 \nabla u_h, \Pi_k^0 v_h)_E, \\ c_h^E(u_h, v_h) &= (\gamma \Pi_k^0 u_h, \Pi_k^0 v_h)_E, \end{aligned} \tag{5}$$

where $S_a^E(u_h, v_h)$ is a symmetric bilinear form that scales in a desired way in order to account for the neglected contributions involving products with $(I - \Pi_k^0)$ and $(I - \Pi_{k-1}^0)$. It is called the stabilization term and for a_h^E it is approximated by

$$S_a^E\left((I - \Pi_{k-1}^0)\nabla u_h, (I - \Pi_{k-1}^0)\nabla v_h)\right) \approx \sum_{n=1}^{N_{DOF}} \chi_n((I - \Pi_{k-1}^0)\nabla u_h)\chi_n((I - \Pi_{k-1}^0)\nabla v_h),$$

where N_{DOF} is the number of degrees of freedom of the shape functions in $V_{k,h}^E$ and χ_n is the functional that evaluates the function at its n^{th} DOF. With this new discrete bilinear forms, the local stiffness matrix can be assembled. The load term is simply computed with a suitably accurate scheme and the rest of the procedure follows closely that of standard FEM.

3.2 VEM for elasticity problems

The first work introducing the Virtual Element Method to elasticity was described by Brezzi et al. (2013), which was followed by a generalization to handle different material models with non elastic behavior and even finite strain problems (see Beirão da Veiga et al. (2015)). As in the case of general second order elliptic equations explained previously, the method takes advantage of a projection operator and uses integration by parts to allow the computation of the stiffness matrix without explicit knowledge of the local shape functions.

The problem is to find displacements $\mathbf{u} : \Omega \to \mathbb{R}^2$, such that

$$\begin{cases} -\text{div}\,\boldsymbol{\sigma} & = \mathbf{f} \quad \text{in } \Omega \\ \mathbf{u} & = 0 \quad \text{on } \Gamma_D \\ \boldsymbol{\sigma}\mathbf{n} & = 0 \quad \text{on } \Gamma_N \end{cases} \tag{6}$$

where Γ_D and Γ_N are the Dirichlet and Neumann boundaries, respectively, and $\mathbf{n}$ is the normal vector to the Neumann boundary. The definitions for three dimensional problems is a straightforward generalization of the plane case.

We assume a given constitutive law at every point $\mathbf{x}$ in the domain $\Omega \subset \mathbb{R}^d$, in the form $\boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{x}, \nabla\mathbf{u})$. This law may also consider the history of the material and uses the "black-box" approach present in many commercial softwares. Similarly as before, the local space V_h^E is defined as

$$V_h^E = \left\{ \mathbf{v}_h \in \left[\mathrm{H}^1(E) \cap C^0(E)\right]^2 : v_{h|\partial E} \in C^0(\partial E),\ v_{h|e} \in \mathbb{P}_1(e)\ \forall e \subset \partial E,\ \Delta v_h = 0 \right\}.$$

Only a linear approximation is available at the moment, but a higher order generalization is foreseeable.

The global space is simply

$$V_h = \left\{ \text{ Allowable displacements } \mathbf{v}_h : \mathbf{v_h}|_E \in V_h^E\ \forall E \text{ in the triangulation} \right\}.$$

The variational form of the problem is then to find an allowable displacement $\mathbf{u}$ such that

$$a(\mathbf{u}, \mathbf{v}) = \int_\Omega \mathbf{f}(\mathbf{x})\mathbf{w}(\mathbf{x})dx \quad \forall \mathbf{w} \text{ in the space of variations,} \tag{7}$$

with

$$a(\mathbf{u}, \mathbf{v}) = \sum_{E \in \tau_h} \int_E \boldsymbol{\sigma}(\mathbf{x}, \nabla\mathbf{v}(\mathbf{x})) : \nabla\mathbf{w}(\mathbf{x})dx. \tag{8}$$

We use a suitably defined projection operator Π_E^0 (see the references for a complete description), that can be computed using only knowledge of the value of the function at the degrees of freedom, and therefore is well defined for the local VEM space. DOF for this problem are vertex values. We define a discrete bilinear form as

$$\begin{aligned} \hat{a}_h(\mathbf{u}_h, \mathbf{v}_h) &= \sum_E \int_E \boldsymbol{\sigma}(\mathbf{x}, \Pi_E^0(\nabla\mathbf{v}_h)(\mathbf{x})) : (\Pi_E^0(\nabla\mathbf{w}_h)(\mathbf{x}))dx \\ &= \sum_E |E|\sigma(\Pi_E^0(\nabla\mathbf{v}_h)) : \Pi_E^0(\nabla\mathbf{w}_h). \end{aligned} \tag{9}$$

In (9), Π_E^0 is a suitably defined projection operator that approximates the gradient of the displacement with a constant value in each element. Once again, this operator can be computed using only knowledge of the value of the function at the DOF, and therefore is well defined for the local VEM space. Note that the VEM space contains functions that may not be polynomials (unlike standard FEM). However, the discrete bilinear form $\hat{a}(\mathbf{u}_h, \mathbf{v}_h)$ is computed exactly when either one of the arguments is a polynomial. In this way, the formulation of the element is such that it guarantees the passing of the patch test. For non-triangular elements (or non-tetrahedral elements in $3D$), this approach may lead to spurious modes in the solution. As in the case of second order elliptic problems, an stabilization form S_h is introduced that *scales* appropriately,

i.e., takes care of the spurious modes and guarantees stability without affecting convergence and accuracy of the solution as it it bounded above and below by the a constant multiples of the a bilinear form (that guarantees consistency).

Finally, the complete global bilinear form becomes: for any $\mathbf{u}_h, \mathbf{v}_h \in V_h$,

$$a_h(\mathbf{u}_h, \mathbf{v}_h) = \hat{a}(\mathbf{u}_h, \mathbf{v}_h) + \alpha(\mathbf{u}_h) S_h((\mathbf{u}_h, \mathbf{v}_h) \tag{10}$$

where $\alpha > 0$ is a parameter depending on the material model and is necessary to adjust the stabilization. Loading terms and boundary conditions are computed as in standard FEM procedures.

4 DISCONTINUOUS INTERFACE ELEMENTS

In the framework of continuous mechanics problems which also account for explicitly considered cracks, the governing equations including the equilibrium equation, the natural and essential boundary conditions as highlighted in Eq. (6), are completed with the traction continuity on the crack surface as follows:

$$\begin{cases} \sigma \mathbf{n}_d^+ & = \mathbf{t}_c^+ \\ \sigma \mathbf{n}_d^- & = \mathbf{t}_c^- \quad \text{on } \Gamma_d \\ \mathbf{t}_c^+ & = -\mathbf{t}_c^- \end{cases} \tag{11}$$

where $\mathbf{t}_c$ is the cohesive traction across the crack line Γ_d, while $\mathbf{n}$ is its unit normal vector.

The above relationship is valid for any kind of discontinuous approach employed for discrete crack analysis. When zero-thickness interface elements are used for this purpose, the displacement field of the upper and lower faces of the element are given by

$$\begin{aligned} &\mathbf{u}^+ = \mathbf{V}^{int} \mathbf{U}^+ \\ &\mathbf{u}^- = \mathbf{V}^{int} \mathbf{U}^- \\ &[[\mathbf{u}]] = \mathbf{u}^+ - \mathbf{u}^- = \mathbf{V}^{int} \left(\mathbf{U}^+ - \mathbf{U}^- \right) \end{aligned} \tag{12}$$

being $\mathbf{V}^{int}$ the matrix of the interface shape functions; $\mathbf{U}^+$ and $\mathbf{U}^-$ denote the nodal displacements of the upper face and lower face, respectively, while $[[\mathbf{u}]]$ the displacement jump vector.

Introducing Eq. (12) into the weak form of equilibrium, the following equations can be derived:

$$\mathbf{f}^{EXT} = \mathbf{f}^{INT} + \mathbf{f}^{int} \tag{13}$$

where $\mathbf{f}^{EXT}$ and $\mathbf{f}^{INT}$ are the classical external and internal forces of a continuous FE, while $\mathbf{f}^{int}$ represents the interface force vector, obtainable as follows:

$$\begin{cases} \mathbf{f}^{int,+} = \int_{\Gamma_d} [\mathbf{V}^{int}]^T \mathbf{t}_c d\Gamma_d, \\ \mathbf{f}^{int,-} = -\int_{\Gamma_d} [\mathbf{V}^{int}]^T \mathbf{t}_c d\Gamma_d. \end{cases} \tag{14}$$

In the framework of the classical theory of plasticity, that considers the extension to interface kinematic field of the Prandtl-Reuss additive decomposition law, the following equations can be written as

$$\dot{\mathbf{u}} = \dot{\mathbf{u}}^{el} + \dot{\mathbf{u}}^{cr}, \tag{15}$$

where $\dot{\mathbf{u}} = [\dot{u}, \dot{v}]^t$ is the vector of relative velocities across the interface and $\dot{\mathbf{u}}^{el}$ and $\dot{\mathbf{u}}^{cr}$ are the elastic and cracking (inelastic) velocity vector components, respectively.

The interface elastoplastic constitutive equation can be expressed as

$$\begin{aligned} \dot{\mathbf{u}}^{el} &= \mathbf{C}^{-1}\dot{\mathbf{t}} \\ \dot{\mathbf{t}} &= \mathbf{C}\left(\dot{\mathbf{u}} - \dot{\mathbf{u}}^{cr}\right) \end{aligned} \tag{16}$$

being $\dot{\boldsymbol{t}} = [\dot{\sigma}_N, \dot{\sigma}_T]^t$ the interface stress rate vector and $\mathbf{C}$ the elastic stiffness matrix.

The rate of normal and tangential interface stresses is derived through the rates of relative displacement vector as

$$\dot{\mathbf{t}} = \mathbf{C}^{ep}\dot{\mathbf{u}} \tag{17}$$

where $\mathbf{C}^{ep}$ is the tangential elastic-plastic operator.

Therefore, the tangent stiffness matrix for a given interface element is given by

$$\mathrm{K}_e^{int} = \begin{bmatrix} \int_{\Gamma_d} [\mathbf{V}^{int}]^T \mathbf{Q}\, \mathbf{C}^{ep} [\mathbf{Q}]^T \mathbf{V}^{int} d\Gamma_d & -\int_{\Gamma_d} [\mathbf{V}^{int}]^T \mathbf{Q}\, \mathbf{C}^{ep} [\mathbf{Q}]^T \mathbf{V}^{int} d\Gamma_d \\ -\int_{\Gamma_d} [\mathbf{V}^{int}]^T \mathbf{Q}\, \mathbf{C}^{ep} [\mathbf{Q}]^T \mathbf{V}^{int} d\Gamma_d & \int_{\Gamma_d} [\mathbf{V}^{int}]^T \mathbf{Q}\, \mathbf{C}^{ep} [\mathbf{Q}]^T \mathbf{V}^{int} d\Gamma_d \end{bmatrix} \tag{18}$$

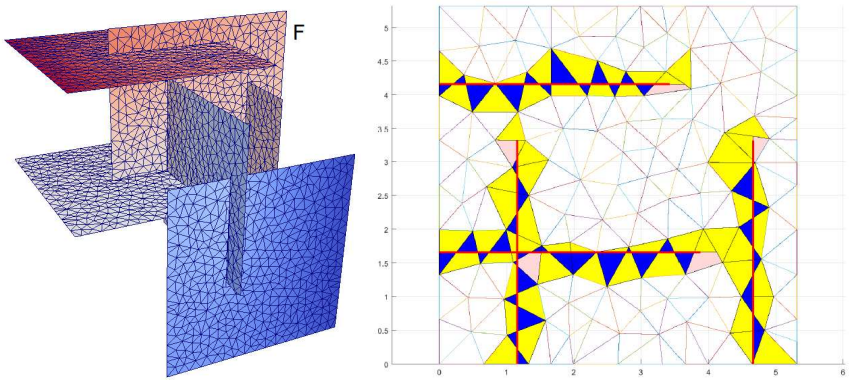
where $\mathbf{Q} = [\mathbf{n},\ \mathbf{s},\ \mathbf{t}]$ is a vector constructed with the unit normal vector of the interface element $\mathbf{n}$ and $\mathbf{s}$, $\mathbf{t}$ are the unit tangential interface vectors.

5 APPLICATIONS AND POTENTIAL DEVELOPMENTS

5.1 Domain decomposition

Among the most immediate applications of the VEM is domain decomposition, in which the domain of the problem is split into several subdomains, each one with its own mesh. The problem is then to assemble the different meshes on the boundaries to obtain the solution for the global problem.

This approach has been successfully tested in the context of solving flow on Discrete Fracture Networks (DFN) in Benedetto et al. (2014). In Figure 3 a, the domains are represented by plane rectangular domains F, each with its own mesh. A base mesh composed by triangles is modified on the intersections between planes and new polygonal elements are created, giving rise to the VEM mesh.

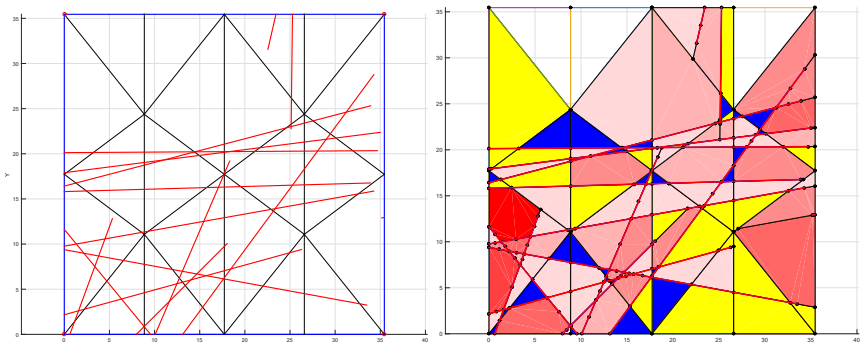


(a) Domains and intersections (b) VEM mesh on domain F

Figure 3: VEM for DFN and domain decomposition.

Since the VEM allows for any arbitrary polygonal to be used as an element, all that is needed to solve the global problem is to combine the problems at the interfaces. This was done by

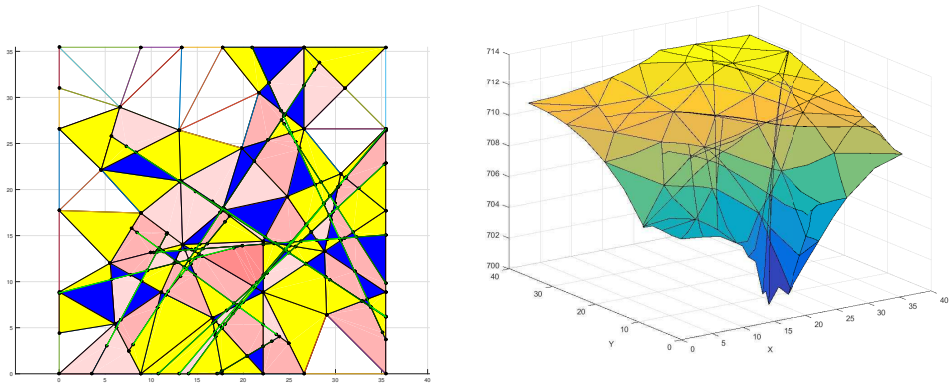
means of a globally conforming approach that shares all nodes on interfaces between domains (see Benedetto et al. (2016b)) or using the well-known Mortar method to impose weak continuity on interfaces (Benedetto et al. (2016a)). Figure 3 b shows the resulting mesh for F, where element coloring indicates number of edges and red lines show intersection segments. An example of a domain in a much larger and complex system is shown in 4 where elements range from triangles to 10-sided polygons.



(a) Original mesh (b) VEM mesh after modifications

Figure 4: Example of a subdivision of a base triangular mesh.

To illustrate the method, a VEM mesh and its solution for a typical fracture in a medium-sized DFN are presented in Figure 5, showing how easily the method can handle complex element geometry and elements with severe shape complications.



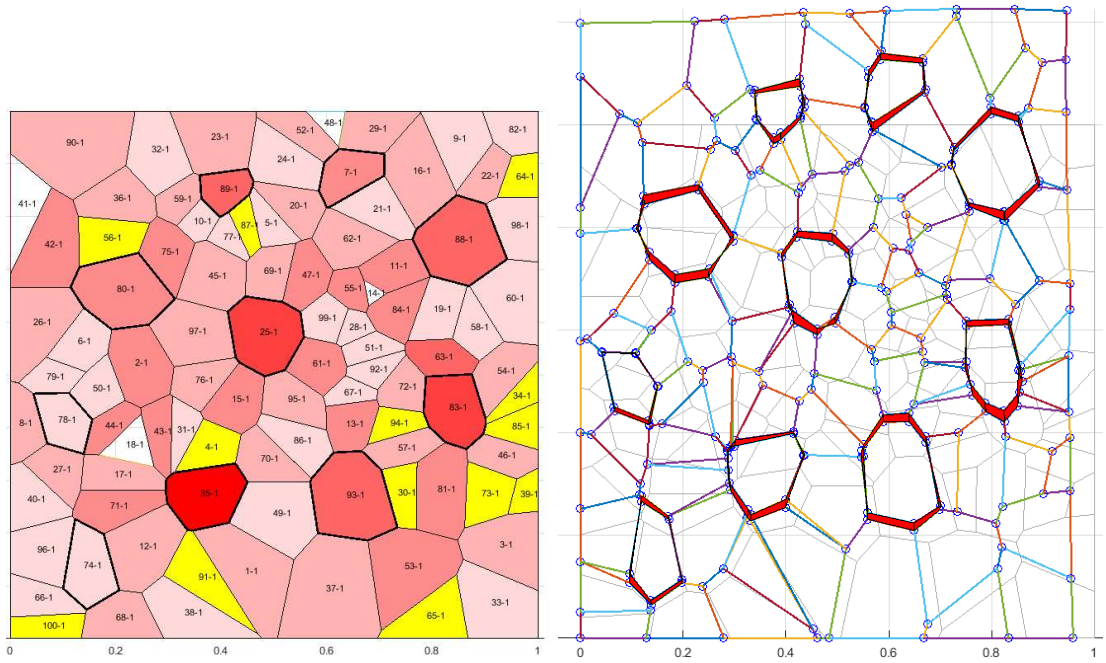
(a) (b)

Figure 5: VEM mesh and obtained pressure head.

5.2 Integrating interface elements for Meso-scale simulations

Meso-scale simulations of heterogeneous materials (like concrete) must represent geometrically the different components of the material, with their different properties. If there exist some component much more rigid than the surrounding material, the versatility of handling arbitrary polynomials can be exploited to reduce the number of degrees of freedom and meshing the rigid element as a single Virtual Element. Since only a relatively small deformation is expected to appear in the more rigid element, not much error is introduced in the solution component with a

clear reduction in degrees of freedom. Since meshing complex aggregate geometry is no longer needed, there will be an improvement in element quality despite the possible presence of small edges and angles. Traditional interface elements are added on edges of the triangulation made up by Virtual Elements. The interfaces can be introduced regardless of the shape of the Virtual Element. Figure 6 illustrates the idea of combining interface elements around the aggregates that are discretized using only one arbitrary-shaped element, where a random polygonal mesh with interface elements (bold lines) is shown before and after the application of a load. Once again, element coloring indicates number of edges, and deformation of interface elements is shown in red.



(a) Before loading (b) After loading

Figure 6: Polygonal mesh with interface elements around aggregates.

One of the limiting factors in cracking analysis is the need to introduce *a priori* crack surfaces, i.e., to propose a potential failure path before the analysis. Complex material models usually require large computational effort and therefore it is desirable to reduce the number of degrees of freedom. This limits the possibility of introducing many possible failure paths in the form of interface elements that increase the computational effort (Oliver et al., 2006; Wells and Sluys, 2001). Thus a new approach for dealing with these problems using virtual elements consists on simply running a test analysis to determine critical areas. Using the same mesh and only introducing interfaces at these areas preserves most of the original mesh, aside from the new polygonal elements included. The key ingredient here is to use the capibilities of the VEM as a domain decomposition method and the ability to introduce fracture paths at any angle without the need to remesh to have all triangular or quadrilateral elements.

6 CONCLUDING REMARKS

Since the introduction of the VEM is very recent, many applications other than the mentioned ones are expected to surface in the foreseeable future. The introduction of this new technology in numerical methods always leads to new insight into the nature of the problem under study.

We believe that the preliminary results and ideas are very promising and could lead to a new viable framework for studying materials and cracking problems while avoiding some of the issues that have historically plagued the study of these types of problems.

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REFERENCES

Baky H.A., Ebead U., and Neale K. Nonlinear micromechanics-based bond slip model for frp/concrete interfaces. *Engineering Structures*, 39(0):11 – 23, 2012.

Beirão da Veiga L., Brezzi F., Cangiani A., Manzini G., Marini L.D., and Russo A. Basic principles of virtual element methods. *Math. Models and Methods in Applied Sciences*, 2013.

Beirão da Veiga L., Lovadina C., and Mora D. A virtual element method for elastic and inelastic problems on polytope meshes. *Computer Methods in Applied Mechanics and Engineering*, 295:327 – 346, 2015.

Beirão da Veiga L., Brezzi F., Marini L.D., and Russo A. Virtual element method for general second-order elliptic problems on polygonal meshes. *Mathematical Models and Methods in Applied Sciences*, 26(04):729–750, 2016.

Benedetto M.F., Berrone S., Borio A., Pieraccini S., and Scialò S. A hybrid mortar virtual element method for discrete fracture network simulations. *Journal of Computational Physics*, 306:148 – 166, 2016a. ISSN 0021-9991. doi:http://dx.doi.org/10.1016/j.jcp.2015.11.034.

Benedetto M.F., Berrone S., Pieraccini S., and Scialò S. The virtual element method for discrete fracture network simulations. *Computer Methods in Applied Mechanics and Engineering*, 280:135–156, 2014.

Benedetto M.F., Berrone S., and Scialò S. A globally conforming method for solving flow in discrete fracture networks using the virtual element method. *Finite Elements in Analysis and Design*, 109:23 – 36, 2016b.

Brezzi F., Beirão da Veiga L., , and Marini L.D. Virtual elements for linear elasticity problems. *SIAM Journal on Numerical Analysis*, 51(2):794–812, 2013. doi:10.1137/120874746.

Brezzi F., Lipnikov K., and Simoncini V. A family of mimetic finite difference methods on polygonal and polyhedral meshes. *Mathematical Models and Methods in Applied Sciences*, 15(10):1533–1551, 2005.

Caggiano A. and Etse G. Coupled thermo–mechanical interface model for concrete failure analysis under high temperature. *Computer Methods in Applied Mechanics and Engineering*, 289:498–516, 2015.

Caggiano A., Etse G., and Martinelli E. Zero-thickness interface model formulation for failure behavior of fiber-reinforced cementitious composites. *Computers & Structures*, 98:23–32, 2012.

Carol I., Prat P.C., and López C.M. Normal/shear cracking model: application to discrete crack analysis. *Journal of engineering mechanics*, 123(8):765–773, 1997.

Chen J., Crisfield M., Kinloch A.J., Busso E.P., Matthews F., and Qiu Y. Predicting progressive delamination of composite material specimens via interface elements. *Mechanics of composite materials and structures*, 6(4):301 – 317, 1999.

Ciancio D., Carol I., and Cuomo M. A method for the calculation of inter-element stresses in

3d. *Computer Methods in Applied Mechanics and Engineering*, 254(0):222 – 237, 2013.

Costa Daguiar S., Modaressi-Farahmand-Razavi A., Dos Santos J.A., and Lopez-Caballero F. Elastoplastic constitutive modelling of soil âĂŞ structure interfaces under monotonic and cyclic loading. *Computers and Geotechnics*, 38(4):430 – 447, 2011.

Häfner S., Eckardt S., Luther T., and Könke C. Mesoscale modeling of concrete: Geometry and numerics. *Computers & structures*, 84(7):450–461, 2006.

Klein R. *Concrete and abstract Voronoi diagrams*. Lecture Notes in Computer Science, Springer-Verlag, Berlin, 1989.

Lei X. Contact friction analysis with a simple interface element. *Computer Methods in Applied Mechanics and Engineering*, 190(15 - 17):1955 – 1965, 2001.

López C.M., Carol I., and Aguado A. Meso-structural study of concrete fracture using interface elements. i: numerical model and tensile behavior. *Materials and structures*, 41(3):583–599, 2008.

Lorefice R., Etse G., and Carol I. Viscoplastic approach for rate-dependent failure analysis of concrete joints and interfaces. 45(9):2686–2705, 2008.

Oliver J., Huespe A., and Sanchez P. A comparative study on finite elements for capturing strong discontinuities: E-fem vs x-fem. *Computer methods in applied mechanics and engineering*, 195(37):4732–4752, 2006.

Stankowski T., Runesson K., and Sture S. Fracture and slip of interfaces in cementitious composites. I: Characteristics, II: Implementation. *ASCE - J. Engrg. Mech.*, 119(2):292–327, 1993.

Wells G. and Sluys L. A new method for modelling cohesive cracks using finite elements. *International Journal for Numerical Methods in Engineering*, 50(12):2667–2682, 2001.

Wriggers P. and Moftah S. Mesoscale models for concrete: Homogenisation and damage behaviour. *Finite elements in analysis and design*, 42(7):623–636, 2006.

Xie D. and Jr. S.B.B. Progressive crack growth analysis using interface element based on the virtual crack closure technique. *Finite Elements in Analysis and Design*, 42(11):977 – 984, 2006.