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MACROSCOPIC DESCRIPTION OF THE FORCED CONVECTION PROCESS IN POROUS MEDIA FLOWS

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Abstract. As in the clear flow case, the forced convection process in porous media flows can be characterized by a thermally developing region and a fully developed region. In this study, microscopic laminar flow simulations are carried out in a bi-dimensional porous medium to study the characteristics of each region. For the two porosities simulated, 65% and 85%, the spatial variation of the macroscopic heat transfer coefficient and the streamwise thermal dispersion is computed for different *Pe* numbers. The dependence of the thermal entry length on the *Pe* number is also computed. This quantity is found to be larger than five Representative Elementary Volumes for *Pe* numbers larger than 2500. This shows that, for high *Pe* numbers, the thermal entrance effect is not limited to the pore-scale and that macroscopic models used in the open literature are only accurate in the fully developed region.

1 INTRODUCTION

Porous media models are often employed to describe systems that are difficult to simulate in detail with the computer power that is available today. For example, the design of a heat exchanger requires the analysis of the heat transfer process in a heterogeneous system formed by a complex solid matrix filled with a fluid. In general, these type of systems require a precise description of geometrical details that makes their simulation computationally expensive. This motivates the continuous development of macroscopic models that do not resolve the small scale of the system yielding a less expensive computational description (e.g. (Takemoto et al., 2010; Pathak and Ghiaasiaan, 2011; Vijay et al., 2015)). The Volume Average Theory (VAT) (Whitaker, 1999) is the most popular tool to derive macroscopic model (i.e. macroscopic partial differential equations) that can be numerically solved to obtain a macroscopic description of the system. One of the simplest but not trivial cases that can be studied from the macroscopic point of view is an isothermal fluid that enters to a constant porosity porous medium with a constant wall temperature different than that in the fluid. From the macroscopic point of view, the momentum equations for this case simple reduce to a constant volume average velocity in the streamwise direction. However, the macroscopic energy equation must be capable to accurately describe the behavior of the fluid-average temperature when the fluid flows and transfers heat from/to the solid structure. Under considerations of steady, incompressible, one-dimensional flow (x-direction) in a constant porosity medium with constant wall temperature, the solid phase is uncoupled from the fluid phase and the transport equation for the macroscopic fluid temperature, findable elsewhere (Nakayama et al., 2006), resumes:

$$\rho C_p \phi \overline{U} \frac{d\overline{T}}{dx} = \frac{d}{dx}[\phi(k_f + k_{D-xx})\frac{d\overline{T}}{dx}] + h_{sf} a_{sf}(T_w - \overline{T}), \tag{1}$$

where the over bar indicates the intrinsic Cellular Average (CA) operation (Quintard and Whitaker, 1994a,b) over the Representative Elementary Volume (REV, V) and, ϕ, $\overline{U}$, a_{sf} are the porosity, streamwise velocity and interfacial area per unit volume respectively. Equation 1 is a convection-diffusion equation with the CA temperature as dependent variable. This equation is characterized by two macroscopic coefficients, the interfacial heat transfer (h_{sf}) and the streamwise thermal dispersion (k_{D-xx}). These two coefficients are generally defined from modeling assumptions and conservation criteria. The interfacial heat transfer, or its equivalent non-dimensional number, Nu_D, is defined to assure the conservation of energy (Nakayama et al., 2006):

$$Nu_D = \frac{h_{sf} D}{k_f} = \frac{D}{k_f} \frac{\frac{1}{V}\int_V [\frac{1}{V}\int_{A_{sf}} k_f \nabla T \cdot d\bar{A}] dV}{a_{sf}(T_w - \overline{T})}. \tag{2}$$

And the streamwise thermal dispersion is defined employing a diffusion hypothesis following the ideas of Taylor (Taylor, 1953) and Aris (Aris, 1956):

$$k_{D-xx} = -\rho C_p \frac{\frac{1}{V_f}\int_V [\frac{1}{V}\int_V {}^i u {}^i T dV] dV}{\nabla_x \overline{T}}, \tag{3}$$

where ${}^i u$, ${}^i T$ are the space fluctuations of the streamwise velocity and temperature respectively, and V_f is the fluid volume inside the REV. These two macroscopic coefficients were studied in detail in the flow configuration under consideration in Teruel (2015) for different porosities than those simulated in this study. In this work, the macroscopic energy Eq. (1) will be tested

evaluating its accuracy to represent the macroscopic fluid temperature in the porous medium. This will be achieved comparing results from the model with those obtained from numerical experiments. Additionally, the common assumption found in the open literature that considers constant values for the macroscopic coefficients Nu_D y k_{D-xx}, will be tested. Under this assumption, the macroscopic model given in Eq. (1) has an analytical solution characterized by an exponential decay:

$$\theta(x^*) = \frac{\overline{T}(x^*) - T_w}{T_i - T_w} = e^{-\alpha x^*}, \tag{4}$$

where T_w and T_i are the solid wall and the inlet fluid temperatures respectively and x^* is the non-dimensional streamwise coordinate. In Eq. (4), the decay rate, α, is a function of the macroscopic coefficients as well as the flow and medium properties:

$$\alpha = \frac{1 - \sqrt{1 + 4AC}}{2A}, A = \frac{\sqrt{1-\phi}}{2Pe_D}(1 + \frac{k_{D-xx}}{k_f}), C = \frac{8Nu_D}{Pe_D}\sqrt{1-\phi}. \tag{5}$$

Therefore, the α parameter can be computed from numerical data and compared with values calculated from model equations.

2 DOMAIN SIMULATED

A schematic diagram of the domain selected for the simulation is shown in Fig. 1. The fluid flows from left to right, entering the porous medium after flowing a distance of H as a clear flow. The porous medium extends in the streamwise direction from $x = 0$ to a location $x = 140H$. Therefore, the porous region extends for 70 REVs in a row (the REV is chosen as a cell of $2H \times H$ in the streamwise and spanwise directions respectively). The fluid-solid interface is set to a different temperature than that in the fluid at the entrance. But this is done at the location $x = 6H$ to achieve a smooth transition in the CA temperature in the entrance region and to allow the flow to develop hydrodynamically. To save computational time, only the bottom half of the REV ($H/2$) is simulated.

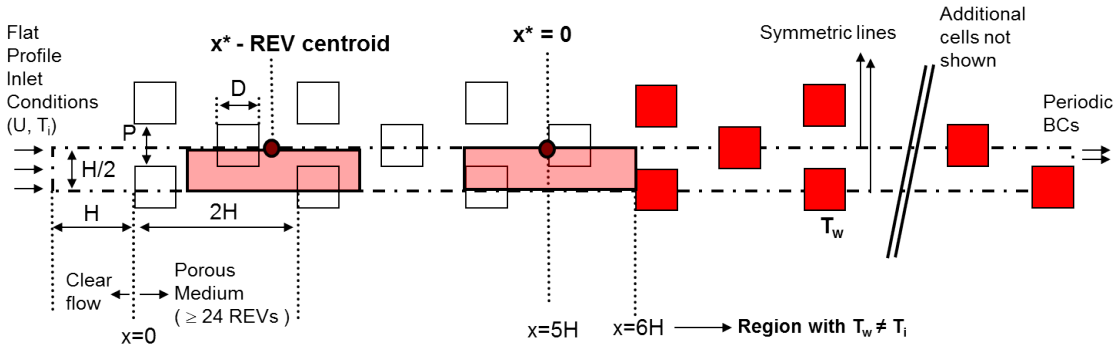


Figure 1: Domain simulated.

The governing equations for the fluid phase (mass, momentum and energy) are solved employing an in-house solver that uses the SIMPLER algorithm (additional details of the solver can be found in Teruel (2015)). Boundary conditions are given as follows. For the velocity, the no-slip BC is employed at the walls and periodicity is employed at the outlet (note that after a hydrodynamically developing length the flow has a $2H$ periodicity). For the thermal field, a

fixed temperature is employed at the interface and periodicity is employed at the outlet (after the flow is thermally developed, the non dimensional temperature field is periodic except for an exponential decay that is iteratively solved (Teruel, 2015)). The domain was discretized using a uniform and structured grid of squares. Macroscopic quantities reported in this study were found to be independent of any further grid refinement. The grid resolution employed for each REV was 180 x 45 ($2H \times H/2$, streamwise x vertical direction). Simulations were carried out for laminar flow conditions with Re_D=50 (based on the Darcy velocity and size of the obstacles). The Pe_D number, defined as $Re_D Pr$, was varied from 200 to 5000. For each simulation carried out, the macroscopic temperature and macroscopic coefficients are obtained applying the CA operation to microscopic results. For instance, for each macroscopic point x (the centroid of a given REV), the CA temperature is calculated as:

$$\overline{T}(x) = \frac{1}{2H}\int_{-H}^{H} d\zeta[\frac{1}{\phi H^2}\int_{-H}^{H} d\eta \int_{0}^{H/2} T(x+\zeta+\eta, y)dy]; \tag{6}$$

and the space-fluctuation of this averaged quantity is defined as:

$$^{i}T(x,y) = T(x,y) - \overline{T}(x). \tag{7}$$

3 DEVELOPING REGION

The microscopic (i.e. real geometry) results are shown in Fig. 2. Contours of the non-dimensional temperature are shown for both porosities simulated and $Pe_D = 1000$. The color pattern clearly shows that the fluid heats faster in the medium with larger interfacial area or lower porosity.

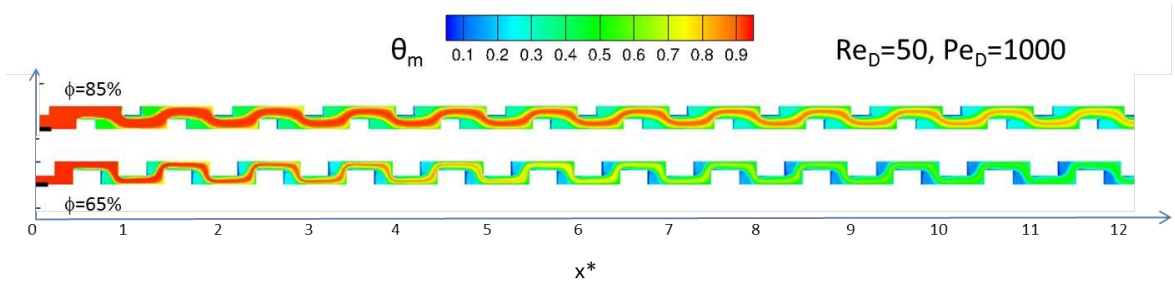
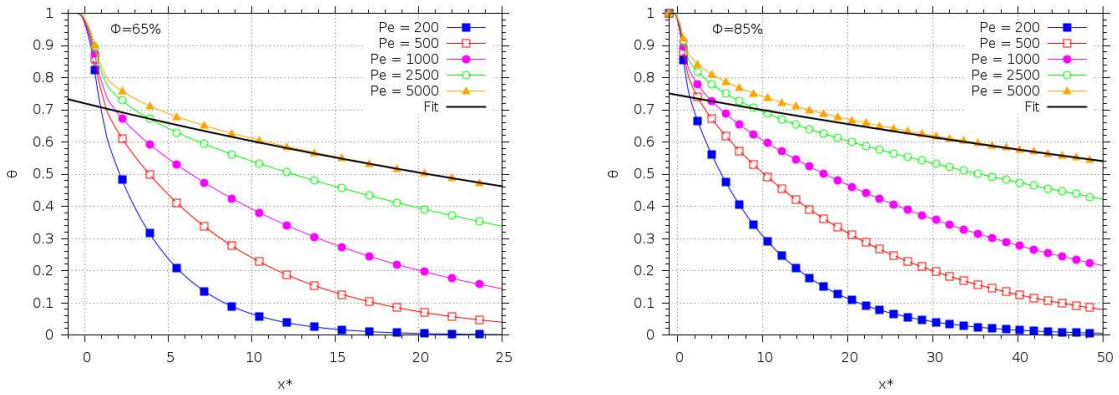


Figure 2: Non-dimensional microscopic temperature θ_m.

Based on microscopic results, the macroscopic temperature is computed carrying out the CA operation. The complete set of macroscopic non-dimensional temperatures are shown in Fig. 3. Qualitatively, it is shown that the decay rate of this macroscopic variable decreases with the porosity and Pe_D. Additionally, it is shown that the behavior of this quantity does not follow an exponential decay from the inlet. This is exemplified showing a line that fits the data corresponding to the last 10 REVs simulated with an exponential decay (as in Eq. (4)). This line is only shown for the cases with the largest Pe_D simulated. The fit deviates from the data near the inlet showing the existence of a developing region.

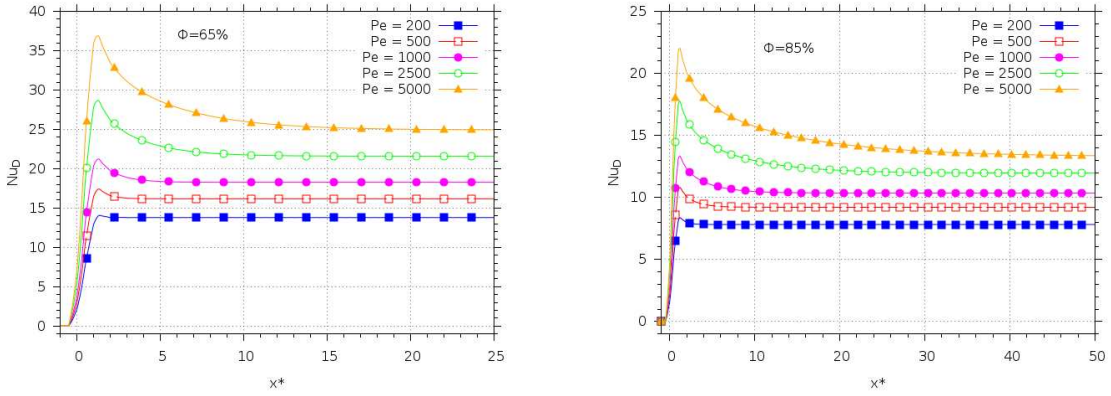
The macroscopic coefficients, Nu_D and k_{D-xx}, can also be computed as a function of the streamwise coordinate employing Eqs. (2) y (3). Fig. 4 shows calculated values for the macroscopic heat transfer coefficient. This quantity, similar to that found in the clear flow case in the developing region, presents a peak at the entrance followed by a decay to its fully developed



(a) 65% porosity. (b) 85% porosity.

Figure 3: Macroscopic non-dimensional temperature. Pe_D as parameter.

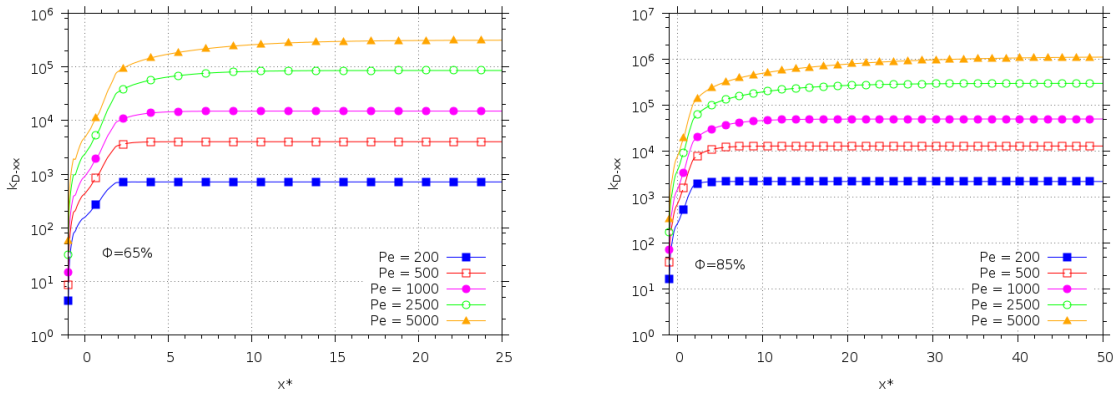
value. A space dependent quantity is also obtained for the streamwise thermal dispersion. This quantity is shown in Fig. 5. This diffusion coefficient increases from zero at the inlet to a fully developed value in a distance that depends on the porosity and Pe_D.



(a) 65% porosity. (b) 85% porosity.

Figure 4: Macroscopic heat transfer coefficient.

The deviation of the macroscopic temperature from the exponential fit and the dependence on the spatial coordinate of the macroscopic coefficients suggest, by comparison with the clear flow case, the need of a definition for a thermally developing region characterized by a thermal entrance length. This quantity can be defined considering the value of the parameter that controls the behavior of the macroscopic temperature in the developed region, α (see Eq. (4)). When this parameter is approximately constant, the macroscopic temperature can be assumed to vary exponentially. The Thermal Entry Length (TEL) is then defined in this study as the position in the porous medium from where the α-parameter, calculated with Eq. (5), shows a difference lower than 5% respect to its fully developed value. Although Eq. (5) is strictly valid when the macroscopic coefficents are constants, it is an excellent approximation when these coefficients are close to their fully developed values. With this definition, the TEL is then calculated for all cases simulated. Fig. 6 shows the dependence of this quantity on Pe_D and porosity. For large Péclet numbers this quantity extends for several REVs in a row (this fact is



(a) 65% porosity. (b) 85% porosity.

Figure 5: Streamwise thermal dispersion coefficient.

emphasized for high porosities). This suggests that the use of a macroscopic model with constant coefficients, as those found in the open literature (Kuznetsov and Nield, 2009; Sano et al., 2011; Alfieri et al., 2012; Ouyang et al., 2013), yields large deviations from the data for flow with high Pe_D numbers.

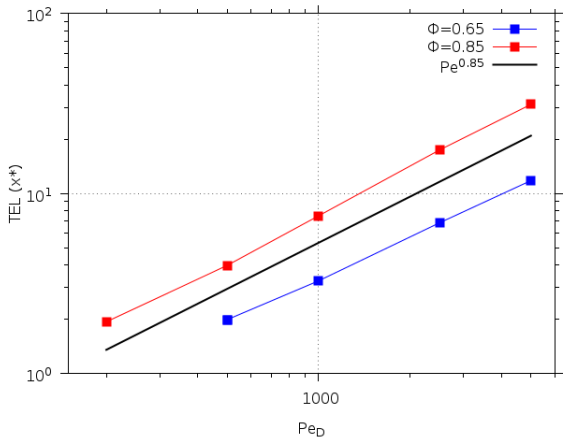


Figure 6: TEL computed for different Pe_D numbers. ϕ as parameter.

4 FULLY DEVELOPED REGION

In the fully developed region the macroscopic coefficients are constant and the macroscopic model predicts an exponential decay rate of the temperature. This decay rate can be compared with that obtained from the data to evaluate the accuracy of the model in the fully developed region. To obtain the experimental decay rate, the macroscopic data for the temperature is fitted near the outlet of the domain, in the last five to ten REVs, depending on the case. The correlation-coefficient for the fit was calculated to be greater than 0.9999 in all cases, insuring that a linear regression in a linear-log scale is an appropriate fitting model. Fig. 7 shows the decay rate computed with both methods, fitting the data and employing Eq. 5. The agreement is excellent (differences are below 1% for all cases simulated) showing that the macroscopic model can accurately predict the behavior of the macroscopic temperature in the fully developed region.

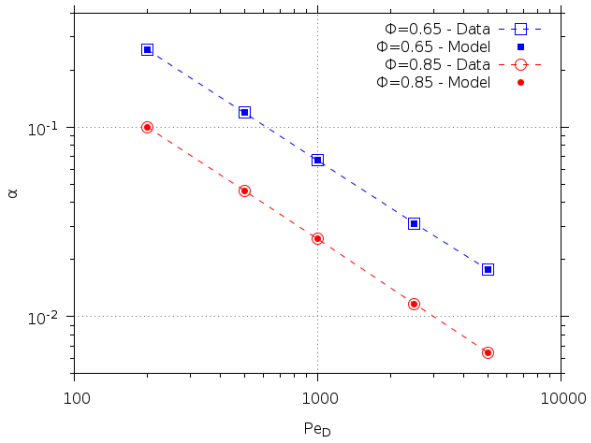


Figure 7: Values for α. Meassured and computed with the model.

5 CONCLUSIONS

The performance of the macroscopic energy equation model for laminar flows through porous media has been analyzed. Two regions have been identified, a developing region where the macroscopic coefficients are space dependent and a fully developed region where the macroscopic temperature decays exponentially. The thermal entry length dependence on Pe_D has been calculated. This parameter extends for several REVs for Péclet numbers larger than 2500. This suggest that the use of a macroscopic model with constant coefficients in flows with high Pe_D is questionable and that novel models are needed to accurately describe the developing region. On the contrary, the macroscopic model was shown to be very accurate in the fully developed region.

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