Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXXVI, págs. 751-760 (artículo completo)
José G. Etse, Bibiana M. Luccioni, Martín A. Pucheta, Mario A. Storti (Eds.)
San Miguel de Tucumán, 6-9 Noviembre 2018

A KRIGING APPROACH FOR THE PROBABILITY OF FAILURE MINIMIZATION IN FTMD DESIGN

Fabio Nascentes[a,b], **Rafael H. Lopez**[a], **Jose Eduardo Souza de Cursi**[c], **Rubens Sampaio**[d] **and Leandro F.F. Miguel**[a]

[a]*Center for Optimization and Reliability in Engineering (CORE), Universidade Federal de Santa Catarina, Rua João Pio Duarte Silva, sn, Florianopolis, Brasil, fabiotrabmat@gmail.com, rafael.holdorf@ufsc.br, leandro.miguel@ufsc.br http://www.core.ufsc.br*

[b]*Departamento de Áreas Acadêmicas, Instituto Federal de Educação, Ciência e Tecnologia de Goiás - IFG, Jataí, 75804-714, Brasil https://www.ifg.edu.br/*

[c]*Department Mecanique, Institut National des Sciences Appliquees (INSA) de Rouen, 76801 Saint Etienne du Rouvray, CEDEX, France, eduardo.souza@insa-rouen.fr, http://www.insa-rouen.fr*

[d]*Departamento de Engenharia Mecanica, PUC-Rio, Rua Marquês de São Vicente, 225, 22453-900, RJ, Brazil, rsampaio@puc-rio.br, http://www.mec.puc-rio.br/dem_perfil.php?id=43*

Keywords: Efficient Global Optimization, Probability of failure Minimization, Multimodal optimization, Friction Tunned Mass Damper.

Abstract. This paper aims at presenting an efficient approach for the optimal design of friction tunned mass damper (FTMD) devices under uncertainties. The objective function of the optimization problem is the maximization of structural reliability, which is approximated here as an out-crossing problem. The solution of the equations of motion of the FTMD system leads to a nonlinear dynamical problem, which coupled with the time dependent reliability problem, requires a substantial computational effort. An additional complexity is that the design of a multiple FTMD system is nonlinear and leads to a multimodal optimization problem. In order to address the issues of computational cost and multi-modality, an Efficient Global Optimization (EGO) method with Expected Improvement as infill criterion is employed. The results showed that the EGO was able to successfully provide the optimum solution of the FTMD design under uncertainty within a reasonable computational effort. For example, using only 80 points, the EGO algorithm was able to consistently find the optimum solution for all the cases analyzed in this paper. One aspect that is worth to be highlighted is that these results were obtained in problems with a relatively high stochastic dimension, e.g. over 30 random variables.

1 INTRODUCTION

A fast increase in development and application of passive energy dissipation devices, such as viscoelastic dampers, friction dampers and tuned mass dampers (TMD) has been observed. One of the main applications of such devices is to reduce the dynamic response of structures subject to earthquake excitation. The design of these vibration attenuation devices must incorporate the uncertainties inherent to the structural system and excitation into the design optimization process (Marano et al., 2010), leading to an optimization under uncertainties problem (Ritto et al., 2011; Lopez and Beck, 2013; Lopez et al., 2014). In civil engineering structures, the main objective in the design of these dampers is the minimization of the structural probability of failure (or maximization of its counterpart, the structural reliability) (Taflanidis et al., 2007; Lopez et al., 2015).

A literature survey reveals that most of the TMD studies is associated with the hypothesis of linear behavior for the structure and for the dampers (Gewei and Basu, 2010). In this sense, researches involving some non linearity of this devices, such as the consideration of a friction damping, which is the case of the friction tuned mass dampers (FTMDs), are significantly reduced (Mantovani et al., 2017). One of the issues that appears with the inclusion of this nonlinearities is the associated high computational cost, even for deterministic analysis. Consequently, a well-known drawback of the coupling of the nonlinear dynamics, optimization procedure and reliability analysis is the high computational cost, which may be inviable. Furthermore, an additional complexity is that the design of a multiple FTMD system leads to a multimodal optimization problem (Fadel Miguel et al., 2016; Mantovani et al., 2017). That is, local optimization method may get stuck in local minima, and global optimization methods are normally required.

In order to find a trade off between computational cost and global optimization, this paper investigates the performance of Efficient Global Optimization (EGO) methods for the probability of failure minimization in FTMD design. The Expected Improvement (EI) infill criterion is employed to guide the EGO search, since it presented robust results in a series of optimization problems (Jones, 2001). Moreover, the time dependent reliability problem is approximated using a out-crossing approach (Melchers and Beck, 2018; Lopez et al., 2015).

The rest of the paper is organized as follows: Section 2 presents the problem of optimal design of passive control devices for probability of failure minimization. The EGO algorithm employed is detailed in Section 3. The design under uncertainty of a 2-FTMD system is analyzed in Section 4. Finally, the main conclusions drawn from this paper are summarized in Section 5.

2 OPTIMAL DESIGN OF PASSIVE CONTROL DEVICES

In this paper, we are interested in designing passive control devices that minimize the probability of failure of the structures subject to seismic excitations. Then, the optimal passive control design optimization problem may be posed as:

$$\text{Find: } \mathbf{d} \in \mathbb{R}^{n_d} \tag{1}$$

that

$$\text{Minimizes } y(\mathbf{d}) = P_f(\mathbf{d}) \tag{2}$$

subject to

$$\mathbf{d}_{\text{min}} \leq \mathbf{d} \leq \mathbf{d}_{\text{max}}, \tag{3}$$

where y is the objective function given by the P_f is the structural probability of failure, $\mathbf{d}$ is the design vector, while $\mathbf{d}_{\text{min}}$ and $\mathbf{d}_{\text{max}}$ are, respectively, the lower and upper bounds of the design variables. For instance, in the FTMD design, $\mathbf{d}$ is comprised by the stiffness (k_{F_i}) and friction force magnitude (f_{F_i}) of each FTMD device, while $\mathbf{d}_{\text{min}}$ and $\mathbf{d}_{\text{max}}$ are the manufacturing limits of these variables.

As already mentioned, we use the design of multiple FTMD system as case study in this paper. A full description of the nonlinear dynamics of the FTMD model may be found in Mantovani et al. (2017). For the reliability analysis and optimization process, we model all the FTMD, structural and excitation parameters as random variables and group them into the random vector $\mathbf{X}$. That is, $\mathbf{X}$ is comprised by: (a) the stiffness, damping and masses of the structure, (b) masses, stiffness and friction forces of the control system ($k_{F_1}, ..., k_{F_{n_F}}, m_{F_1}, ..., m_{F_{n_F}}, f_{F_1}, ..., f_{F_{n_F}}$), and (c) parameters of the excitation (S_0, ξ_g and ω_g). In such a situation, the design variables of the problem are the mean value of the stiffness and friction forces of the FTMDs, i.e. $\mathbf{d} = \left(\mathbb{E}[k_{F_1}], ..., \mathbb{E}[k_{F_{n_F}}], \mathbb{E}[f_{F_1}], ..., \mathbb{E}[f_{F_{n_F}}]\right)$.

The evaluation of P_f of a building subject to seismic loads leads to a time dependent reliability problem (Melchers and Beck, 2018; Lopez et al., 2015). Thus, in order to compute Eq. (2), we employ the out-crossing rate approach, which is detailed in the next subsection.

2.1 Time-dependent reliability analysis

The time-variant reliability problem for the random system response displacement can be formulated as follows. During a zero mean excitation event of specified duration t_E, the response of the oscillator should not exceed the specified limit - or barrier - $\pm b$. This barrier b can be the relative displacement between floors, displacement of top floor or any other critical measure. For a linear system excited by a zero mean Gaussian process, the response is Gaussian and the up-crossing rate can be evaluated as

$$v_z^+(\mathbf{d}, \mathbf{X}) = \frac{\sigma_{\dot{z}}(\mathbf{d}, \mathbf{X})}{\sigma_z(\mathbf{d}, \mathbf{X})} \frac{1}{2\pi} \exp\left(-\frac{b^2}{2\left(\sigma_z(\mathbf{d}, \mathbf{X})\right)^2}\right), \tag{4}$$

where σ_z and $\sigma_{\dot{z}}$ are the standard deviation of the displacement and of the velocity response, respectively. These quantities are obtained from the solution of the nonlinear dynamics model presented in Mantovani et al. (2017). In Eq. (4), we make explicit the dependence of the crossing rate on the design variables $\mathbf{d}$ as well as the random parameters $\mathbf{X}$ of the problem. Thus, considering a stationary excitation, the probability of a failure event F of a given duration t_E may be computed as

$$P(F|\mathbf{d}, \mathbf{X}, t_E) = 1 - \exp\left(-2\int_0^{t_E} v_z^+(\mathbf{d}, \mathbf{X})dt\right), \tag{5}$$

$$= 1 - \exp\left(-2t_E v_z^+(\mathbf{d}, \mathbf{X})\right). \tag{6}$$

$$\tag{7}$$

The structural loading from an earthquake, which is the application topic, is described by the arrival of an unknown number of events, which is modeled here as a Poisson process. Consequently, the probability of failure, for a design life t_D with a number of events n_e, may be evaluated as

$$P(F|\mathbf{d}, \mathbf{X}, t_D) = \sum_{i=1}^{\infty} P(F|\mathbf{d}, \mathbf{X}, t_E, n_e = i)\; P(n_e = i|t_D), \tag{8}$$

where

$$P(F|\mathbf{d}, \mathbf{X}, t_E, n_e = i) = 1 - (1 - P(F|\mathbf{d}, \mathbf{X}, t_E))^i, \tag{9}$$

$$P(n_e = i|t_D) = \frac{(\nu\, t_D)^i \exp(-\nu\, t_D)}{i!}, \tag{10}$$

in which ν is the arrival rate of events. Note that the probability in Eq. (8) still depends on the random vector $\mathbf{X}$, characterized by its joint probability density function $f_{\mathbf{X}}$. Consequently, in order to compute the resulting structural failure probability (P_f), we must then employ the Total Probability Theorem, leading to

$$P_f(\mathbf{d}) := \mathbb{E}_{\mathbf{X}}[P(F|\mathbf{d}, \mathbf{X}, t_D)] \tag{11}$$

$$= \int_{\mathbf{X}} P(F|\mathbf{d}, \mathbf{x}, t_D) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}, \tag{12}$$

where $\mathbb{E}$ is the expected value operator and $P_f(\mathbf{d})$ is the objective function to be minimized in the optimization process. For the computational implementation, we may approximate Eq. (12) using MCI by

$$P_f(\mathbf{d}) \approx \hat{P}_f(\mathbf{d}) = \frac{1}{n_r} \sum_{i=1}^{n_r} P\left(F|\mathbf{d}, \mathbf{x}_{(i)}, t_D\right), \tag{13}$$

where $\mathbf{x}_{(i)}$ are samples of $\mathbf{X}$ that comprise the sample set $\{\mathbf{x}_{(1)}, ..., \mathbf{x}_{(n_r)}\}$ drawn from $f_{\mathbf{X}}$. Then, we must set a sample size n_r for the estimation of $\hat{P}_f$. For this purpose, we employ the following procedure widely adopted in the literature of robust design (Capiez-Lernout and Soize, 2008; Soize et al., 2008; Ritto et al., 2011; Lopez et al., 2015; Miguel et al., 2016b,a). First, we construct a plot of the estimation of $\hat{P}_f$ with respect to the sample size (n_r). Then, we set n_r as the sample number around the value that $\hat{P}_f$ becomes stable.

In the numerical analysis section, to make it easier to visualize the results, we present them in terms of reliability index, instead of probability of failure. The relation between them is given by

$$\hat{\beta}(\mathbf{d}) := -\Phi^{-1}(\hat{P}_f(\mathbf{d}), \tag{14}$$

and the problem becomes the maximization of $\hat{\beta}$.

One may easily see that the computation of Eq. (13) is demanding, since it requires hundreds of calls of the FE dynamics code. Hence, to mitigate the computational burden of the reliability maximization, we employ the EGO approach described in the next section.

3 EFFICIENT GLOBAL OPTIMIZATION (EGO)

According to Jones (2001), EGO methods generally follow these steps:

1. Construction of the initial sampling plan;
2. Construction of the Kriging metamodel;
3. Addition of a new infill point to the sampling plan and return to step 2.

Steps 2 and 3 are repeated until a stop criterion is met, e.g., maximum number of function evaluations. The manner in which the infill points are added in each iteration is what differs the different EGO approaches. In the next subsections, these steps are detailed in order to set the basis of the proposed approach.

3.1 Initial sampling plan

In the first step, a Kriging sampling plan $\mathbf{\Gamma}$ containing n_s points is created, i.e.

$$\mathbf{\Gamma} = \{\mathbf{d}^{(1)}, \mathbf{d}^{(2)}, \dots, \mathbf{d}^{(n_s)}\}. \tag{15}$$

A Latin Hypercube scheme is usually employed for this purpose. Then, the objective function value J of each of these points is evaluated using the original model, obtaining

$$\mathbf{y} = \{y^{(1)}, y^{(2)}, \dots, y^{(n_s)}\}, \tag{16}$$

where $y^{(i)} = \hat{\beta}\left(\mathbf{d}^{(i)}\right)$.

3.2 Deterministic Kriging

Step 2 constructs a prediction model $\hat{y}$ based on the available information of the current sampling plan - $\mathbf{\Gamma}$ and $\mathbf{y}$ - using Kriging Sacks et al. (1989). The basic idea behind Kriging is to construct a metamodel whose response at any point $\mathbf{d}$ is modeled as a realization of a stationary stochastic process. Thus, at any point on the design domain, we have a Normal random variable with mean μ and variance σ^2. Considering an initial sampling plan $\mathbf{\Gamma}$, the covariance between any two input points $\mathbf{d}^{(i)}$ and $\mathbf{d}^{(j)}$ is:

$$\text{Cov}\left[\mathbf{d}^{(i)}, \mathbf{d}^{(j)}\right] = \sigma^2\, \mathbf{\Psi}\left(\mathbf{d}^{(i)}, \mathbf{d}^{(j)}\right), \tag{17}$$

where $\mathbf{\Psi}$ is the correlation matrix, which has the form:

$$\mathbf{\Psi}\left(\mathbf{d}^{(i)}, \mathbf{d}^{(j)}\right) = \sum_{k=1}^{n} \exp\left(-\theta_k \left|d_k^{(i)} - d_k^{(j)}\right|^{p_k}\right). \tag{18}$$

The unknown parameters θ_k and p_k may be found by Maximum Likelihood Estimate (MLE), which then gives us the mean value - or average trend - and variance of the approximation:

$$\hat{\mu} = \frac{\mathbf{1}^T \mathbf{\Psi}^{-1} \mathbf{y}}{\mathbf{1}^T \mathbf{\Psi}^{-1} \mathbf{1}} \tag{19}$$

and

$$\widehat{\sigma^2} = \frac{(\mathbf{y} - \mathbf{1}\hat{\mu})^T \mathbf{\Psi}^{-1} (\mathbf{y} - \mathbf{1}\hat{\mu})}{n_s}, \tag{20}$$

where **1** is the identity matrix. With the estimated parameters, the Kriging prediction at a given point $\mathbf{d}_u$ is:

$$\hat{y}(\mathbf{d}_u) = \overbrace{\hat{\mu}}^{\text{Trend}} + \overbrace{\mathbf{r}^T \mathbf{\Psi}^{-1}(\mathbf{y} - \mathbf{1}\hat{\mu})}^{\text{Model uncertainty}}, \tag{21}$$

where $\mathbf{r}$ is the vector of correlations of $\mathbf{d}_u$ with the other n_s Kriging sampled points. The second term in the righ-hand-side of Eq. (21) may be view as the model uncertainty since its value is inferred based on the function value of the points of the sampling plan.

One of the key benefits of kriging and other Gaussian process based models is the provision of an estimated error in its predictions. The Mean Squared Error (MSE), derived by Sacks et al. (1989) using the standard stochastic process approach reads:

$$s^2(\mathbf{d}) = \widehat{\sigma^2}\left[1 - \mathbf{r}^T\mathbf{\Psi}^{-1}\mathbf{r} + \frac{(1 - \mathbf{1}^T\mathbf{\Psi}^{-1}\mathbf{r})^2}{\mathbf{1}^T\mathbf{\Psi}^{-1}\mathbf{1}}\right]. \tag{22}$$

Equation (22) has the intuitive property that it is zero at already sampled points. In other words, Kriging acts as a regression model which exactly interpolates the observed input/output data, i.e. $\hat{y}(\mathbf{d}^{(i)}) = y^{(i)}$.

3.3 Expected improvement (EI) infill criterion

The idea behind the EGO infill criteria is to use the information about the uncertainty of the model given by the Kriging interpolation to guide the optimization search. In this paper, we employ the EI infill criterion, which estimates the amount of improvement expected at a given point in the domain. Such an improvement at a given design point $\mathbf{d}$ is evaluated using its Kriging prediction $\hat{y}(\mathbf{d})$ and variance $s^2(\mathbf{d})$. If the best solution found up to the current iteration is $f_{\min} = \min\left\{y^{(1)}, y^{(2)}, \cdots, y^{(n)}\right\}$, then a improvement I may be measured as $I(\mathbf{d}) = (f_{\min} - \hat{y}(\mathbf{d}))$. As shown in Jones et al. (1998), the EI is analytically tractable and given by

$$E(I(\mathbf{d})) = [f_{\min} - \hat{y}(\mathbf{d})]\,\Phi\left(\frac{f_{\min} - \hat{y}(\mathbf{d})}{s(\mathbf{d})}\right) + s(\mathbf{d})\phi\left(\frac{f_{\min} - \hat{y}(\mathbf{d})}{s(\mathbf{d})}\right), \tag{23}$$

where $\Phi(\cdot)$ e $\phi(\cdot)$ are the Gaussian cumulative and probability density functions, respectively. Thus, at each iteration of the EGO algorithm, the point on the design domain that maximizes Eq. (23) is added to the sampling plan $\mathbf{\Gamma}$.

4 NUMERICAL EXAMPLE

In this section, we investigate the efficiency and robustness of the EGO algorithm for the probability of failure minimization of FTMD design. The optimization problem analyzed in this section is nonconvex and multimodal. We compared the performance of EGO to the Firefly Algorithm (FA) algorithm Fadel Miguel et al. (2016), which was applied for the optimum design of passive control devices Fadel Miguel et al. (2016).

The efficiency of the optimization methods is measured here as the number of objective function evaluations (OFE), which also is set as the stopping criterion for all algorithms. It is important to point out that all the algorithms employed in this section depend on random quantities. Therefore, the results obtained are not deterministic and may change when the algorithm is run several times. For this reason, when dealing with stochastic algorithms, it is appropriate to present statistical results over a number of algorithm runs Gomes et al. (2018). Thus, statistics over 10 independent runs are presented, such as the average, standard deviation (std), best and worst results.

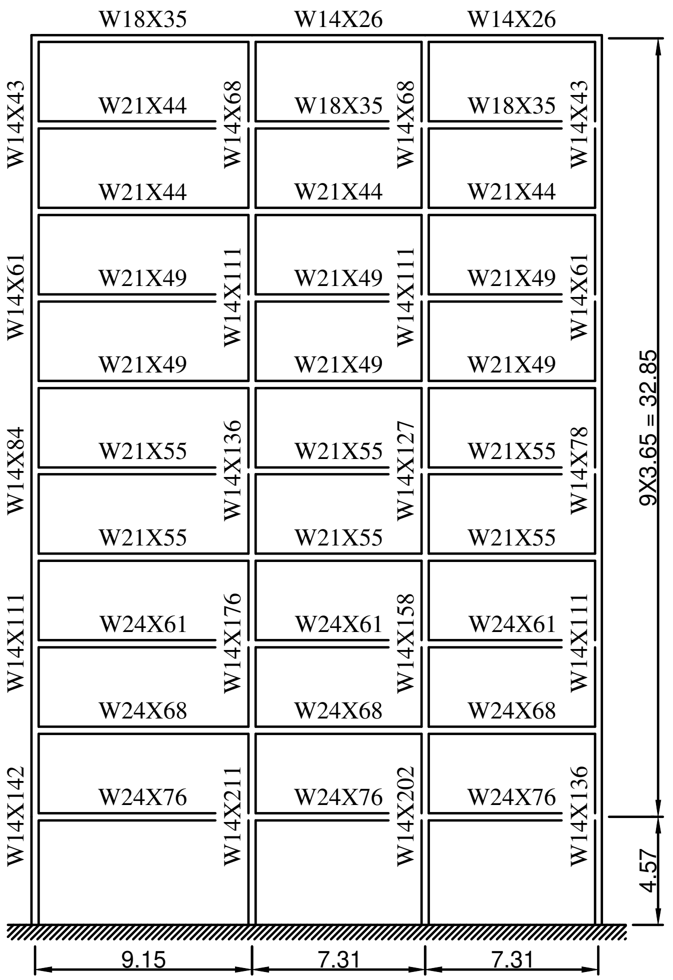

Figure 1: Planar steel frame (dimensions in m): Adapted from Bertero and Kamil (1975)

The design of a 2-FTMD passive control system is analyzed. The structure, taken from Bertero and Kamil (1975) and illustrated in Figure 1, is modeled as a planar steel building frame, with ten stories (37.42 m high) and three spans (23.77 m wide). The linear elastic finite element model is discretized in 70 elements and 44 nodes, totalizing 132 degrees of freedom. It is adopted a consistent mass matrix and the classical Rayleigh proportional damping matrix.

As already mentioned, the seismic excitation, structural and FTMD parameters are modeled as random variables and grouped into the vector $\mathbf{X}$, whose mean value and coefficient of variation are detailed in Table 1.

For the optimization problem, the design vector is $\mathbf{d} = (\mathbb{E}[k_{F_1}], \mathbb{E}[f_{F_1}], \mathbb{E}[k_{F_2}], \mathbb{E}[f_{F_2}])$, and its lower and upper bounds are respectively $\mathbf{d}_{\text{min}} = (0 \text{ kN/m}, 0 \text{ kN/m}, 0 \text{ kN}, 0 \text{ kN})$ and $\mathbf{d}_{\text{max}} = (66.5 \text{ kN/m}, 2.50 \text{ kN}, 66.5 \text{ kN/m}, 2.50 \text{ kN})$. In addition, we set the design life as $t_D = 50$ years in Eq. (8), and the earthquake occurrence rate as $\nu = 0.1$ (1 event every 10 years) in Eq. (10) accordingly to Lopez et al. (2015). Regarding the barrier b in Eq. (4), it is chosen as the top floor horizontal displacement limit. Its value is set here as 0.467 m, following the results obtained by Curadelli and Amani (2014) from a static nonlinear analysis (pushover), and also

Table 1: Statistical information about the random variables

Random variable	Probability distribution	Mean value	Coefficient of variation (%)
Building frame			
Elasticity modulus (steel)	Gamma	200 GPa	5.0
Specific mass (steel)	Gamma	7500 kg/m^3	5.0
Additional mass per story	Gamma	44 t	5.0
Damping ratio	Gamma	0.05	10.0
FTMD			
Random variable	Probability distribution	Mean value	Coefficient of variation (%)
Mass	Gamma	1.94 t	5.0
Stiffness	Gamma	$\mathbb{E}[k_{F_i}]$	10.0
Friction force magnitude	Gamma	$\mathbb{E}[f_{F_i}]$	10.0
Seismic excitation			
Random variable	Probability distribution	Mean value	Coefficient of variation (%)
Natural frequency of the filter	Normal	37.3 rad/s	20.0
Damping ratio of the filter	Normal	0.30	20.0
PGA	Log-normal	0.500g	20.0

employed by Mantovani et al. (2017).

For selecting the sample size required to approximate the failure probability, we employ the procedure briefly described at the end of subsection 2.1. Thus, Figure 2 shows an example of the behavior of $\hat{\beta}$ with respect to n_r in a given point of the domain. The estimator $\hat{\beta}$ becomes stable after approximately 400 simulations. Then, throughout the search, we set $n_r = 400$ and sample $\mathbf{x}_{(i)}$ always with the same seed of the random number generator.

Table 2 presents the statistics of the objective function reached by each algorithm over independent 10 runs. It presents the results of the FA having as stopping criterion OFE = 100, i.e. Eq. (14) is run 100 times, while for EGO, we set OFE = 60 and 80. We may easily see from this table that the EGO algorithm outperforms the FNM even for the OFE = 60 case.

Table 2: 2-FTMD - Statistics of the results for different optimization methods over 10 independent runs.

Optimization method	Objective function (β)			
	mean	std	best	worst
FA (OFE = 100)	3.94	0.08	4.12	3.85
EGO (OFE = 60)	4.15	0.15	4.25	3.80
EGO (OFE = 80)	4.19	0.11	4.25	3.89

5 CONCLUDING REMARKS

This paper aimed at presenting an efficient approach for the design of FTMD under uncertainties. The objective function was set as the maximization of structural reliability, which was evaluated using an out-crossing approach. The solution of the equations of motion of the FTMD system led to a nonlinear dynamical problem, which coupled with the time dependent reliability problem, required a substantial computational effort. Moreover, the design of a multiple FTMD system is well-known to be a nonlinear and multimodal optimization problem. In order

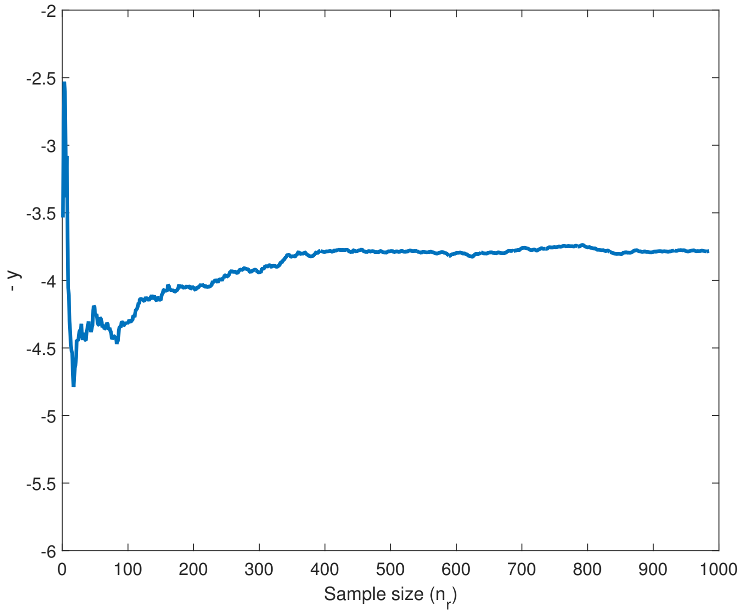

Figure 2: 2-FTMD problem: $\hat{\beta}$ with respect to n_r.

to address the issues of computational cost and multi-modality, an EGO method with EI as infill criterion was employed.

In the numerical example section, the design of a 2-FTMD system was studied. The results showed that the EGO was able to successfully provide the optimum solution of the FTMD design under uncertainty within a reasonable computational effort. For example, using only 80 points, the EGO with EI algorithm was able to consistently find the optimum solution for all the cases analyzed in this paper. One aspect that is worth to be highlighted is that these results were obtained in problems with a relatively high stochastic dimension, e.g. over 30 random variables.

The results obtained in this paper are promising and indicate further analysis of the performance of EGO on FTMD design. Natural extensions of the present work include dealing with non stationary excitations, nonlinear structural behavior and the modification of the objective function to deal with consequences of failure, i.e. risk optimization.

REFERENCES

Bertero V.V. and Kamil H. Nonlinear seismic design of multistory frames. *Canadian Journal of Civil Engineering*, 2(4):494–516, 1975.

Capiez-Lernout E. and Soize C. Robust design optimization in computational mechanics. *Journal of Applied Mechanics, Transactions ASME*, 75(2):021001–1–021001–11, 2008.

Curadelli R.O. and Amani M. Integrated structure-passive control design of linear structures under seismic excitations. *Engineering Structures*, 81:256–264, 2014.

Fadel Miguel L.F., Lopez R.H., Miguel L.F.F., and Torii A.J. A novel approach to the optimum design of mtmds under seismic excitations. *Structural Control and Health Monitoring*, 23(11):1290–1313, 2016.

Gewei Z. and Basu B. A study on friction-tuned mass damper: harmonic solution and statistical linearization. *Journal of Vibration and Control*, 17(5):721–731, 2010.

Gomes W.J., Beck A.T., Lopez R.H., and Miguel L.F. A probabilistic metric for comparing metaheuristic optimization algorithms. *Structural Safety*, 70:59 – 70, 2018. ISSN 0167-4730.

Jones D. A Taxonomy of Global Optimization Methods Based on Response Surfaces. *Journal of Global Optimization*, 21(4):345–383, 2001. doi:doi:10.1023/a:1012771025575.

Jones D.R., Schonlau M., and William J. Efficient Global Optimization of Expensive Black-Box Functions. *Journal of Global Optimization*, 13:455–492, 1998. ISSN 09255001. doi:10.1023/a:1008306431147.

Lopez R., Ritto T., Sampaio R., and de Cursi J.S. A new algorithm for the robust optimization of rotor-bearing systems. *Engineering Optimization*, 46(8):1123–1138, 2014. doi:10.1080/0305215X.2013.819095.

Lopez R.H. and Beck A.T. *Optimization of Structures and Components*, chapter Optimization Under Uncertainties, pages 117–141. Springer, New York, 2013.

Lopez R.H., Miguel L.F.F., and Beck A.T. *Tuned Mass Dampers for Passive Control of Structures Under Earthquake Excitations*, pages 3814–3823. Springer Berlin Heidelberg, Berlin, Heidelberg, 2015. ISBN 978-3-642-35344-4. doi:10.1007/978-3-642-35344-4_215.

Mantovani G.Z., Fadel Miguel L.F., Lopez R.H., Miguel L.F.F., and Torii A.J. Optimum design of multiple friction tuned mass dampers under seismic excitations. *Proceedings of the 6th international symposium on solid mechanics, Joinville*, pages 521–535, 2017.

Marano G.C., Greco R., and Sgobba S. A comparison between different robust optimum design approaches: Application to tuned mass dampers. *Probabilistic Engineering Mechanics*, 25(1):108–118, 2010.

Melchers R.E. and Beck A.T. *Structural Reliability Analysis and Prediction*. John Wiley & Sons, 2018.

Miguel L.F., Lopez R.H., Torii A.J., Miguel L.F., and Beck A.T. Robust design optimization of tmds in vehicle–bridge coupled vibration problems. *Engineering Structures*, 126:703 – 711, 2016a. ISSN 0141-0296. doi:https://doi.org/10.1016/j.engstruct.2016.08.033.

Miguel L.F.F., Miguel L.F.F., and Lopez R.H. Failure probability minimization of buildings through passive friction dampers. *Engineering Optimization*, 25(17):869–885, 2016b. doi:10.1080/0305215X.2018.1428318.

Ritto T.G., Lopez R.H., Sampaio R., and Cursi J.E.S.D. Robust optimization of a flexible rotor-bearing system using the campbell diagram. *Engineering Optimization*, 43(1):77–96, 2011.

Sacks J., Welch W.J., Mitchell T.J., and Wynn H.P. Design and analysis of computer experiments. *Statist. Sci.*, 4(4):409–423, 1989. doi:10.1214/ss/1177012413.

Soize C., Capiez-Lernout E., and Ohayon R. Robust updating of uncertain computational models using experimental modal analysis. *Mechanical Systems and Signal Processing*, 22(8):1774–1792, 2008.

Taflanidis A.A., Beck J.L., and Angelides D.C. Robust reliability-based design of liquid column mass dampers under earthquake excitation using an analytical reliability approximation. *Engineering Structures*, 29(12):3525–3537, 2007.