Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXXVI, págs. 931-944 (artículo completo)
José G. Etse, Bibiana M. Luccioni, Martín A. Pucheta, Mario A. Storti (Eds.)
San Miguel de Tucumán, 6-9 Noviembre 2018

ONE ALONE MAKES NO COUPLING

Roberta Lima[a], **Rubens Sampaio**[a] **and Peter Hagedorn**[b]

[a]*Department of Mechanical Engineering, PUC-Rio, Rua Marquês de São Vicente, 225, Gávea, 22451-900 RJ, Brazil*

[b]*Vibrations and Dynamics Group, Numerical Methods in Mechanical Engineering, fnb, TU Darmstadt, Dolivostr. 15, 64293, Darmstadt, Germany*

Keywords: Electromechanical systems, coupled systems, parametric excitation, nonlinear dynamics.

Abstract. Electromechanical systems deal with the mutual interaction between electromagnetic and mechanical parts. In this paper, it simply means the connection of a DC motor and a mechanical part by some mechanism. This interaction is called coupling. The mechanical and the electromagnetic subsystems interact. To properly represent the dynamics of a coupled system, it is necessary to properly characterize this interaction between the parts. Any change in the modeling of the interaction affects the behavior of the entire system. Typically, the coupling between electromagnetic and mechanical parts is expressed by a set of coupled differential equations. The dynamics of the coupled system is given by an initial value problem comprising this set of coupled differential equations. In this paper, we discuss three mistakes found in the literature on electromechanical systems. The three mistakes somehow decouple the system. They maim the initial value problem of the coupled system, in a way that it looses one differential equation and the initial condition related to the lost equation. The remaining equations represent only the dynamics of the mechanical part. The dynamics of the motor is ignored in a way that the electromagnetic part is decoupled from the system. Apparently they are useful hypotheses, since they simplify the problem greatly. However they lead to wrong results, as is shown in this paper. To exemplify how the hypotheses mislead and change the dynamics, numerical simulations are performed for an simple electromechanical system. Observing the results, one sees, immediately, the inadequacy of them. The oldest of these misleading hypotheses was first made at least 75 years ago still persist in the literature. It seems that lately these hypotheses are used more than ever.

1 INTRODUCTION

Coupled systems present an interesting behavior characterized by the mutual influence between the parts of the system Dantas et al. (2014), Dantas et al. (2016), Clerkin and Sampaio (2017), Manhães et al. (2018). Each part of the system affects the behavior of the other, i.e., they interact. To properly represent the dynamics of a coupled system, it is necessary to properly characterize this interaction between the parts. Any change in the modeling of the interaction affects the behavior of the entire system.

This paper focus on a specific type of coupling, an electromechanical coupling. We analyze systems with an electromagnetic and a mechanical part coupled as sketched in Fig. 1.

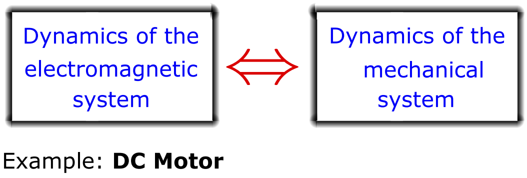


Figure 1: Sketch of the mutual interaction between electromagnetic and mechanical parts of an electromechanical system.

The dynamics of these systems are described by an initial value problem (IVP) comprising a set of coupled differential equations. Observe that the configuration space of an electromechanical system must contains electromagnetic as well as mechanical variables. Mechanical variables only cannot describe the coupling.

In this paper, we discuss three mistakes found in the literature on electromechanical systems. The first one appears in classical books since at least 1943, as Rocard (1943), Kononenko (1969) and Nayfeh and Mook (1979), and has being propagated in several papers, see, for exemple, the recent works Avanço et al. (2017), Rocha et al. (2018) and Gonçalves et al. (2016). This first mistake is about a hypothesis that affirms that the coupling between the electrical and mechanical part can be represented by a linear function. With simple numerical examples, we show that this hypothesis can mislead the results. It is valid only if the electromechanical system reaches a steady state, in which the electromagnetic and mechanical variables do not vary with time (they are constant). In this case, the dynamics vanishes, and the coupling is reduced to a static problem. If a steady state is not reached, the coupling varies with the coupling conditions, it is not a functional relation and depends on the initial conditions. To better understand the difference between the static and dynamical cases, see the Figs. 3.1 and 5.1. The lack of a functional relation is the essence of the coupling.

The second mistake discussed can be frequently found in the literature, please see Danuta and Maciej (2006), González-Carbajal and Domínguez (2017), Gonçalves et al. (2014) and Cveticanin et al. (2017). These books and papers supposedly deal with electromechanical systems, however they do not even mention electromagnetic variables, as current and electrical charge. They use only mechanical variables to describe the system. One form this mistake appears is when one wants to construct a Lagrangian for an electromechanical system using only mechanical variables, as happens in Danuta and Maciej (2006) and Gonçalves et al. (2016). They formulate a Lagrangian only for the mechanical part, disregarding the electrical subsystem. It is interesting to observe that in Cveticanin et al. (2017) terms related with electromagnetic variables just appear in the introduction and in the penultimate chapter of the book. In the mathematical models, there is no mention of them.

The third mistake discussed is a hypothesis found in the literature that affirms that it is possible to neglect a term in the initial value problem that describes the dynamics of an electromechanical system without changing the interaction between the electromagnetic and mechanical parts. The hypothesis is based only on parameters values, it does not depend on the system being studied and, also, does not depend on the type of electromechanical system analyzed. Apparently it is a useful hypothesis, since it simplifies the problem greatly. However it can lead to wrong results, as is shown in this paper. This hypothesis leads to the decoupling of the subsystems. The dynamics of the electromagnetic part is ignored, in a a way that the the configuration space is maimed (it just has the variables of the mechanical part). To exemplify how the hypothesis misleads and changes the dynamics, simulations are performed for an electromechanical system neglecting the term and not neglecting it Lima and Sampaio (2018). Comparing the results, one sees, immediately, the big difference between the two dynamics.

2 GEOMETRICAL CONSTRAINT BETWEEN THE ELECTROMAGNETIC AND MECHANICAL PARTS

Typically, in an electromechanical system, there is a geometrical constraint between the electromagnetic and mechanical parts. To illustrate, we present two simple electromechanical systems composed by a cart coupled to a DC motor, see Figs. 2 and 2. The difference between these two systems is the mechanism that couples the electromagnetic and mechanical parts. In Fig. 2, it is shown a mechanism called scotch yoke and in Fig. 2, the slider crank mechanism. Both of them relate the horizontal cart motion x with the motor rotational motion α, i.e., introduce a geometrical constraint between these two variables.

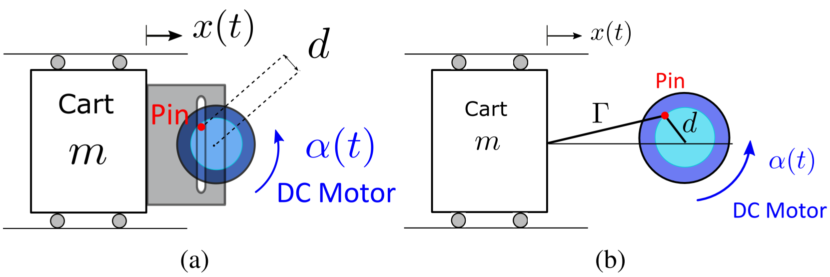


Figure 2: (a) Electromechanical system with a scotch yoke mechanism. (b) Electromechanical system with a slider crank mechanism.

In this work, we focus on the system with the scotch yoke mechanism. To translate the results to the other case is trivial. Due to the system geometry, $x(t)$ and $\alpha(t)$ are related by the constraint

$$x(t) = d\cos(\alpha(t)) . \tag{1}$$

3 DYNAMICS OF AN ELECTROMECHANICAL SYSTEM

To determine the dynamics of the electromechanical system sketched in Fig. 2, first we derive the equations of the dynamics of each part of the system, electromagnetic (DC motor) and mechanical (cart). After that we couple the equations by the coupling torque that exists between the parts and the geometric constraint given by Eq. (1).

The mathematical modeling of DC motors is based on the Kirchhoff's law Karnopp et al. (2006). The dynamics of a DC motor is given by the following initial value problem (IVP).

Given a source voltage ν, find $(\alpha,\, c)$ such that, for all $t > 0$,

$$l\dot{c}(t) + r\; c(t) + k_e \dot{\alpha}(t) = \nu \;, \tag{2}$$

$$j_m \ddot{\alpha}(t) + b_m \dot{\alpha}(t) - k_e\; c(t) = -\tau(t) \;, \tag{3}$$

with the initial conditions

$$\dot{\alpha}(0) = \dot{\alpha}_0 \quad , \quad \alpha(0) = \alpha_0 \quad , \quad c(0) = c_0 \,, \tag{4}$$

where t is the time, c is the electric current, $\dot{\alpha}$ is the angular speed of the motor, l is the electric inductance, j_m is the inertia moment of the motor, b_m is the damping ratio in the transmission of the torque generated by the motor to drive the coupled mechanical system, k_e is the motor electromagnetic force constant and r is the electromagnetic resistance. The available torque delivered to the coupled mechanical system is represented by τ, that is the component of the torque vector $\boldsymbol{\tau}$.

3.1 Steady state: solution of a static problem

Assuming that τ and ν are constants, the motor reaches a steady state in which the electric current and the angular speed approach constants. By Eqs. (2) and (3), the angular speed of the motor shaft and the current in steady state, respectively $\dot{\alpha}_{steady}$ and c_{steady}, are written as

$$\dot{\alpha}_{\text{steady}} = \frac{-\tau\; r + k_e\; v}{b_m\; r + k_e^2} \qquad , \qquad c_{\text{steady}} = \frac{v}{r} - \frac{k_e}{r}\left(\frac{-\tau\; r + k_e\; v}{b_m\; r + k_e^2}\right) \;. \tag{5}$$

Remark that if τ and ν are constants, there is a functional relation between $\dot{\alpha}_{\text{steady}}$ and τ and between $\dot{\alpha}_{\text{steady}}$ and c_{steady}. The graphs of the curves are shown in Figs. 3.1 and 3.1. These graphs are usually called motor map and usually are provided by engine manufacturers.

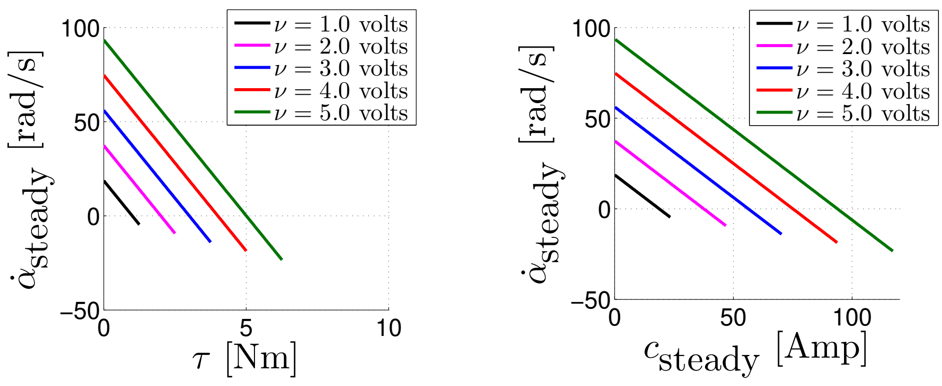


Figure 3: Graph of $\dot{\alpha}_{\text{steady}}$ (a) as function of τ and, (b) as function of c_{steady} for different values of ν. The values of j_m, k_e, r and b_m are given in Table 1.

3.2 Transient state: solution of a dynamical problem

When τ or ν are not constants, the angular speed of the motor shaft and the current do not reach a steady state. Figures 3.2 and 3.2 show the results considering for example that $\tau = \tau_0 \cos{(\omega\, t)}$ and ν constant.

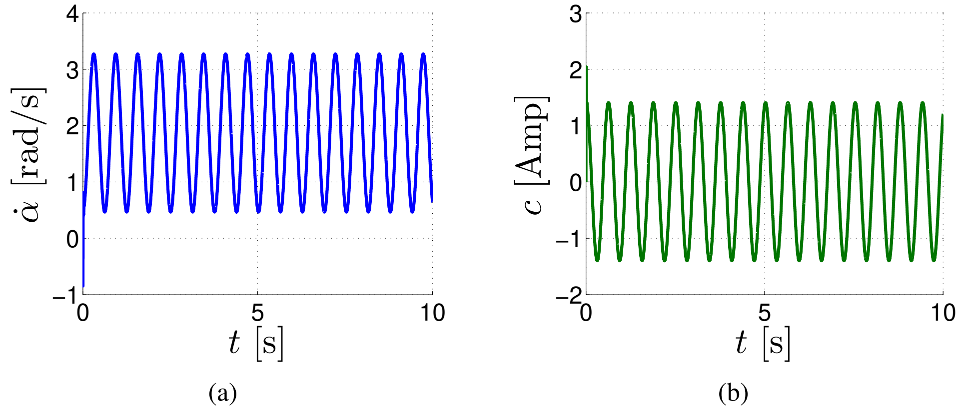


Figure 4: (a) Angular speed of the motor, without mechanical subsystem attached, and (b) current over time. It is considered that $\nu = 0.1$ V, $\tau_0 = 0.075$ N/m and $\omega = 10$ rad/s. The values of j_m, k_e, r and b_m are given in Table 1. It is considered $\dot{\alpha}(0) = 0$ rad/s, $\alpha(0) = 0$ rad and $c(0) = \nu/r$ Amp.

Another situation in which $\dot{\alpha}$ and c do not reach a steady state is when a mechanical system is coupled to the motor. In this case, $\dot{\alpha}$ and c vary in time in a way that the dynamics of the motor will be influenced by the coupled mechanical system. To model the coupling between the motor and the mechanical system, the motor shaft is assumed to be rigid. Thus, the available torque vector to the coupled mechanical system, $\boldsymbol{\tau}$, can be written as

$$\boldsymbol{\tau}(t) = \boldsymbol{d}(t) \times \mathbf{f}(t)\,, \tag{6}$$

where $\boldsymbol{d} = (d\ \cos\alpha(t),\ d\ \sin\alpha(t),\ 0)$ is the vector related to the eccentricity of the pin, and where $\mathbf{f}$ is the coupling force between the DC motor and the cart. Assuming that there is no friction between the pin and the slot, the vector $\mathbf{f}$ only has a horizontal component, f (the horizontal force that the DC motor exerts in the cart). The available torque τ is written as

$$\tau(t) = -f(t)\ d\ \sin\alpha(t)\,. \tag{7}$$

Due to constraints, the cart is not allowed to move in the vertical direction. The cart mass is m and the horizontal cart displacement is represented by x. Since the cart is modeled as a particle, it satisfies the equation

$$m\ \ddot{x} = f(t)\,. \tag{8}$$

Observe that Eqs. (2) and (3) are related with the dynamics of the electromagnetic part (DC motor) and Eq. (8) is related with the dynamics of the mechanical part (cart). Substituting Eqs. (6) to (1) into Eqs. (2) and (3), we obtain the initial value problem for the coupled system, i.e., the motor-cart system, that is written as follows. Given a source voltage ν, find $(\alpha,\ c)$ such that, for all $t > 0$,

$$l\ \dot{c}(t) + r\ c(t) + k_e\ \dot{\alpha}(t) = \nu\,, \tag{9}$$

$$\ddot{\alpha}(t)\ \left[j_m + md^2(\sin\alpha(t))^2\right] + \dot{\alpha}\ \left[b_m + md^2\dot{\alpha}(t)\cos\alpha(t)\ \sin\alpha(t)\right] - k_e\ c(t) = 0\,, \tag{10}$$

with the initial conditions

$$\dot{\alpha}(0) = \dot{\alpha}_0 \quad , \quad \alpha(0) = \alpha_0 \quad , \quad c(0) = c_0\,. \tag{11}$$

Observe that the dynamics of the coupled system is given by an initial value problem comprising a set of two coupled differential equations. Observe also that the problem has three initial values, one for the current, one for the angular position of the motor, and one for the angular velocity of the motor. To determine how the variables c and α change over time, it is necessary to integrate the set of differential equations. It is not possible to deal with the equations separately and this characterizes the coupling between the electromagnetic and mechanical parts. Each part influences the other, i.e., there is a mutual influence.

4 MISTAKES COMMONLY FOUND IN THE LITERATURE THAT DEALS WITH ELECTROMECHANICAL SYSTEMS

As explained in the paper Introduction, several mistakes can be found in the literature on electromechanical systems. In this Section, three of these mistakes are discussed. It is shown with simple numerical examples that they can produce wrong results.

The main discussion is around the third mistake presented in the Introduction, that is, the hypothesis that affirms that it is possible to neglect a term in the initial value problem that describes the dynamics of an electromechanical system without changing the interaction between the electromagnetic and mechanical parts. The same results used to demonstrate that this hypothesis is a misstep show immediately that the first hypothesis presented in the Introduction (coupling represented by a linear function) is also a misstep. The second mistake represents clearly a nonsensical way to deal with electromechanical systems. If the system is electromechanical, it must have an electromechanical part. Of course, one alone makes no coupling.

The third mistake can be found in the recent article published by Avanço et al. (2018), and Belato (2002); Avanço (2015); Belato et al. (2001). These papers and doctoral thesis claim that the inductance of the armature can be neglected due to the fact that the electrical time constant of the motor l/r is usually much smaller than the mechanical time constant $r j_m/{k_e}^2$.

With the hypothesis, it is possible to obtain from Eq. (9) a functional relation between $\dot{\alpha}$ and c, given by

$$\cancel{l\dot{c}(t)} + \ r\,c(t) + k_e\dot{\alpha}(t) = \nu \qquad \Longrightarrow \qquad c(t) = \frac{\nu - k_e\dot{\alpha}(t)}{r}\,. \tag{12}$$

Substituting this functional relation into Eq. (10), the initial value problem for the coupled system is altered. Given ν, find (α) such that

$$\begin{aligned} \ddot{\alpha}(t) \quad & \left[j_m + m\,d^2(\sin\alpha(t))^2\right] - k_e\,\tfrac{\nu(t)-k_e\dot{\alpha}(t)}{r} + \\ & +\dot{\alpha}\ \left[b_m + m\,d^2\dot{\alpha}(t)\cos\alpha(t)\ \sin\alpha(t)\right] = 0\,, \end{aligned} \tag{13}$$

with initial conditions

$$\dot{\alpha}(0) = \dot{\alpha}_0 \quad , \quad \alpha(0) = \alpha_0\,. \tag{14}$$

Remark that this is a hypothesis based only on parameters values, it does not depend on the system being studied, and it does not depend on the type of mechanism that couples the mechanical end electromagnetic parts. Apparently it is a useful hypothesis, since it simplifies the problem greatly. When the hypothesis is made, the differential equation related to the dynamics of the electromagnetic part becomes an algebraic equation given by Eq. (12). Thus, the initial value problem (IVP) is maimed, it looses one differential equation and the initial condition related with the lost equation, i.e., the initial condition for the current. The initial value of the current becomes $c(0) = \frac{\nu - k_e \dot{\alpha}_0}{r}$, it is not an independent value anymore, i.e., $c(0)$ is related with $\dot{\alpha}(0)$. It is important to remark that the remaining equation in the IVP represents only the dynamics of the mechanical part. The dynamics of the motor is ignored, as sketched in Fig. 4. The current is computed after the integration of the IVP.

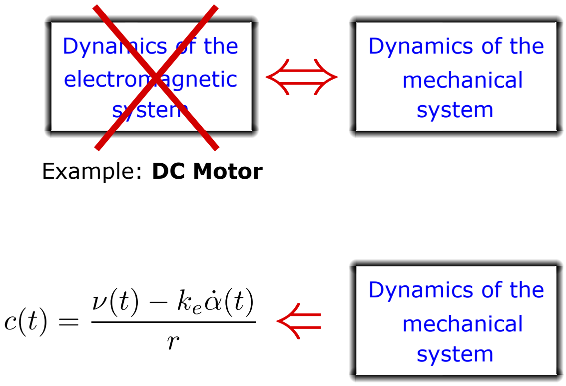


Figure 5: Sketch of the decoupling between electromagnetic and mechanical parts with the inductance neglected.

To exemplify how it misleads, we perform simulations neglecting the inductance and not neglecting it and comparing the two results. One sees, immediately, the big difference between the two dynamics.

5 RESULTS OF NUMERICAL SIMULATIONS

For computation, the initial value problem defined by Eqs. (9) to (14) was integrated in a range of [0.0, 6.0] seconds. The 4th-order Runge-Kutta method is used for the time integration scheme with a time-step equal to 10^{-6}. The motor parameters used in all simulations are listed in Table 1. The cart mass is 5.0 kg and, it is considered $\dot{\alpha}(0) = 0$ rad/s, $\alpha(0) = 0$ rad and $c(0) = \nu/r$ Amp. Observe that, with these values, one has $\frac{l}{r} = 6.12 \times 10^{-4}$ and $\frac{r j_m}{{k_e}^2} = 1.31 \times 10^{-2}$.

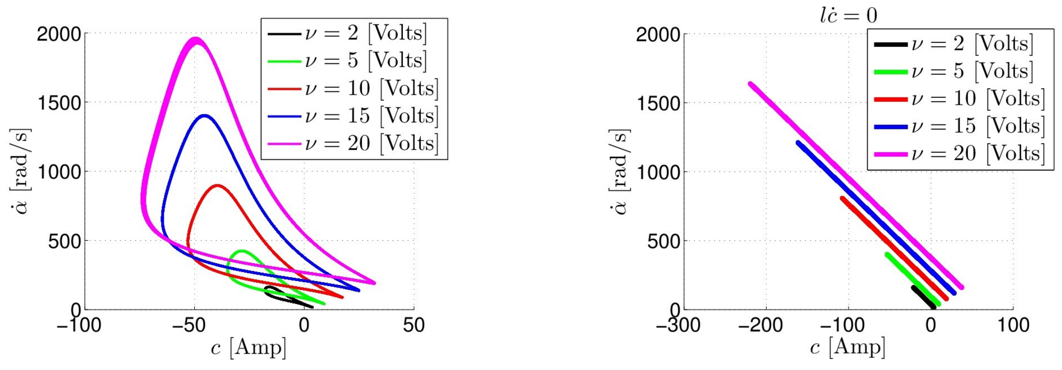
Parameter	Value
l	1.880×10^{-4} H
j_m	1.210×10^{-4} Kg m^2
b_m	1.545×10^{-4} Nm/(rad/s)
r	$0.307\ \Omega$
k_e	5.330×10^{-2} V/(rad/s)

Table 1: Values of the motor parameters used in simulations.

To demonstrate how neglecting the inductance can mislead the results, we perform simulations neglecting and not neglecting it and we compare the results. The simulations were computed for different values of ν, the source voltage, and of d, the pin eccentricity.

5.1 Results varying the source voltage

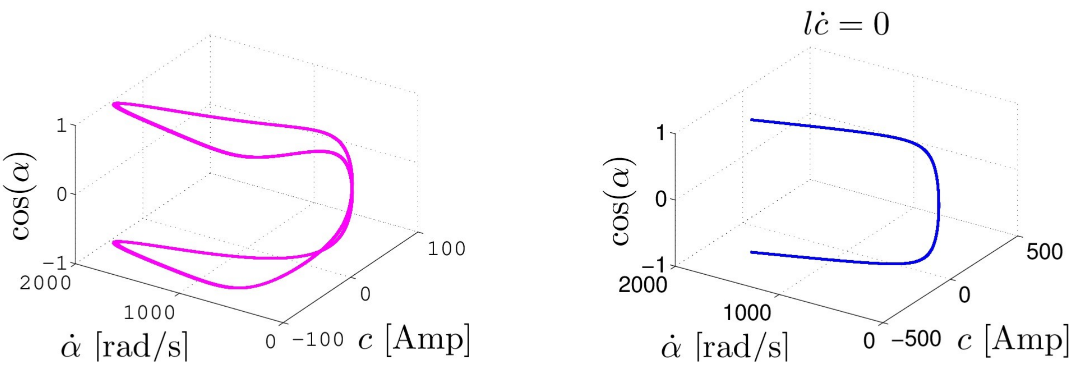
Figures 5.1 and 5.1 show the phase portrait of $\dot{\alpha}$ graph as function of c for different values of ν. In these simulations $d = 0.05$ [m]. Observe that the results are different. When the inductance is neglected, there is a linear relation between $\dot{\alpha}$ and c. This can be verified by Eq. (12). Observe that when the inductance is not neglected, there is no functional relation between $\dot{\alpha}$ and c. The relation depends on initial conditions.



(a) (b)

Figure 6: Phase portrait of the $\dot{\alpha}$ as function of c (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.

As explained previously, when $l\dot{c}(t)$ is neglected, the configuration space is maimed. It looses one dimension. To observe this flattening we ploted the phase portrait of $\cos(\alpha)$ as function of $\dot{\alpha}$ and c for $\nu = 20$ V neglecting and not neglecting the inductance (see Figs. 5.1 and 5.1). Observing them, it is clear that the 3D phase space becomes a 2D phase space.



(a) (b)

Figure 7: Phase portrait of the $\cos(\alpha)$ as function of $\dot{\alpha}$ and c for $\nu = 20$ V (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.

Observing the graphs of τ as function of $\dot{\alpha}$ shown in Figs. 5.1 and 5.1, it is possible to verify that the maximum and minimum values of torque changes if the inductance is neglected. Besides this, Figs. 5.1 shows that there is no functional relation between $\dot{\alpha}$ and τ. This is an important result. It shows that the hypothesis found in the literature since, at least, 1943 (please see Rocard (1943), Kononenko (1969), Nayfeh and Mook (1979)) that affirms that the coupling between the electrical and mechanical part can be represented by a linear function is not always correct. The lack of a functional relation is the essence of the coupling!

Analyzing the graphs of c as function of x shown in Figs. 5.1 and 5.1, it is possible to

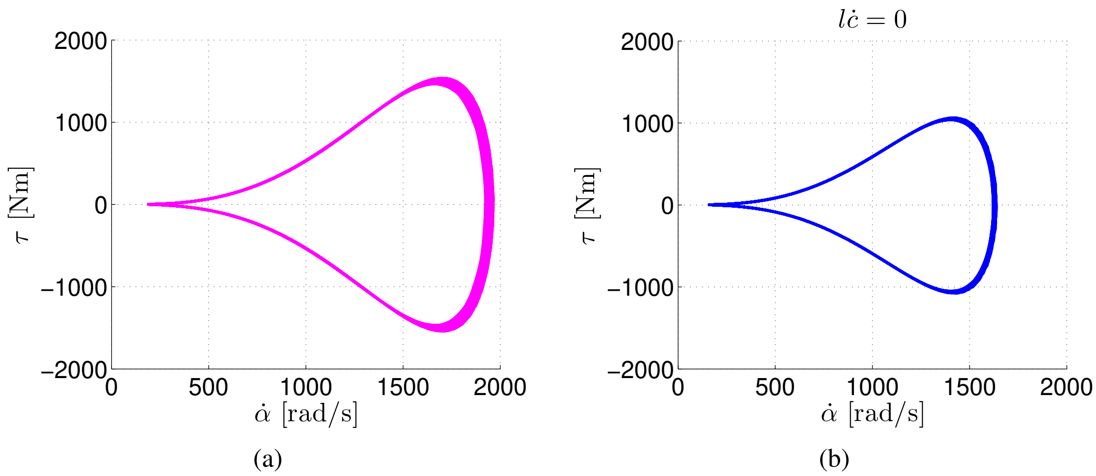


Figure 8: Phase portrait of the τ as function of c (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.

observe that when the cart position is near the origin, the values of the current neglecting and not neglecting the inductance are similar. However, as the cart moves away from the origin, the values of the current become very different. While the the minimum value of c not neglecting the inductance is around -74.0 Amp, the minimum value of c neglecting it is -220.0 Amp (almost 300% lower). To understand the reason of this, we should observe the graph of $\dot{\alpha}$ as function of x shown in Figs. 5.1 and 5.1. Remark that the maximum values of $\dot{\alpha}$ occur when the cart changes its direction of movement, i.e., when $x = d$ or $x = -d$. Recalls also that when the inductance is neglected, there is a linear relation between $\dot{\alpha}$ and c (see Eq. (12)). Thus, as $\dot{\alpha}$ grows, c decreases linearly. Comparing Figs. 5.1 and 5.1, it is also possible to verify that when $l\dot{c}$ is neglected, the motor speed is lower. While the the maximum value of $\dot{\alpha}$ not neglecting the inductance is around $2,000.0$ rad/s, the maximum value neglecting it is around $1,626.0$ rad/s (almost 20% lower).

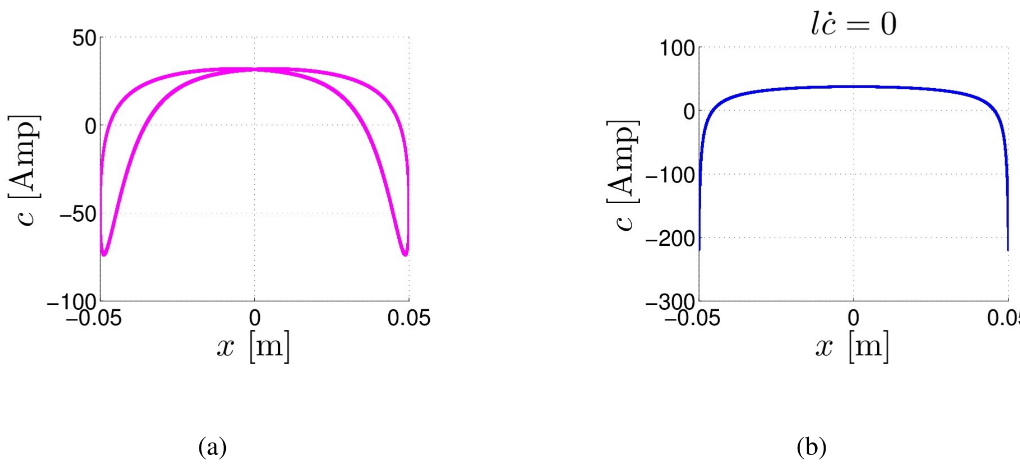
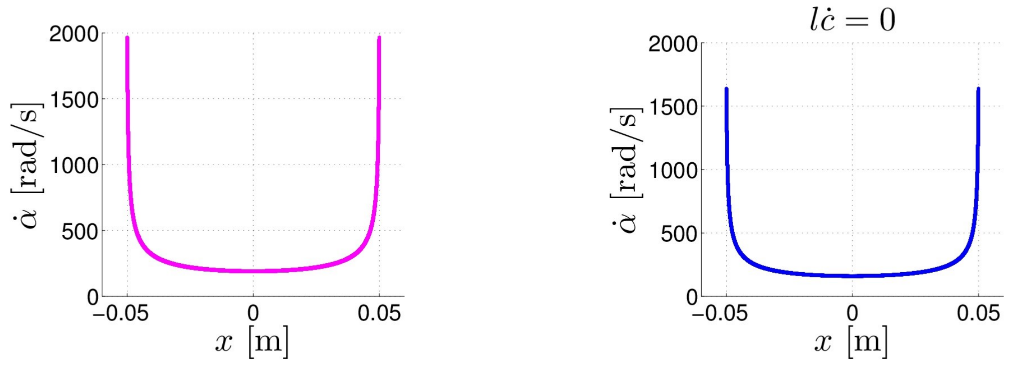


Figure 9: Phase portrait of the c as function of x for $\nu = 20$ V (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.

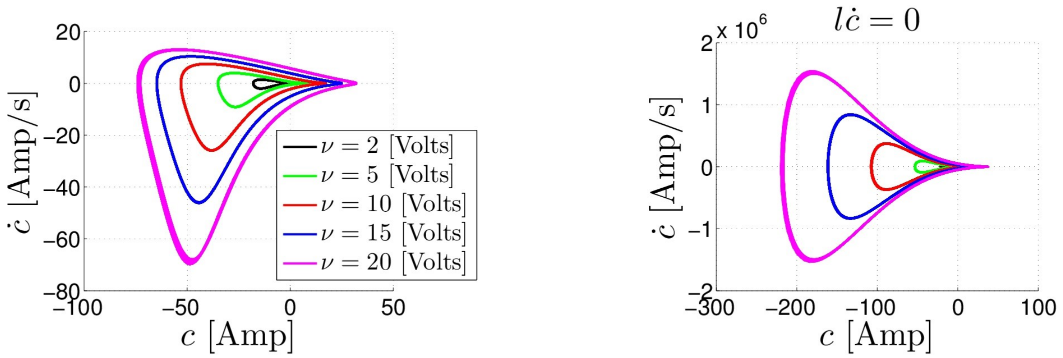
Another interesting result can be verified comparing $\dot{c}$ as function of c not neglecting the in-



(a) (b)

Figure 10: Phase portrait of the $\dot{\alpha}$ as function of x for $\nu = 20$ V (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.

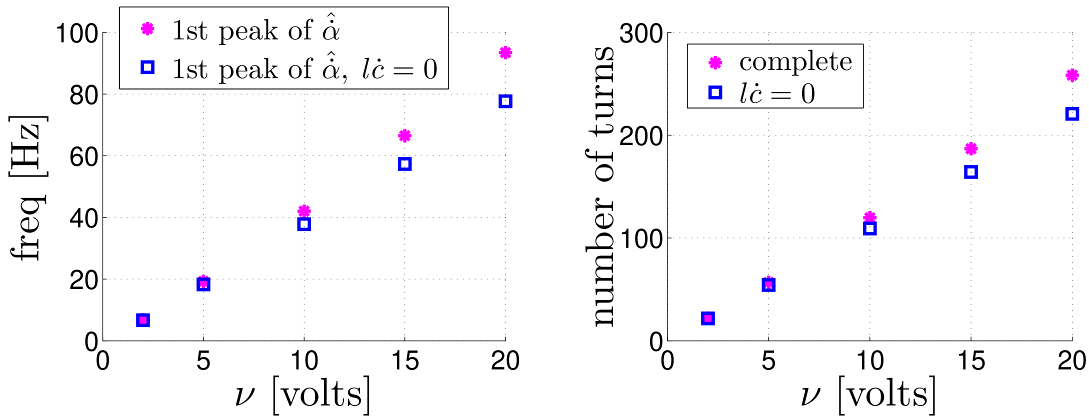
ductance and neglecting it. These graphs are shown in Figs. 5.1 and 5.1. Remark the hypothesis $l\dot{c} = 0$ introduces a symmetry in relation to the $\dot{c} = 0$.



(a) (b)

Figure 11: Phase portrait of the $\dot{c}$ as function of c (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.

To quantify how neglecting inductance misleads the results, and also to enrich the analysis, we computed the Fast Fourier Transform (FFT) of the current and motor speed over time, $\hat{c}$ and $\hat{\alpha}$. This tool has been used in the analysis of electromechanical systems (see Lima and Sampaio (2015), Lima and Sampaio (2012)). It provides important information of the signals in the frequency domain. The FFT was computed for the cases in which the inductance is neglected and is not neglected. Figure 5.1 shows the value of the frequency which corresponds to the first peak of the FFT of the motor speed for different values of ν. In these simulations it was considered that $d = 0.05$ m. Observe that as the value of ν grows, the difference between the frequencies of the first peak neglecting and not neglecting the inductance also grows. For $\nu = 20$ V, while the frequency of the peak is 93.41 Hz not neglecting it, it is 77.64 Hz neglecting it (almost 17% lower). This confirms the results presented previously in Figs. 5.1 and 5.1. Figure 5.1 shows the total number of turns of the disk in the range of integration $[0.0, 6.0]$ seconds. As expected, the total number of turns is bigger if the inductance is not neglected (almost 15.0% bigger).



(a) (b)

Figure 12: (a) Values of the first peak (not neglecting and neglecting the inductance) for the FFT of the motor speed. (b) Total number of turns of the disk in the range of integration $[0.0, 6.0]$ seconds.

5.2 Results varying the nominal eccentricity of the pin

Lima and Sampaio (2015) discuss the influence of the nominal eccentricity of the pin, the parameter d, in the dynamics of the system. It is shown that this parameter is related with the nonlinearity of the system. When d is small, the initial value problem of the motor-cart system tends to a linear system, But as the eccentricity grows, the nonlinearities become more pronounced.

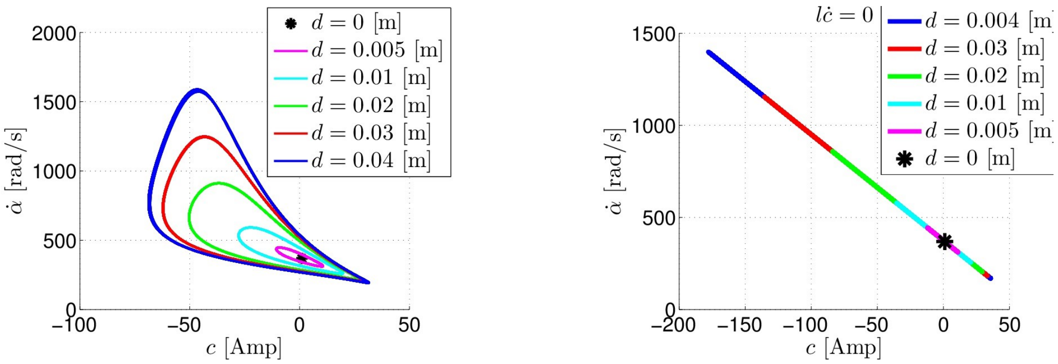
To show how neglecting inductance misleads the results, specially when the nonlinearities are more pronounced, we perform simulations neglecting and not neglecting it and we compare the results for different values of d. Figures 5.2 and 5.2 show the phase portrait of $\dot{\alpha}$ graph as function of c for different values of d. In these simulations $\nu = 20.0$ [V]. Figures 5.2 and 5.2 show the value of the frequency which corresponds to the first peak of the FFT of the current and motor speed for different values of ν. In these simulations $\nu = 20.0$ Volts. Observe that as the value of ν grows, the difference between the frequencies of the first peak neglecting and not neglecting the inductance also grows.

6 CONCLUSIONS

This paper discuss three mistakes found in the literature dealing with electromechanical systems. The oldest hypothesis was first made at least 75 years ago, Rocard (1943), and still persists in the literature, as for example in a recent paper Avanço et al. (2018).

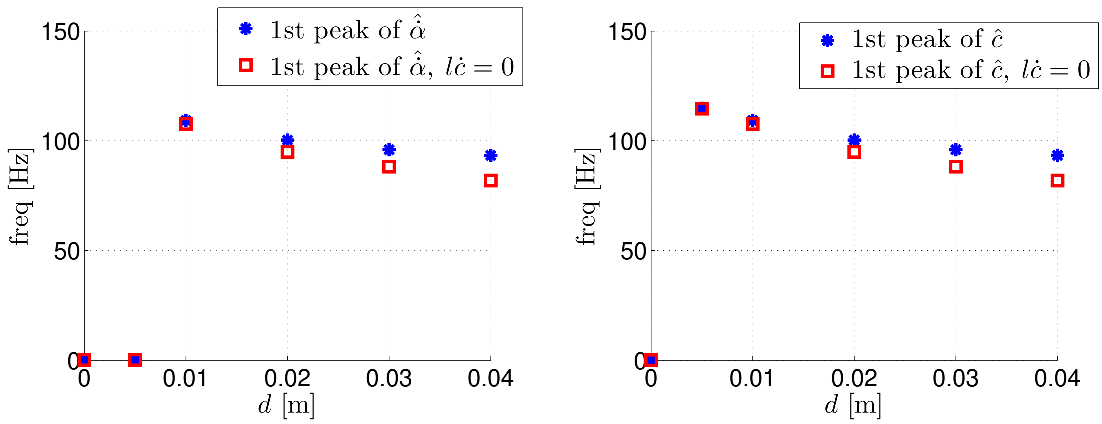
To exemplify how the three hypotheses can mislead and change the dynamics, we performed simulations in a simple electromechanical system. The results shows immediately the misleading.

One of the discussed hypothesis is the claim that the inductance of a DC motor can be neglected due to the fact that the electromagnetic time constant of the motor is usually much smaller than the time constant related to the mechanical part of the system. Some references,



(a) (b)

Figure 13: Phase portrait of the (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.



(a) (b)

Figure 14: Values of the first peak (not neglecting and neglecting the inductance) for the FFT of (a) current (b) motor speed.

as Belato et al. (2001), affirm also that this hypothesis does not affect the interaction between the electromagnetic and mechanical parts of a coupled system. Comparing the results of the numerical simulations neglecting the inductance and not neglecting, it is possible to verify, immediately, the big difference between the two dynamics. Hence, we showed that he hypothesis that the inductance can be neglected based only on the values of the system parameters can not be made in general. The neglect introduces in the system a linear algebraic relationship between $\dot{\alpha}$ and c. Since there is no functional relation between these two variables, this hypothesis can not be right. The lack of a functional relation is the essence of the coupling.

The dynamics of the coupled system is given by an initial value problem comprising a set of coupled differential equations. Any change in this set of coupled differential equations modifies the interaction between electromagnetic and mechanical parts, affecting the behavior of the entire system. The hypothesis that the inductance can be neglected has been used as a strategy to reduce the number of equations in the initial value problem. However, with the neglect, the system is decoupled! The dynamics of the electromagnetic part is ignored. We believe that, in this case, the system should not be called an electromechanical system. One alone makes no coupling!

Beyond the literature dealing with electromechanical systems neglecting inductance, there are books and papers dealing with electromechanical systems without even mentioning electromagnetic variables, as current and charge (please see, Danuta and Maciej (2006), González-Carbajal and Domínguez (2017), Gonçalves et al. (2014) and Cveticanin et al. (2017)).

Some others references supposedly dealing with electromechanical systems assume a linear algebraic relationship between τ and $\dot{\alpha}$ (Avanço et al. (2017), Nayfeh and Mook (1979), Rocard (1943), Kononenko (1969)). In the case of a DC motor in a steady state, this is correct (see Fig. 3.1 and Eq. (5)), however Fig. 5.1 shows that this is not true if there is no steady state. Observe that in the motor-cart system, there is no functional relation between $\dot{\alpha}$ and τ. It should be remarked that in books as Kononenko (1969) and Nayfeh and Mook (1979), the disarray is such that the response of a DC motor in a steady state is used to characterize a system that is not in a steady state. To be precise, see page 225 and figure 4.33 of Nayfeh and Mook (1979). On this page it is said that "to account the influence of the motion on the performance of the motor, one needs to to know the characteristics of the motor, which are the net driving torques developed by the motor". Furthermore, the characteristics of the motor are presented in a form of a graph, similar to the one presented in Fig. 3.1.

ACKNOWLEDGEMENTS

The authors acknowledge the support given by FAPERJ, CNPq, and CAPES-DAAD, through a Probral, ID 57390840 - 15.03.2018

REFERENCES

Avanço R. *Análise da dinâmica não-linear de pêndulos com excitação paramétrica por um mecanismo biela-manivela*. Ph.D. thesis, USP-São Carlos, São Carlos, S.P., Brazil, 2015.

Avanço R., Tusset A., Balthazar J., Nabarrete A., and Navarro H. On nonlinear dynamics behavior of an electro-mechanical pendulum excited by a nonideal motor and a chaos control taking into account parametric errors. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 40:23, 2018.

Avanço R., Tusset A., Suetake M., Navarro H., Balthazar J., and Nabarrete A. The pendulum dynamic analysis with dc motors and generators for sea waves energy harvest. In *Proceedings* *of 14th International Conference in Dynamical Systems Theory and Applications (Dynamical Systems Theory and Applications)*. Łódź, Poland, 2017.

Belato D. *Análise não linear de sistemas dinâmicos holonômos não ideais*. Ph.D. thesis, Unicamp, Campinas, S.P., Brazil, 2002.

Belato D., Weber H., Balthazar J., and Mook D. Chaotic vibrations of a nonideal electromechanical system. *Nonlinear Dynamics*, 38:1699–1706, 2001.

Clerkin E. and Sampaio R. A bifurcation and symmetry discussion of the sommerfeld effect. In *Proceedings of 14th International Conference in Dynamical Systems Theory and Applications (Dynamical Systems Theory and Applications)*. Łódź, Poland, 2017.

Cveticanin L., Zukovic M., and Balthazar J. *Dynamics of Mechanical Systems with Non-Ideal Excitation*. Springer, 2017.

Dantas M., Sampaio R., and Lima R. Asymptotically stable periodic orbits of a coupled electromechanical system. *Nonlinear Dynamics*, 78:29–35, 2014.

Dantas M., Sampaio R., and Lima R. Existence and asymptotic stability of periodic orbits for a class of electromechanical systems: a perturbation theory approach. *Zeitschrift für angewandte Mathematik und Physik*, 67:2, 2016.

Danuta S. and Maciej J. Nonlinear oscillations of a coupled autoparametrical system with ideal and nonideal sources of power. *Mathematical Problems in Engineering*, 2006:1–20, 2006.

Gonçalves P., Silveira M., Petrocino E., and Balthazar J. Double resonance capture of a two-degree-of-freedom oscillator coupled to a non-ideal motor. *Meccanica*, 51:2203–2214, 2016.

Gonçalves P., Silveira M., Pontes Junior B., and Balthazar J. The dynamic behavior of a cantilever beam coupled to a non-ideal unbalanced motor through numerical and experimental analysis. *Journal of Sound and Vibration*, 333:5115–5129, 2014.

González-Carbajal J. and Domínguez J. Non-linear vibrating systems excited by a nonideal energy source with a large slope characteristic. *Mechanical Systems and Signal Processing*, 96:366–384, 2017.

Karnopp D., Margolis D., and Rosenberg R. *System Dynamics: Modeling and Simulation of Mechatronic Systems*. John Wiley and Sons, 4th edition, New-York, USA, 2006.

Kononenko V.O. *Vibrating Systems with a Limited Power Supply*. London Iliffe Books LTD, England, 1969.

Lima R. and Sampaio R. Analysis of an Electromechanic Coupled System with Embarked Mass. *Mecânica Computacional,*, XXXI:2709–2733, 2012.

Lima R. and Sampaio R. Two parametric excited nonlinear systems due to electromechanical coupling. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, DOI:10.1007/s40430-015-0395-4, 2015.

Lima R. and Sampaio R. Pitfalls in the dynamics of coupled electromechanical systems. In *Proceeding Series of the Brazilian Society of Computational and Applied Mathematics (CNMAC 2018)*. Campinas, Brazil, 2018.

Manhães W., Sampaio R., Lima R., Hagedorn P., and Deü J. Lagrangians for electromechanical systems. In *Mecom 2018*. Argentina, 2018.

Nayfeh A.H. and Mook D.T. *Nonlinear Oscillations*. John Wiley and Sons, USA, 1979.

Rocard Y. *Dynamique Générale des Vibrations (General Dynamics of Vibrations)*. Masson et Cie., Éditeurs, Paris, France, 1943.

Rocha R., Balthazar J., A. T., and Quinn D. An analytical approximated solution and numerical simulations of a non-ideal system with saturation phenomenon. *Nonlinear Dynamics*, Published online:10.1007/s11071–018–4369–9, 2018.