

## AN EVALUATION OF A CLASS OF INCREMENTAL PHENOMENOLOGICAL THEORIES OF FERROELECTRICITY AND FERROMAGNETISM

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**Abstract.** Various phenomenological theories of ferroelectricity and ferromagnetism for polycrystalline ferroic solids have been proposed in recent years. A particularly attractive class of multi-axial theories, which allows for variational formulations of boundary-value problems, hinges upon an additive decomposition of the strain and the dipole density into reversible internal variables associated with elasticity and dipole perturbations, on the one hand, and irreversible internal variables associated with dipole alterations or switching, on the other hand, whose evolution laws are discretized in time following an implicit Euler scheme. It has been recently recognized, however, that these theories can provide unreliable predictions for certain —yet unexceptional— loading histories. The source of the problem was pinned down to the non-convex dependence of the internal energy on the irreversible variables, which renders constitutive relations numerically unstable. The purpose of the present study is to further evaluate the genuine capabilities of the theories to provide reliable predictions. It is found that predictions for complex loading histories can be reliable provided the mechanical stress remains below a certain threshold, which can be in the order of a few megapascals in the case of typical ferroelectric ceramics, and well below a megapascal in the case of typical ferromagnetic alloys. Even though alternative functional dependences have not been considered in this study, the non-convexity seems inevitable if the set of internal variables describing the permanent dipole distribution is to be restricted to a single vectorial variable as in this class of theories. This is because the physics of ferroelectricity and ferromagnetism then require that the strain tensor associated with permanent dipoles be an even function of such vectorial variable, and the elastic energy be a quadratic form of such strain tensor. It is thus concluded that robust phenomenological theories for ferroelectricity and ferromagnetism require the use of enriched sets of internal variables to describe the distribution of permanent dipoles within the solid.