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LATTICE STRUCTURES DESIGN BY MEANS OF TOPOLOGY OPTIMIZATION

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Abstract. In recent years, research and development of lattice structures have gained much attention because of their unique functional characteristics such as high compliance, high specific strength, and enhanced heat transfer. One promising strategy for new developments is to use topology optimization as a design tool to obtain new unit cells for lattice material. However, this approach may lead to optimal topologies with highly complex geometries and traditional manufacturing processes may no longer be feasible in these cases. Additive manufacturing (AM) techniques may fill the gaps between topology optimization and product by removing the geometric complexity restriction to a large extent. Even so, their manufacturing constraints, such as feature size and maximum inclination angle for building a product, need to be considered in the optimization process. In the effort to address the design of lattice infill, different approaches have been developed. A simple approach, called variable-density lattice structure optimization, is to perform a single-scale topology optimization and fill the interior with a prescribed base shaped unit lattice. The two-scale topology optimization approach optimizes both the macro and micro-scale, allowing the unit cell freely change. In this paper, we present a study on the variable-density lattice structure optimization.

1 INTRODUCTION

Cellular materials consist of an array of periodic structures called cells. These cells are a small compartment which can be closed or opened and can be grouped by periodic or stochastic processes (Gibson and Ashby, 1997; Mazur et al., 2017) in 2D or 3D structures, like honeycombs, corks, sponges, and foams among others. The different properties of the cellular structures attend to three principles: the first one is the material from which it is made, the second one is the topology and shape of the cell, and the third one is the relative density (Ashby, 2006; Scheffler et al., 2005). These principles grant different properties such as thermal insulation, thermal expansion (Gu et al., 2012; R Lakes, 1996; Takezawa and Kobashi, 2017; Wu et al., 2017), product packaging, structural applications (static and dynamic) (Hohe et al., 2012), buoyancy, among others (Gibson and Ashby, 1997). These applications are suitable for almost all industries.

Topology optimization (TO) has gained much attention in the design process, being this a subfield of the structural optimization. This technique is presented as a problem with prescribed boundary conditions, constraints, and loads, to reach the best material distribution. To do this, many methods have been developed such as density-based (Bendsøe and Sigmund, 1999) and level set (LSM) (Wang et al., 2003). These methods have proved to have a great potential to generate complex and novel forms, which has revolutionized structural design. Due to the complexity of this new forms, a manufacturing problem emerges that often cannot be solved with the traditional manufacturing methods.

ASTM: F2792 – 12 defines the AM technique as the “process of joining materials to make objects from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing methodologies, such as traditional machining.” This technology is characterized by having a great compatibility with different materials and enabling the construction of highly complex structures, hence being greatly compatible with the problem described above (Martin et al., 2014). This manufacturing technique has drawbacks as well (Gaynor et al., 2014; Gaynor and Guest, 2016; Jun et al., 2016; Langelaar, 2016; Strano et al., 2013; Tejera, 2009; Wu et al., 2016; Zhao et al., 2017); particularly, being structurally self-supporting (maximum overhang printing angle and subtraction of internal supports) during the manufacturing process is one of them. However, these restrictions can be accounted early in the design process without sacrificing the versatility when building complex solids.

2 LATTICE INFILL DESIGN

In the AM process, supports are often needed to enable the construction of structures where the overhang angles are larger than maximum self-supporting angle. Overhang-free topology design has been achieved by different approaches (Gaynor and Guest, 2016; Wu et al., 2016), totally eliminating the need for support. However, most of these approaches drastically change the optimal topology, compromising the structural performance. AM process is a layer-by-layer process where structures with porous infill can be successfully built. Taking advantage of that, several overhang-free infill topology optimization methods have been developed (Wu et al., 2016, 2018). In this section, we briefly explain some of these methods and expand the works oriented to the AM.

2.1 Single-scale topology optimization

Another way to create a cellular solid is to perform the optimization process on the solid, and then fill it with a prescribe cellular shape, such Octet Truss (Deshpande et al., 2001; Fuller, 1961) among others (Panesar et al., 2018; Robbins et al., 2016), using the output density from the TO. This process is called variable-density approach.

In the work done by (Primo et al., 2017) a hybrid solution between the TO and a Lattice Approach (LA) is presented. The focus of their work is the optimization study of the C-clip Geometry. Key Performance Indicators (KPI) have been introduced to compare the solution's behavior. These indicators comprise four factors: the displacement (mm), the von Misses stress (MPa), structure weight (kg) and the printing time. The process to meet the objective consists of the following steps: the first step is the problem specification, the second step is the parallel work with the TO and Lattice approaches and the final step is to merge these solutions into a final form. Figure 1 shows two different solutions obtained by (Primo et al., 2017), the solution for a TO (a), and the hybridization solution called HA03 (b).Considering the weight and the displacement values (shown below each example with their respective KPI values in Figure 1, the HA03 is presented as a strong option when compared to the TO approach. However, the best solution depends on the final application. The results from this study indicate that LA is a good option to generated solid objects because different solutions can be used in different fields, thus adapting the result according to what is needed.

(a) (b)

Figure 1: Concept solutions adapted from (Primo et al., 2017)

In the study done by (Panesar et al., 2018), SIMP method was used to obtain optimal topologies with discrete values and a grayscale density for a cantilever beam domain. The output of this optimization process is combined with a lattice using different strategies, as shown in Figure 2, for a volume fraction of 0.5. The first one is made with an intersection between the cellular lattice and the TO, called intersected lattice (a). The second one uses the grayscale from TO and maps a lattice varying its material onto the density solution with a lower value equal to zero and upper value equal to one, this process is called "Graded Scale" (b). The third one is a re-scale of the value of the TO between a maximum and a minimum, for example, between 0.1-0.9, this strategy is called "scaled lattice" (c). The last one consists of filling the entire domain with a prescribed lattice called uniform lattice (d). In their work is used the support structure requirements, the computational efforts, and the Desing-to-Manufacture discrepancy as performance indicators. They concluded that the lattice strategies, in general, are around of 40-50% superior in comparison of the TO results, when is considered the high resilience to loading variations, and that the most robust strategy was the graded lattice. They observed that the complexity of the manufacture efforts and the support requirements increase; however, the latter can be inverted if the unit cell is self-supported. The authors acclaims that this strategy can be implemented more generally because it is independent from the design parameters chosen.

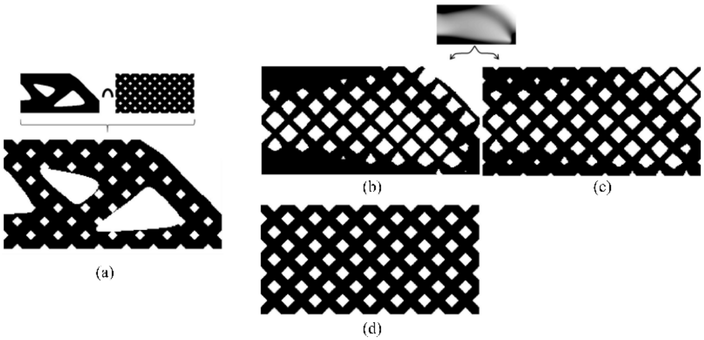

Figure 2: Lattice concept structures adapted from (Panesar et al., 2018)

As a general conclusion about the variable-density approach has as advantages that it can be easily applied as indicated by (Panesar et al., 2018) and partially guarantees the fully self-supported condition, depending on the cell selection (Panesar et al., 2018). Nonetheless , this approach also increase the complexity of the geometry, and since the lattice cannot freely change to according to force variation, the approach can fall into a sub-optimal solutions. Despite these limitation the variable-density approach is very popular in the industry (Arabnejad Khanoki and Pasini, 2012; Liu et al., 2018; Wang, Xu, et al., 2016).

2.2 Two-scale topology optimization

The multiscale optimization method takes in account two scales (macro and micro), in the macroscale is defined optimal spatial material layout distribution and simultaneously find the optimal local use of the cellular material at the microscopic scale. To deal with these different scales, the optimization needs an approach to characterize the material properties, called homogenization (Xia, 2016), which is also used in multiphase flow analysis for either Newtonian or non-Newtonian fluids (Hornung, 1997). Multiscale optimization process can also be classified as homogenous (Liu et al., 2018, 2008; Niu et al., 2009; Wang, Wang, et al., 2016) and heterogeneous (Coelho et al., 2008; Li et al., 2018; Rodrigues et al., 2002; Sivapuram et al., 2016).

Du (Du et al., 2017) improve the shear performance of the hexagonal cells. To do this, the energy homogenization method was used to maximize the shear modulus, using the optimality criteria (OC) (Bendsøe and Sigmund, 2004; Zhou and Rozvany, 1991) algorithm with a heuristic polarization operator for the approximation $0/1$. The algorithm of this work was tested with different volumes fractions and different mesh sizes; it was found that the entirety of these settings had influence in the final topological shapes. Those under the same volume fraction showed similar elasticity matrices and a common diagonal direction in the numerical tests with a simple load beam. They found that their method performs better when it is compared with a classical honeycomb sandwich structure having a lower strain under the same load conditions. The derivation of a mathematical model for the transverse shear deformation and flexibility of the cell wall is shown in their work, based on the previous work by (Gibson and Ashby, 1997).

The research done by (Wang, Xu, et al., 2017) uses the Asymptotic Homogenization method applied to four different unit cells and different relative densities. They found that the effective stiffness matrix of two of the four cellular structures correspond to orthogonal-isotropic materials with the same Young's modulus (x and y-direction) and a different shear modulus for the other two, which corresponds to orthotropic materials with different Young's modulus. Those unit cells were made from the same base material.

The work of (Li et al., 2018) consists of a topology optimization process with multi-patch microstructures. This method optimizes a representative microstructure for a whole family of microstructures having the same volume fraction, in order to improve the computational cost. To solve the compliance problem, the parametric level set method was used in combination with asymptotic homogenization method. In the microscale, each macro element can be regarded as an individual microstructure. The method consists in the calculation of the effective elasticity tensor with the induced displacement field in the microscale through the homogenization method. With these new properties, the macroscale displacement field is calculated and the macro design variables are updated through an Optimality Criteria (OC) algorithm (Bendsøe and Sigmund, 2004; Zhou and Rozvany, 1991). Next, the micro design variables are updated with another OC algorithm until the problem converges. In order to produce a solid that features manufacturing feasibility, kinematical connectors were added in the same position in all the microstructures. These connectors are tiny portions of the macro element that function as a non-design area for the microstructure, guaranteeing the connectivity between macro elements. This strategy can be used in either two- or three-dimensional space. They concluded that the strategy of grouping the densities reduced the computational cost and improved the structural performance when compared with a conventional one-scale structural design. Figure 3 shows the optimal solution for the cantilever problem.

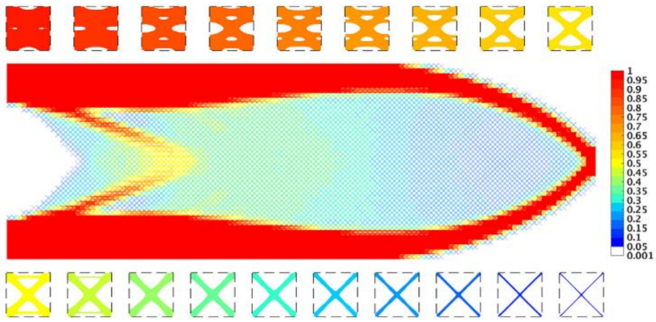

Figure 3: Optimization result for a cantilever beam (Li et al., 2018).

As a general conclusion, the multiscale method can find both global and a local optimal material distribution, and as a disadvantages, the homogenization process can encounter difficulties when handling with a nonlinear problems, have high computational burden (Xia, 2016) that demand parallel computing (Coelho et al., 2008; Wang et al., 2010; Wang, Chen, et al., 2017). Furthermore, the multiscale approach bears some discontinuities problems due to the absence of limitations in the material variation (Li et al., 2018; Zhou and Li, 2008) discontinuities problems may occur.

3 NUMERICAL RESULT

In our study, we use the well-known MBB problem to illustrate the variable-density method. The design domain and boundary conditions are provided in Figure 4 (a). Using `PolyMesher`, we construct a 180×60 uniform quadrilateral mesh and `PolyTop` (Talischi et al., 2012) was employed to minimize the compliance of the structure. The volume fraction is prescribed as $V = 50\%$ of the initial volume, the applied load is $F = 1000\ N$, the Young's modulus is $E = 210GPa$, the Poisson's ratio is ν = 0.3, and the radius of the filter is $R = 0.05$.

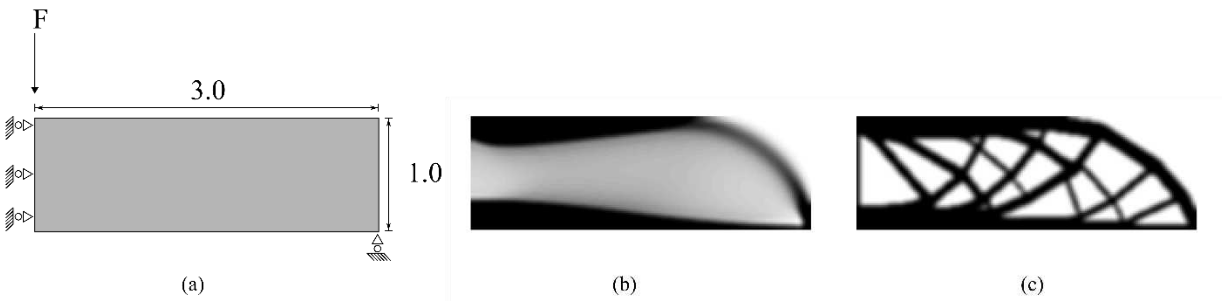

Figure 4: MBB problem: (a) geometry and boundary conditions, (b) TO with $p = 1$ (c) TO with $p = 1 - 3$

Figure 4 (b) and (c) show, respectively, the final topologies using the SIMP approach with a fixed penalization factor $p = 1$ and with an increasing penalization factor from 1 to 3, with increments of 1. Using the final topology shown in Figure 4 (b), the lattice solution (graded lattice) is obtained by replacing a group of $5\ x\ 5$ elements by a unit cell with the average of the volume fraction values, as illustrated in Figure 5.

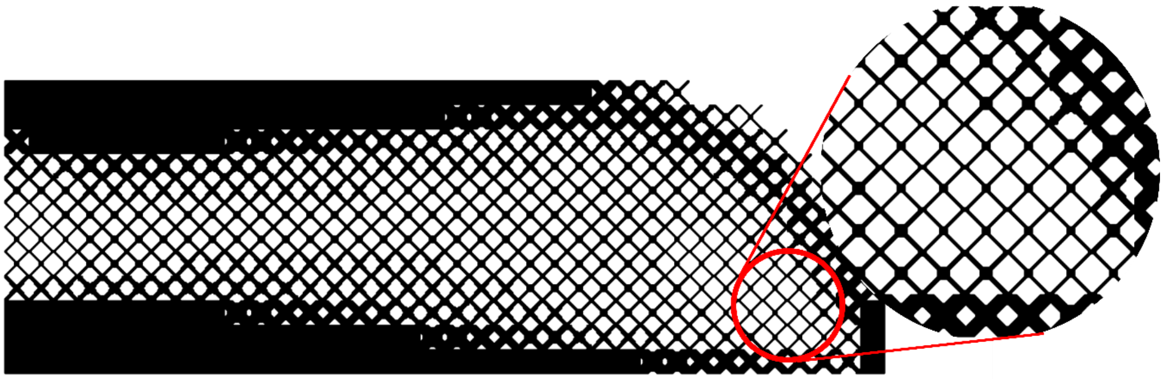

Figure 5: Lattice solution using the variable-density method.

After topology optimization, the boundaries of final structures were obtained using the `contour` function from MATLAB (MATLAB,2016a) and new meshes were obtained using the open-source code `mesh2d` (Engwirda, 2014) for both solid (as seen in Figure 4 (c)) and graded lattice (as seen in Figure 5). Figure 6 shows the results given by Ultimaker cura (Braam, 2018) software where the support material is represented by red color.

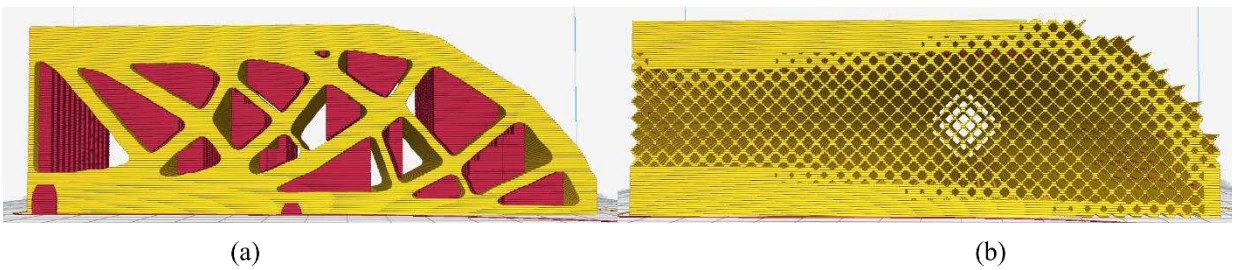

Figure 6 Solid and lattice printing simulation giving by Ultimaker Cura (Braam, 2018).

The volume of the structures after the post-treatment slight increased as presented in Table 1. The compliance and support material for both solid and lattice structures are presented in Table 1. We note that for almost the same material consumption the solid result performs better in stiffness (smaller compliance). One possible reason for this observation is that in the variable density approach, the lattice cannot freely change to account for the varying directions and magnitudes of the principal stresses, leading to a sub-optimal result. We also note that the support material in the lattice structure is significantly lower than the solid one.

Table 1 MBB problem: statistics of the lattice solution using the variable-density method.

	Volume (%)	Normalized compliance	Support (%)	Normalize printing time
Lattice	53.48	1.14	0.326	1.016
Solid	50.9	1	8.333	1

4 CONCLUSIONS

In this study, the variable-density method was applied in the MBB beam problem to obtain a lattice structure. This method starts with a single-scale topology optimization followed by filling the interior with a prescribed base shaped unit lattice. Because of this restriction in the design space, the structural mechanical performance was compromised, leading to a 14 % increase of the compliance. On the other hand, most of the lattice structure is self-support and no post-processing is needed to remove the supports. In summary, lattice material design is a promising strategy and has much room for improvement.

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