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BOUNDARY-LAYER RECEPTIVITY ANALYSIS OF A NACA-0012 AIRFOIL WITH WALL SUCTION

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Abstract. In this work, a receptivity analysis of the boundary layer evolving along a NACA-0012 wing profile with suction at wall surface is considered. The environmental perturbations that arrive from the free stream and enter the boundary layer, such as acoustic or vortical waves, do not have the correct wave parameters (i.e., the frequency and the wavelength) to excite the instability wave. Thus, a wave-modulation mechanism is needed in order to create the modal instability. The interaction between the external disturbance and the perturbation induced in the boundary layer by suction at the wing surface can activate this mechanism. In the model proposed here, the Navier-Stokes equations that govern the problem are linearized around a basic flow which is determined by the boundary-layer equations. Besides, considering that the boundary layer evolves with two different length scales, the theory of multiple scales is employed to solve the receptivity problem. A solvability condition, given by the orthogonality relation between the adjoint eigenfunction (solution of the adjoint eigenvalue problem) and the non-homogeneous term of the receptivity differential equation, allows the solution of the non-homogeneous problem. After imposing the solvability condition, a first-order differential equation with variable coefficients is obtained. The analytical expression of this equation gives the solution of the complex multiplicative coefficient that allows the wave-amplitude estimation as well as the obtention of the here-denominated receptivity function which gives an estimation of how receptive is the airfoil boundary layer to the considered disturbances.

1 INTRODUCTION

One of the observations reported by Reynolds (1883) during his early experiments on the pipe flow was that the amount of "noise" present in the external environment (e.g. wall pipe vibrations) as well as the roughness of the pipe surface, strongly affected the position along the pipe where the flow abandoned its laminar state to change to a turbulent one. Many experimentations during the years have demonstrated that the transition point, now characterized by the transition Reynolds number, is strongly influenced, as correctly predicted by Reynolds, by the specific natural characteristics of the freestream disturbances.

At the beginning of the sequence of events that lead to the breakdown to turbulence in unbounded open flows, there is a region of relatively low local-Reynolds number where instability waves, represented by primary modes, are generated. This zone extends approximately from the vicinity of the body nose to the area around the lower branch of the neutral-stability curve where the onset of wave amplification occurs. Environmental disturbances present in the free stream, such as vorticity or sound waves, enter the boundary layer under the form of steady or unsteady low-amplitude fluctuations of the basic state. Moreover, external disturbances propagate at the sound velocity relative to the fluid or convect at the freestream speed. At the same time, instability waves have phase velocities that are a fraction of the freestream velocity and consequently the energy to be transmitted to the boundary layer, in order to excite wave instabilities, is concentrated at wave numbers that are significantly different from the instability wave number. Therefore, a wave-modulation mechanism is needed in order to guarantee the unstable wave amplification. This process that establishes the initial conditions for the breakdown of laminar flow is referred to as the problem of boundary-layer receptivity to external disturbances or also to as the natural receptivity problem. The "adaptation mechanism" can be activated into the boundary layer by interaction between external disturbances and surface inhomogeneities caused by wall roughness, wall suction or wall vibration. This interaction generates a "new" forcing term that has the wave parameters tuned in the correct frequency and wavelength of the instability wave permitting in this way the instability excitement.

This aspect of the transition process, the receptivity process, was clearly formulated for the first time by Morkovin (1969), but it was during the late seventies, eighties and early nineties that receptivity theory expanded and a series of theoretical as well as experimental works came to light (Goldstein, 1981, 1983, 1984, 1985; Nishioka and Morkovin, 1986; Corke et al., 1986; Goldstein and Hultgren, 1987, 1989; Kerschen, 1990; Saric, 1993). Asymptotic analysis, employing matched asymptotic expansions (by means of the multiple-deck approach), was introduced by Goldstein for the study of leading-edge receptivity problems as one of the first effective theoretical approaches. The cardinal point of this theory resides on dividing the boundary layer into different regions in the wall-normal direction called *decks*, allowing the solution of simplified governing equations in each region. On the other hand, Zavol'skii et al. (1983) and then Choudary and Street (1992) contemporaneously with Crouch (1992a,b, 1994) proposed the finite-Reynolds-number theory that is, in fact, an evolution of the asymptotic theory. The first receptivity problems studied with this theory were localized receptivity problems including the modelization of localized inhomogeneities such as bumps or suction holes employing the technique of residue calculation. Posterior works presented the solution for distributed receptivity analysis. In the end of the nineties, Hill (1995, 1997) based on the Grosch and Salwen (1978, 1981) works, presented a new approach of the local receptivity theory introducing the adjoint problem as an alternative to the residue calculation. Other relevant boundary-layer receptivity works were published by Luchini and Bottaro (1998, 2001). The first work deals with the problem of Görtler vortices appearing in concave surfaces. The receptivity analysis was solved by means of a backward-in-time approach of the adjoint problem. The second one studies the Stokes layer produced by an impulsively started plate and was treated using two different adjoint approaches, a backward-in-time and a multiple-scale approach. One of the first applications of multiple-scale approach in stability analysis is due to Saric and Nayfeh (1975).

In the study of transition to turbulence, one of the most common applications is focused on aircraft industry, more precisely on wing boundary layers. With the objective of maintaining the boundary-layer flow in a laminar state, researchers developed control techniques based on wall suction along the wing surface. Even though theory predicts that application of wall suction can stabilize the flow, environmental disturbances impinging the boundary layer could interact with surface perturbations induced in the boundary layer by suction, activating a wave modulation mechanism capable of exciting modal instabilities.

The objective of this work is to investigate, in a theoretical way, the receptivity of the boundary layer developed over a classical NACA-0012 wing profile employing the multiple-scale technique. From a practical point of view, the extention of the receptivity analysis from the classical flat plate to the airfoil configuration can give more realistic results regarding aeronautic applications. The study is focused on the airfoil boundary layer when a disturbance induced by wall suction interacts with two possible external perturbations, namely an acoustic or a vorticity wave. This study could contribute to make clear on the phenomenon of receptivity generated on the boundary layer developed around convex surfaces of slender bodies such a wing profile.

2 AIRFOIL ANALYSIS

Although the problem of the boundary layer evolving over a wing profile can be treated as a flat-plate boundary layer, it differs from the flat-plate problem by some particular aspects that should be taken into account. Due to each particular airfoil geometry a given pressure distribution along the streamwise direction is present and should be contemplated for the basic flow calculation in order to introduce the correct outer boundary condition at each point along the airfoil. The use of the boundary-layer equations for the calculation of the unperturbed basic state, in this case the Prandtl formulation, presents the advantage that equations reduce to a parabolic form. In this way, the problem can be solved in a marching-pass scheme along the streamwise direction.

Moreover, from a geometrical point of view, the coordinates of the problem must be parametrized as a function of the airfoil coordinates in order to take into account the curved form of the surface where the boundary layer develops. Then, via parametrization, the streamwise coordinate (namely the x^* coordinate) will be treated as a coordinate whose base vector lays tangent to the wing surface in each point along the geometry profile while the y^* coordinate will be considered normal to the wing surface. Following this assumption, x^* is calculated as a function of the airfoil coordinates ($x^* = f(x_p^*, y_p^*)$). Subscript p means coordinates referred to a Cartesian reference frame in which for a symmetric airfoil the origin is placed at the leading edge of the profile, the x_p axis coincides with the chord line and the y_p coordinate gives the position of the airfoil surface along the chord in order to describe the airfoil geometry.

The analysis can be started from the full Navier-Stokes equations for an incompressible flow

$$
\begin{aligned}
u^*_{x^*} + v^*_{y^*} + w^*_{z^*} &= \quad 0\,, \\
u^*_{t^*} + u^* u^*_{x^*} + v^* u^*_{y^*} + w^* u^*_{z^*} &= -\frac{p^*_{x^*}}{\rho^*} + \nu^* (u^*_{x^*x^*} + u^*_{y^*y^*} \, + \, u^*_{z^*z^*})\,, \\
v^*_{t^*} + u^* v^*_{x^*} + v^* v^*_{y^*} + w^* v^*_{z^*} &= -\frac{p^*_{y^*}}{\rho^*} + \nu^* (v^*_{x^*x^*} \, + v^*_{y^*y^*} \, + \, v^*_{z^*z^*})\,, \\
w^*_{t^*} + u^* w^*_{x^*} + v^* w^*_{y^*} + w^* w^*_{z^*} &= -\frac{p^*_{z^*}}{\rho^*} + \nu^* (w^*_{x^*x^*} + w^*_{y^*y^*} + w^*_{z^*z^*})\,.
\end{aligned}
\tag{1}
$$

Asterisks mean dimensional quantities. It is possible to reduce the set of Eqs. 1 to a dimensionless form assuming proper reference quantities. For this purpose we assume as reference length $L^* = \nu^*/U^*_e$ with ν^* the kinematic viscosity and U^*_e the outer velocity at each x position. The pressure is rendered dimensionless with $\rho^* U_e^{*2}$ where ρ^* is the fluid density and the L^*/U^*_e relation is employed to put t in a dimensionless form. Besides, a global Reynolds number $R_c = U^*_\infty\, x^*_c/\nu^*$ and a global reference length $\delta_c = \sqrt{x^*_c\; \nu^*/U^*_\infty}$ are defined, where length x^*_c is equal to the chord length of the airfoil and U_∞ is the velocity of the unperturbed free stream.

At this point, the Navier-Stokes equations reduce to a dimensionless form:

$$
\begin{aligned}
u_x + v_y + w_z &= \quad 0\,, \\
u_t + u u_x + v u_y + w u_z &= -p_x + (u_{xx} + u_{yy} \, + u_{zz}\,)\,, \\
v_t + u v_x + v v_y + w v_z &= -p_y + (v_{xx} \, + v_{yy} \, + v_{zz}\,)\,, \\
w_t + u w_x + v w_y + w w_z &= -p_z + (w_{xx} + w_{yy} + w_{zz})\,.
\end{aligned}
\tag{2}
$$

Six boundary conditions close the problem, three at the wall and three at the infinity:

$$
\begin{aligned}
u &= u_0 \quad \text{at} \quad y = 0\,, \quad & u &= u_\infty \quad \text{for} \quad y \to \infty\,, \\
v &= v_0 \quad \text{at} \quad y = 0\,, \quad & w &= w_\infty \quad \text{for} \quad y \to \infty\,, \\
w &= w_0 \quad \text{at} \quad y = 0\,, \quad & p &= p_\infty \quad \text{for} \quad y \to \infty\,.
\end{aligned}
$$

Following the linear stability theory, the problem must be linearized around a basic state given by the solution of the boundary-layer equations. Our interest is centered in the growth or decay of a perturbation produced by the interaction of two disturbances. Following this direction, the velocity field shall be therefore decomposed in different velocity contributions and the previous Navier-Stokes system linearized about the base flow. As outlined in the introduction, in a steady incompressible boundary layer evolving over a flat plate or a wing surface, disturbances can come from the upstream external flow as acoustic waves or vortical waves and can act at the surface like wall roughness, wall suction or wall vibration. Each excitation source produces a contribution to the velocity field at different orders of magnitude and their interaction produces a resonant phenomenon at a higher order exciting the Tollmien-Schlichting wave (from now on TS). Two small disturbances $\epsilon \underline{v}_\epsilon(\underline{x}) e^{-i\omega_\epsilon t}$ and $\delta \underline{v}_\delta(\underline{x}) e^{-i\omega_\delta t}$ are introduced. The general vector $\underline{v}_{(.)} = [u_{(.)}, v_{(.)}, w_{(.)}]$, represents an unsteady wave amplitude of $O(\epsilon)$ and $O(\delta)$, generated by a general unsteady excitation source behaving as $e^{-i\omega_\epsilon t}$ and $e^{-i\omega_\delta t}$ respectively. These two perturbations are superimposed to a two-dimensional steady base flow $\underline{V} = [U(x,y), V(x,y), 0]$ and their interaction generates other beating waves $\epsilon\delta \underline{v}_{\epsilon\delta}(\underline{x}) e^{-i(\omega_\epsilon + \omega_\delta)t}$ and $\epsilon\delta \underline{v}_{\epsilon\delta}(\underline{x}) e^{-i(\omega_\epsilon - \omega_\delta)t}$ at order $\epsilon\delta$ respectively, plus other waves at higher orders ϵ^2 and δ^2. The waves at order ϵ and order δ do not have the right spatial wavelength and time frequency which characterize the TS waves. However, their interaction produced at order $\epsilon\delta$, could generate the proper conditions to excite the TS waves. Assuming that the resonant wave is characterized by $\epsilon\delta\underline{v}_{\epsilon\delta}(\underline{x})e^{-i(\omega_\epsilon+\omega_\delta)t}$, its amplitude is much larger than the other possible one, $\epsilon\delta\underline{v}_{\epsilon\delta}(\underline{x})e^{-i(\omega_\epsilon-\omega_\delta)t}$, so that the latter can be neglected in the analysis. The full expression for the velocity into the boundary layer is therefore,

$$\begin{aligned} \underbrace{\underline{v}(\underline{x},t)}_{\text{Total Vel.}} = \underbrace{\underline{V}(\underline{x})}_{\text{Base Flow Contrib.}} + \underbrace{\epsilon\,\underline{v}_\epsilon(\underline{x})e^{-i\omega_\epsilon t}}_{\text{External Pert. Contrib.}} + \underbrace{\delta\,\underline{v}_\delta(\underline{x})e^{-i\omega_\delta t}}_{\text{Wall Pert. Contrib.}} + \\ + \underbrace{\epsilon\delta\,\underline{v}_{\epsilon\delta}(\underline{x})e^{-i(\omega_\epsilon+\omega_\delta)t}}_{\text{Resonant Contrib.}} + O(\epsilon^2) + O(\delta^2) + \ldots \end{aligned} \tag{3}$$

Now, introducing Eq. 3 into Eqs. 2 and holding the terms of interest like order ϵ, order δ and order $\epsilon\delta$; three linear problems at different orders are found:

$$\begin{aligned} \mathbf{L}_\epsilon\ \mathbf{f}_\epsilon &= \mathbf{y}_\epsilon &&\rightarrow O(\epsilon), \\ \mathbf{L}_\delta\ \mathbf{f}_\delta &= \mathbf{y}_\delta &&\rightarrow O(\delta), \\ \mathbf{L}_{\epsilon\delta}\ \mathbf{f}_{\epsilon\delta} &= \mathbf{y}_{\epsilon\delta} &&\rightarrow O(\epsilon\delta). \end{aligned} \tag{4}$$

$\mathbf{L}$ is a linear operator and $\mathbf{f} = [\underline{v}, p]$ is the unknown array of the system. A general form of the obtained linear system is shown in the following set of equations:

$$\begin{aligned} u_x + v_y + w_z &= a\,, \\ u_t + Uu_x + Vu_y + uU_x + vU_y + p_x - \Delta u &= b\,, \\ v_t + Uv_x + Vv_y + uV_x + vV_y + p_y - \Delta v &= c\,, \\ w_t + Uw_x + Vw_y + p_z - \Delta w &= d\,. \end{aligned} \tag{5}$$

The known terms $\mathbf{y}_\epsilon$ and $\mathbf{y}_\delta$ of Eqs. 4 are originated at their corresponding orders ϵ and δ by the non-homogeneous boundary conditions at wall or at infinity, while at order $\epsilon\delta$, $\mathbf{y}_{\epsilon\delta}$ represents a forcing term originated by the coupling of the $\mathbf{f}_\epsilon$ and $\mathbf{f}_\delta$ solutions as shown in the next expression.

$$\mathbf{y}_{\epsilon\delta} = \begin{bmatrix} a_{\epsilon\delta} \\ b_{\epsilon\delta} \\ c_{\epsilon\delta} \\ d_{\epsilon\delta} \end{bmatrix} = \begin{bmatrix} 0 \\ u_\epsilon(u_\delta)_x + u_\delta(u_\epsilon)_x + v_\epsilon(u_\delta)_y + v_\delta(u_\epsilon)_y + w_\epsilon(u_\delta)_z + w_\delta(u_\epsilon)_z \\ u_\epsilon(v_\delta)_x + u_\delta(v_\epsilon)_x + v_\epsilon(v_\delta)_y + v_\delta(v_\epsilon)_y + w_\epsilon(v_\delta)_z + w_\delta(v_\epsilon)_z \\ u_\epsilon(w_\delta)_x + u_\delta(w_\epsilon)_x + v_\epsilon(w_\delta)_y + v_\delta(w_\epsilon)_y + w_\epsilon(w_\delta)_z + w_\delta(w_\epsilon)_z \end{bmatrix}$$

Eqs. 4 are three linear systems that model at order ϵ and order δ, the evolution in the boundary layer of the external and wall perturbations respectively and at order $\epsilon\delta$ the solution of the evolution of the unstable wave "created" by the interaction between the external and wall perturbation. The forcing term $\mathbf{y}_{\epsilon\delta}$ shows the contribution of each solution at precedent orders. On the other hand, from a mathematical point of view, the receptivity problem is no more a generalized eigenvalue problem with homogeneous boundary conditions (as the classical stability problem), but it is now a non-homogeneous problem with non-homogeneous boundary conditions. Moreover, being this a wave propagation problem, the dispersion relation that must be satisfied implies that $\det\left[\mathbf{L}_{\epsilon\delta}(\underline{\mathrm{k}},\omega,\underline{V})\right] = 0$, so that the solution cannot be reached in a direct form inverting the matrix of the discretized operator and not even as a generalized eigenvalue problem due to the non-homogeneous forcing term. Thus, in order to overcome this obstacle and based on the characteristics of the boundary-layer problem the multiple-scale approach shall be introduced in the analysis.

3 THE MULTIPLE-SCALE APPROACH

The method of multiple scales is an asymptotic approach which has been applied in many different branches of physics. If a problem differs from an already solved one, just due to certain parameters that in the solved problem are constant and in the unsolved one are substituted by slowly varying functions, then, the multiple-scale theory can be applied as a possible approach to solve it. Under this consideration, one expects to find out that the arbitrary constants, present in the constant-coefficient problem, now become also slowly varying functions. The problem reduces, therefore, to find these "variable constants" by means of an asymptotic development of power series with a proper "slow" parameter. Linear-stability analysis makes use just of the linear version of the multiple-scale approach (e.g. Saric and Nayfeh (1975)) but also non-linear applications exist. The linear approach, employed at the beginning of the last century in quantum mechanics, came to light under the name of theory of adiabatic perturbations and was introduced by Born and Fock (1928). The term adiabatic stays here for the slowly varying concept, making reference to the thermodynamic processes.

3.1 The Orr-Sommerfeld formulation

From a computational point of view, the linear system of Eqs. 5 can be reduced to a more amenable formulation that leads to the so-called Orr-Sommerfeld and Squire equations. This kind of formulation is based on a hybrid formulation involving primitive and non-primitive variables (namely the velocity field $\underline{v}$ and the vorticity field $\underline{\Omega}$) instead of the primitive variables $\underline{v}$ and p. Furthermore, under consideration of the Squire's theorem (Squire, 1933) the most destabilizing perturbation for a two-dimensional flow is also a two-dimensional disturbance. Thus, the analysis here proposed shall be accomplished for two-dimensional perturbations. After a few algebraic operations on Eqs. 5 and having in mind that we are dealing with a two-dimensional case (U_x = -V_y and u_x = -v_y); a general expression for the classic Orr-Sommerfeld equation (henceforth OS) can be obtained:

$$\begin{aligned}(\Delta v)_t + U(\Delta v)_x + V(\Delta v)_y + U_x\Delta v - V_x\Delta u -\\ - \Delta U v_x - \Delta V v_y + v(\Delta V)_y + u(\Delta V)_x -\\ - \Delta(\Delta v) = c_{xx} - b_{xy}\,.\end{aligned} \tag{6}$$

It is easy to find, from Eq. 6, the traditional OS equation assuming the conditions for a parallel flow (i.e. $U(y)$ and $V \equiv 0$).

3.2 Application of the multiple-scale approach

In order to apply this type of approach to the airfoil boundary-layer problem, the first step is to identify the scale of the problem that varies in a slowly form. As a well-known aspect of the boundary-layer subject, the parameters of this physical problem evolve in the chordwise direction (i.e. the x coordinate) in a slower form than in the wallnormal direction (namely the y coordinate). At this point, following which proposed by the multiple-scale theory, one is constrained to rescale the x coordinate as $X = \xi x$ for both the basic flow and the perturbation. Hence, the y coordinate shall indicate a fast-varying direction while the x coordinate a slow-varying one.

The general expression for the perturbation vector is represented by applying the normal mode hypothesis

$$\underline{v}(x,y,z,t) = \Big[\sum_{n=0}^{n=\infty} \xi^n\, \underline{\mathrm{v}}_n(X,Y,Z,T)\Big]\, e^{i\Theta(X,Y,Z,T)} \;. \tag{7}$$

Considering a two-dimensional perturbation, $\underline{v}(x,y,t)$=$[u(x,y,t),v(x,y,t)]$, convected by the base flow in the streamwise direction, the asymptotic expansion reduces to

$$\underline{v}(x,y,t) = \Big[\sum_{n=0}^{n=\infty} \xi^n\, \underline{\mathrm{v}}_n(X,Y)\Big]\, e^{i\Theta(X,T)} \;. \tag{8}$$

Thus, the scales for the different variables are

$$X = \xi x, \quad Y = y, \quad T = t\,, \tag{9}$$

and the respective derivatives are,

$$(\cdot)_x = \xi(\cdot)_X, \quad (\cdot)_y = (\cdot)_Y \quad \text{and} \quad (\cdot)_t = (\cdot)_T\,. \tag{10}$$

The phase function $\Theta(X,T)$ is constituted by

$$\Theta(X,T) = \theta_0(X)/\xi - \omega T\,, \tag{11}$$

where its derivatives can be expressed as

$$\frac{\partial\Theta(X,T)}{\partial x} = \alpha(x) \quad \text{and} \quad \frac{\partial\Theta(X,T)}{\partial t} = -\omega\,. \tag{12}$$

Provided that modal instabilities evolving in a boundary layer are convective instabilities a spatial analysis must be conducted. This introduces a complex wave number $(\alpha_r + i\alpha_i)$ where the real part represents the wave number and the imaginary part furnishes the growth rate of the wave. The angular pulsation of the problem ω is a real number.

Considering the base flow as $\underline{V} = [U(x,y), \xi V(x,y)]$ and their respective derivatives as

$$U_x = \xi U_X, \quad U_y = U_y, \quad \xi V_x = \xi^2 V_X, \quad \xi V_y = \xi V_Y\,, \tag{13}$$

and introducing the above expressions into Eq.6, the OS equation reads,

$$\begin{aligned}
&(\Delta v)_t + U(\Delta v)_x + \xi V(\Delta v)_y + \xi U_X \Delta v - \frac{i\xi^2}{\alpha} V_X (\Delta v)_y - \\
&- (\xi^2 U_{XX} + U_{YY}) v_x - (\xi^3 V_{XX} + \xi V_{YY}) v_y + \\
&+ v(\xi^3 V_{XX} + \xi V_{YY})_Y + \frac{i\xi}{\alpha}(\xi^3 V_{XX} + \xi V_{YY})_X\, v_y - \\
&- \Delta(\Delta v) = \xi\, \underbrace{(c_{xx} - b_{xy})}_{y_{OS}}\,.
\end{aligned} \tag{14}$$

Now, injecting Eq. 8 into Eq. 14 and retaining the correspondent orders of magnitude, it is possible to find:

$$\begin{aligned}&\text{at } O(\xi^0), \quad (-i\omega + i\alpha\, U)\Delta \mathrm{v}_0 - i\alpha\, U_{YY}\mathrm{v}_0 - \Delta(\Delta \mathrm{v}_0) = 0\,,\\ &\text{at } O(\xi^1), \quad (-i\omega + i\alpha\, U)\Delta \mathrm{v}_1 - i\alpha\, U_{YY}\mathrm{v}_1 - \Delta(\Delta \mathrm{v}_1) = y_{OS}\, e^{-i\Theta(X,T)} - F(\mathrm{v}_0).\end{aligned} \tag{15}$$

The four boundary conditions for Eqs. 15 are:

$$\begin{array}{llll} \mathrm{v} = 0, & \mathrm{v}_y = 0 & \text{for} & y = 0 \\ \mathrm{v} = 0, & \mathrm{v}_y = 0 & \text{for} & y \to \infty\,. \end{array} \tag{16}$$

It is useful to note that $\Delta(\mathrm{v})$ and $\Delta\Delta(\mathrm{v})$ are now $(\mathrm{v}_{YY} - \alpha^2\, \mathrm{v})$ and $(\mathrm{v}_{YYYY} - 2\alpha^2\, \mathrm{v}_{YY} + \alpha^4\, \mathrm{v})$ respectively. Now, looking at Eqs. 15, the left-hand-side term is the traditional OS linear operator for both the equations. The first equation is a homogeneous differential equation and represents an eigenvalue problem where α is the eigenvalue and v_0 its corresponding eigenvector, while the second one is a non-homogeneous differential equation. Moreover, the $\boldsymbol{F}$ operator contains the order ξ^1 terms having the $\mathbf{v}_0$ vector, which is obtained 'a priori' from the solution of the eigenvalue problem at order ξ^0. It can be written in the following form:

$$\boldsymbol{F}(\mathbf{v}_0) = \boldsymbol{G}(\mathbf{v}_0) + \boldsymbol{H}(\mathbf{v}_{0_X})\,. \tag{17}$$

Where $\boldsymbol{G}$ and $\boldsymbol{H}$ are

$$\begin{aligned} \boldsymbol{G}(\mathbf{v}_0) = \Big[&\omega\alpha_X(\cdot) - 3\alpha\alpha_X U(\cdot) + 4i\alpha^2\alpha_X(\cdot) - (U_X)_{YY}(\cdot) + \\ &+ (U_X)_Y(\cdot)_Y - 2i\alpha_X\Delta(\cdot) + V\Delta(\cdot)_Y\Big]\mathbf{v}_0\,, \\ \boldsymbol{H}(\mathbf{v}_{0_X}) = \Big[&2\omega\alpha(\cdot) + U\Delta(\cdot) - 2\alpha^2 U(\cdot) - U_{YY}(\cdot) - 4i\alpha\Delta(\cdot)\Big]\mathbf{v}_{0_X}\,. \end{aligned} \tag{18}$$

The phase exponent $i\Theta(X,T)$ can be expressed as

$$i\Theta(X,T) = i\theta(X)/\xi - i\omega T = i\int_{x_0}^{x} \alpha(x')\, dx' - i\omega t\,. \tag{19}$$

The solution at order $\epsilon\delta$ can be obtained imposing the solvability condition.

3.3 The solvability condition

The solution of the eigenvalue problem of Eqs. 15 provides a spectrum of $\alpha_j(x)$ and their corresponding eigenvectors v_{0_j}. On the other hand, the non-homogeneous problem at $\mathrm{O}(\xi^1)$, has in the left-hand-side term the same OS operator which satisfies, for the $\mathrm{O}(\xi^0)$ problem, the $\det(\mathbf{A}(X) + \alpha_j\mathbf{B}(X)) = 0$ condition. Hence, the matrix of the discretized operator of the non-homogeneous problem is non-invertible. The way of finding the solution at order ξ^1 and at next orders is to impose a compatibility condition.

Using the general condition which establishes for a matrix $\mathbf{A}$ that

$$\mathbf{A}^{-1} = \sum_{k=1}^{k=N} \lambda_k^{-1} \mathbf{u}_k \mathbf{u}_k^+ , \tag{20}$$

where $\mathbf{u}_k$ and $\mathbf{u}_k^+$ are respectively the k^{th} right and left eigenvectors of the matrix $\mathbf{A}$ and λ_k is the corresponding eigenvalue. The general non-homogeneous linear system $(\lambda \mathbf{I} - \mathbf{A})\,\mathbf{x} = \mathbf{y}$ can be solved employing Eq. 20,

$$\mathbf{x} = (\lambda \mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{y} = \sum_{k=1}^{k=N} (\lambda - \lambda_k)^{-1} \mathbf{u}_k \mathbf{u}_k^+ \cdot \mathbf{y}, \tag{21}$$

Now, looking at Eq. 21 as a function of the parameter λ, when $\lambda \to \lambda_k$ the solution $\mathbf{x} \to \infty$ because $(\lambda - \lambda_k)^{-1} \to \infty$. The problem can be then surmounted imposing that the term $(\lambda - \lambda_k)^{-1} \mathbf{u}_k \mathbf{u}_k^+ \cdot \mathbf{y}$ be equal to zero. This is that $\mathbf{u}_k^+ \cdot \mathbf{y} = 0$, which means that the known term $\mathbf{y}$ must be orthogonal to the left eigenvector $\mathbf{u}_k^+$.

Thus, the application of the compatibility condition in order that Eq. 15 admits a solution for v_1 is expressed as

$$\mathrm{v}_0^+ \cdot \left(\mathbf{y}_{OS} e^{-i \int_{x_0}^{x} \alpha(x')\, dx' + i\omega t} - \boldsymbol{G}\, \mathrm{v}_{0_X} - \boldsymbol{H}\, \mathrm{v}_0 \right) = 0\,. \tag{22}$$

The j^{th} eigenvector can be defined as $\mathrm{v}_{0_j} = c_j(x) \widetilde{\mathrm{v}}_{0_j}$, where $c_j(x)$ is a multiplicative factor also evolving in a slow form and $\widetilde{\mathrm{v}}_{0_j}$ is the j^{th} eigenvector normalized in a certain arbitrary way. In our case, we are interested in a specific mode of the spectrum which is the most unstable mode identified as TS mode. Inserting $c(x)\widetilde{\mathrm{v}}_0$ into Eq. 22 and expanding the terms we obtain,

$$\frac{dc(x)}{dx} + \underbrace{\frac{\mathrm{v}_0^+ \cdot \left(\boldsymbol{G}\, \widetilde{\mathrm{v}}_{0_X} - \boldsymbol{H}\, \widetilde{\mathrm{v}}_0 \right)}{\mathrm{v}_0^+ \cdot \boldsymbol{G}\, \widetilde{\mathrm{v}}_0}}_{\mathbf{a}(\mathrm{x})} c(x) = \underbrace{\frac{\mathrm{v}_0^+ \cdot \mathbf{y}_{OS}}{\mathrm{v}_0^+ \cdot \boldsymbol{G}\, \widetilde{\mathrm{v}}_0}}_{\mathbf{b}(\mathrm{x})} e^{-i \int_{x_0}^{x} \alpha(x)\, dx + i\omega t} \quad , \tag{23}$$

$$\frac{dc(x)}{dx} + \mathbf{a}(x)\, c(x) = \mathbf{b}(x). \tag{24}$$

Eq. 23 is a first-order differential equation with slowly-varying coefficients and its solution can be expressed as,

$$c(x) = \int \mathbf{b}(x)\, e^{\int_{x_0}^{x} \mathbf{a}(x')\, dx'}\, dx\; e^{-\int_{x_0}^{x} \mathbf{a}(x')\, dx'}. \tag{25}$$

3.4 The wave amplitude expression

The solution of the multiplicative coefficient provided by Eq. 25 allows to compute the solution of the problem at order $\epsilon\delta$ and consequently to obtain an expression for the amplitude value of the modal perturbation (i.e. the TS wave) excited by a particular forcing term generated by the interaction between the external disturbance and the wall perturbation.

Considering that,

$$u_{\epsilon\delta}(x, y, t) = c(x)\, \widetilde{\mathrm{u}}_0\, e^{i \int_{x_0}^{x} \alpha(x')\, dx' - i\omega t} + O(\xi^1) \tag{26}$$

and introducing into Eq. 26 the expression obtained for $c(x)$ in Eq. 25, it is possible to obtain the solution for $u_{\epsilon\delta}(x_f, y, t)$ as,

$$u_{\epsilon\delta}(x_f, y, t) = \underbrace{\int_{x_0}^{x_f} \mathbf{b}(x) e^{\int_{x_0}^{x} \mathbf{a}(x')dx'} dx \, e^{-\int_{x_0}^{x_f} \mathbf{a}(x')dx'}}_{c(x_f)} \, \widetilde{\mathrm{u}}_0(x_f, y) \, e^{i\int_{x_0}^{x_f} \alpha(x')dx' - i\omega t} . \tag{27}$$

Now, reordering Eq. 27 and substituting $\mathbf{b}(x)$ from Eq. 23 the solution reads

$$u_{\epsilon\delta}(x_f, y, t) = \widetilde{\mathrm{u}}_0(x_f, y) \int_{x_0}^{x_f} \frac{\mathrm{v}_0^+ \cdot \mathbf{y}_{OS}}{\mathrm{v}_0^+ \cdot \boldsymbol{G}\,\widetilde{v}_0} e^{\int_{x_0}^{x} \mathbf{a}(x') - i\alpha(x')dx'} dx \, e^{i\int_{x_0}^{x_f}(\alpha(x') - \mathbf{a}(x'))dx' - i\omega t} . \tag{28}$$

The amplitude of the modal unstable wave is,

$$|u_{\epsilon\delta}(x_f, y, t)| = \left| \widetilde{\mathrm{u}}_0(x_f, y) \int_{x_0}^{x_f} \frac{\mathrm{v}_0^+ \cdot \mathbf{y}_{OS}}{\mathrm{v}_0^+ \cdot \boldsymbol{G}\,\widetilde{\mathrm{v}}_0} e^{\int_{x_0}^{x} \mathbf{a}(x') - i\alpha(x')dx'} dx \right| . \tag{29}$$

Defining the amplitude value for a general x position as $\mathcal{A}(x) = \max|u_{\epsilon\delta}(x, y, t)|$ and normalizing the eigenvectors in order that $\max|\widetilde{\mathrm{u}}_0(x, y)| = 1$ the final amplitude expression becomes,

$$\mathcal{A}(x_f) = \max|u_{\epsilon\delta}(x_f, y, t)| = \left| \int_{x_0}^{x_f} \frac{\mathrm{v}_0^+ \cdot \mathbf{y}_{OS}}{\mathrm{v}_0^+ \cdot \boldsymbol{G}\,\widetilde{\mathrm{v}}_0} \, e^{\int_{x_0}^{x} \mathbf{a}(x') - i\alpha(x')dx'} dx \right| . \tag{30}$$

3.5 Wall suction receptivity

In the precedent analysis the wave amplitude was related to a forcing term $\mathbf{y}_{OS}$ that is function of both the external and the wall disturbance. Having in mind that our objective is to study the influence on the unstable wave that is originated by a certain wall suction profile $v_w(x)$ and some external disturbance, it is possible to include $v_w(x)$ in a direct form into the calculation of the amplitude of the TS wave. The velocity $v_w(x)$ is a function that describes the wall suction profile in the streamwise direction and is the only component different from zero in the right-hand-side term of the order δ system of the set of Eqs. 4. The other components of the $\mathbf{y}_\delta$ forcing vector are zero. As was shown for the multiple-scale analysis, the x variable is a slow variable respect to y. Then, the $O(\delta)$ problem can be analyzed by means of the classical OS equation as a slowly-varying problem in the streamwise direction and can be solved at each x_i position as a function of the y coordinate. The differential problem can be written for each x_i station as $\boldsymbol{OS}\ \boldsymbol{v_\delta}(y) = \boldsymbol{y_\delta}$ with homogeneous boundary conditions except for $v_\delta(y = 0) = v_w(x_i)$. Therefore, as $v_w(x_i)$ is only function of x, it acts as a constant value respect to the y coordinate where the perturbation changes in a fast way. Consequently, the solution can be obtained from $\boldsymbol{OS}\,\boldsymbol{v_{\delta_0}}(y) = \boldsymbol{y_{\delta_0}}$ with the non-homogeneous boundary condition $v_{\delta_0}(y = 0) = 1$ instead of $v_\delta(y{=}0){=}v_w$, where $\boldsymbol{v_\delta} = v_w(x)\,\boldsymbol{v_{\delta_o}}$. Keeping in mind that the forcing term involved at order $\epsilon\delta$ is obtained as a combination of the order ϵ and order δ solutions, it can be written for each x_i position as $\mathbf{y}_{OS}(y) = v_w(x)\,\mathbf{y}_w(y)$. Then, replacing into Eq.30, the equation for the final amplitude reads,

$$\mathcal{A}(x_f) = \left| \int_{x_0}^{x_f} \underbrace{v_w(x) \underbrace{\frac{\mathrm{v}_0^+ \cdot \mathbf{y}_w}{\mathrm{v}_0^+ \cdot \boldsymbol{G}\,\widetilde{\mathrm{v}}_0}}_{\mathrm{g_w}} e^{-\int_{x}^{x_f} \mathbf{a}(x') - i\alpha(x')dx'}}_{\mathrm{G_w}} dx \right| . \tag{31}$$

Following Zucher (2001), the term $\mathrm{g_w}$ is called *receptivity coefficient* and gives an idea of the receptivity of the boundary layer to a localized inhomogeneity placed at a certain x. It can be identified for instance with the "efficiency function" Λ employed by Choudary and Street (1992) calculated as the residue contributions or also with the "response function" Λ presented by Crouch (1992a) and used to evaluate the response residue K. On the other hand, Hill (1995) proposed an adjoint receptivity analysis for a Blasius boundary layer where a coefficient Λ for the localized receptivity is introduced. The model proposed in this work permits to obtain, in an easy way, the results of the above cited works using a different technique. Here, the receptivity coefficient is calculated, by means of the solvability condition, using the solution of the adjoint OS problem which in the algebraic transposition of the problem coincides with left eigenvector v_0^+. In this way, the residue calculation proposed in other works is avoided.

On the other hand, the term $\mathrm{G_w}$, here denominated *receptivity function*, allows to analyze the distributed receptivity taking into account the non-local effects. Moreover, non-parallel boundary-layer corrections are introduced in the analysis by means of the operator $\mathbf{a}(x)$ involved in $e^{-\int_x^{x_f} \mathbf{a}(x) - i\alpha(x)\, dx}$.

The final amplitude at order $\epsilon\delta$ can be evaluated in a compact form as,

$$\mathcal{A}(x_f) = \left| \int_{x_0}^{x_f} v_w(x)\, \mathrm{G_w}(x)\; dx \right|. \tag{32}$$

Assumed a specific freestream disturbance at order ϵ, the amplitude of the TS-wave can be easily computed for different wall-suction velocity profiles (i.e. the receptivity function $\mathrm{G_w}$ can be weighted for different wall-suction conditions). Besides, in order to let $\mathrm{G_w}(x)$ be independent of the final position x_f and assuming that the receptivity function vanishes for $x \to$ -∞ and $x \to x_f$, the integral of the exponential term $e^{-\int_x^{x_f} \mathbf{a}(x) - i\alpha(x)\, dx}$ can be separated into two integrals using the neutral point x_n:

$$\mathcal{A}(x_f) = \left| \int_{-\infty}^{+\infty} v_w(x)\, \frac{\mathrm{v}_0^+ \cdot \mathbf{y}_w}{\mathrm{v}_0^+ \cdot \boldsymbol{G}\, \tilde{\mathrm{v}}_0}\; e^{-\int_x^{x_n} \mathbf{a}(x') - i\alpha(x')\, dx'}\; dx\, e^{-\int_{x_n}^{x_f} \mathbf{a}(x') - i\alpha(x')'\, dx'} \right|. \tag{33}$$

4 FREESTREAM AND WALL DISTURBANCES

As was already mentioned in the previous sections, the external disturbance as well as the wall suction disturbance do not satisfy the dispersion relation of the OS equation because they do not have the correct wave parameters. Two possible environmental disturbances are studied, acoustic waves or vortical waves which arrive from the external upstream flow and enter the boundary layer. Both the disturbances, acoustic waves and vorticity waves, are unsteady waves with a frequency ω_ϵ and wave number α_ϵ. On the other hand, the wall suction disturbance is treated as a steady perturbation. Although suction devices are made in order to reduce the propagation of disturbances into the flow, manufacturing imperfections or clogging of suction holes cause the creation of a broad-band wavenumber spectrum which offers to the external disturbance a wide wavenumber 'assortment'.

4.1 Acoustic wave

Under the hypothesis of low Mach number, the acoustic wave present in the outer flow can be supposed as a plane wave ($u_{\epsilon_\infty} = 1$ and $v_{\epsilon_\infty} = 0$) characterized by a time harmonic pulsation

ω_ϵ where its wave number $\alpha_\epsilon = 0$ (see for instance Ackerberg and Phillips (1972)). Starting the analysis from the linearized Navier-Stokes equations at order ϵ and taking into account the above established conditions, the equations reduce to the following system

$$\begin{aligned} (v_\epsilon)_y &= 0\,, \\ -i\epsilon u_\epsilon - (u_\epsilon)_{yy} &= 0\,, \\ -(p_\epsilon)_y &= 0\,. \end{aligned} \tag{34}$$

with the following four boundary conditions,

$$u_\epsilon = 0\,,\; v_\epsilon = 0 \quad \text{at} \quad y = 0 \quad \text{and} \quad u_\epsilon \to 1\,,\; v_\epsilon \to 0 \quad \text{for} \quad y \to \infty. \tag{35}$$

At this point the $O(\epsilon)$ problem has an analytical solution which is the Stoke's flow where

$$\begin{aligned} &u_\epsilon = 1 - \exp\left[- \sqrt{-i\omega_\epsilon}\, y \right], \\ &v_\epsilon = 0\,. \end{aligned} \tag{36}$$

4.2 Vorticity wave

Another type of freestream disturbance that can enter the boundary layer can be described by a vortical field. It is possible to imagine the vorticity wave as a gust which is convected in the free stream where the wing profile is immersed (see for example Rogler and Reshotko (1975)). Different from the acoustic wave, the vortical wave is characterized not only by its angular pulsation ω_ϵ but also by a finite wave number α_ϵ which is different from zero. The external flow admits disturbances with a certain vorticity value that act like $e^{i(\alpha_\epsilon x - \omega_\epsilon t)}$ for $\alpha_\epsilon = \omega_\epsilon / U_\infty$. This disturbance enters the boundary layer producing a perturbation which is governed by the viscous equation. The boundary conditions at wall are homogeneous while at the outer edge of the boundary layer can be established by means of an asymptotic analysis. The problem can be treated using the OS formulation. Considering that the basic flow outside the boundary layer $(y \to \infty)$ is constant the OS operator reduces to

$$\begin{aligned} i\alpha_\epsilon u_\epsilon + v_{\epsilon_y} &= 0\,, \\ (-i\omega_\epsilon + i\alpha_\epsilon U_\infty)\Delta v_\epsilon - \Delta(\Delta v_\epsilon) &= 0\,, \end{aligned} \tag{37}$$

which is a constant-coefficient ordinary differential equation and its solution may be expressed as:

$$v_\epsilon = c_1\, e^{\lambda_1 y} + c_2\, e^{\lambda_2 y} + c_3\, e^{\lambda_3 y} + c_4\, e^{\lambda_4 y}\,, \tag{38}$$

where

$$\begin{aligned} \lambda_1 &= \alpha, & \lambda_3 &= \sqrt{\alpha^2 + i\alpha U_\infty - i\omega}, \\ \lambda_2 &= -\alpha, & \lambda_4 &= -\sqrt{\alpha^2 + i\alpha U_\infty - i\omega}. \end{aligned} \tag{39}$$

In order to model freestream disturbances entering the boundary layer we must require the solution of Eq. 37 to have a finite (non-vanishing) amplitude in the limit $y \to \infty$. This is possible only if two of the characteristic values λ's are purely imaginary, or in other words if $\mathrm{Re}[\lambda(\alpha_\epsilon, \omega_\epsilon, U_\infty)] = 0$. However, when the wave is convected in the free stream experiences certain attenuation and consequently the possibilities of reaching the boundary layer grow if its wavelength is large enough compared to the boundary-layer thickness. Then, a fairly choice could be to suppose $\lambda = 0$, which allows to calculate the wave number α_ϵ from Eq. 39 as

$$\alpha_\epsilon = \frac{-iU_\infty + i\sqrt{U_\infty^2 - 4i\omega_\epsilon}}{2}\,. \tag{40}$$

This permits to solve

$$\left[(-i\omega_\epsilon + i\alpha_\epsilon\, U)(\mathcal{D}^2 - \alpha_\epsilon^2) - i\alpha_\epsilon\, U_{yy} - (\mathcal{D}^4 - 2\alpha_\epsilon^2\mathcal{D}^2 + \alpha_\epsilon^4)\right] \boldsymbol{v}_\epsilon = \boldsymbol{y}_\epsilon\,. \tag{41}$$

The asymptotic condition can be expressed as

$$\begin{aligned} v_\epsilon\, e^{i(\alpha_\epsilon x - \omega_\epsilon t)} &= (c_1 e^{-\alpha_\epsilon y} + c_3 + c_4 y)\, e^{i(\alpha_\epsilon x - \omega_\epsilon t)} \\ & \qquad\qquad\qquad\qquad \text{for} \quad y \to \infty \\ (v_\epsilon)_y\, e^{i(\alpha_\epsilon x - \omega_\epsilon t)} &= (-\alpha_\epsilon\, c_1 e^{-\alpha_\epsilon y} + c_4)\, e^{i(\alpha_\epsilon x - \omega_\epsilon t)} \end{aligned} \tag{42}$$

with c_1 and c_3 free constants and $c_4 = 1$, while the wall boundary conditions are

$$v_\epsilon = 0 \quad \text{and} \quad (v_\epsilon)_y = 0 \quad \text{at} \quad y = 0. \tag{43}$$

The problem is a non-homogeneous differential system where the ‘forcing’ array is determined by the terms introduced by the asymptotic boundary conditions. In this case, the matrix is not singular and can be solved simply inverting the matrix of the discretized operator. The streamwise component of the perturbation can be recovered as $\boldsymbol{u}_\epsilon = i(\boldsymbol{v}_\epsilon)_y/\alpha_\epsilon$, by means of the continuity equation, after obtained the solution of Eq. 41.

4.3 Wall suction disturbance

As was already anticipated in paragraph 3.5, the wall disturbance induced into the boundary layer may be also obtained using the OS formulation. Under the assumption of a steady disturbance the wave adopts the following form $v(x, y)\, e^{i\int \alpha_\delta(x')dx'}$. The suction velocity profile along the wall is described by the function $v_w(x)$. A wide wavenumber spectrum is present when this function is decomposed in Fourier series but we are interested in a particular wavenumber value α_δ which combined with the freestream wavenumber along the x direction permits to create at order $\epsilon\delta$ the resonant condition which satisfies the dispersion relation of the unstable wave. As previously mentioned, the wall perturbation problem can be solved for each streamwise position independently of the value that $v_w(x)$ adopts. For this pourpose one has to solve the following system:

$$\left[i\alpha_\delta\, U(\mathcal{D}^2 - \alpha_\delta^2) - i\alpha_\delta\, U_{yy} - (\mathcal{D}^4 - 2\alpha_\delta^2\mathcal{D}^2 + \alpha_\delta^4)\right] \boldsymbol{v}_{0_\delta} = \boldsymbol{y}_{\boldsymbol{w}_\delta} \tag{44}$$

with boundary conditions:

$$\begin{aligned} v_{0_\delta}(x, 0) &= 1\,, \quad v_{0_{\delta\,y}}(x, 0) = 0 \qquad y = 0 \\ v_{0_\delta}(x, y) &= 0\,, \quad v_{0_{\delta\,y}}(x, y) = 0 \qquad y \to \infty. \end{aligned} \tag{45}$$

Moreover, the solution $\boldsymbol{v}_\delta(x_i, y)$ can be easily obtained as $\boldsymbol{v}_\delta = v_w\, \boldsymbol{v}_{0_\delta}$ for each x_i position. The forcing term $\boldsymbol{y}_{\boldsymbol{w}_\delta}$ is zero everywhere except for boundary conditions. The problem does not fulfill the dispersion relation of the OS equation and can be solved in a direct form inverting the matrix of the discretized operator.

5 RECEPTIVITY ANALYSIS

In this section the numerical results referred to the receptivity analysis will be presented. For the calculation of basic flow, the Prantl's equation was solved using a finite-difference approach. A second-order centered shceme was employed for the discretization of the terms in the y direction while a second-order backward shceme was used in the x direction. Both the grids used in the x and y direction were non-uniform spaced grids, permitting a finer approximation of the solution near the leading edge and near the wing surface. For the solution of the eigenvalue problem it was employed the inverse iteration algorithm that is a modified version of the direct iteration algorithm. The inverse iteration algorithm permits to calculate a particular eigenvalue from the spectrum even if it is a complex eigenvalue. Besides, this numerical technique requires of a lesser number of iterations than the direct approach. The perturbation problems were solved using a centered second-order finite-difference scheme along the y direction.

The results presented belong to three different cases of mean wall-suction boundary conditions applied to the boundary-layer basic flow of the NACA-0012 airfoil, $< V_w >=$ -1 $\cdot\, 10^{-5}$, -5 $\cdot\, 10^{-5}$ and -1 $\cdot\, 10^{-4}$. The Reynolds number was $R_c = 18 \cdot 10^6$ and the corresponding adimensional frequencies were $F = \omega\nu/U_e^2 = 28 \cdot 10^{-6},\ 22 \cdot 10^{-6},\ 15 \cdot 10^{-6}$ respectively. The frequency values correspond to the most unstable case for each mean wall-suction condition. The most unstable frequency is intended to be the frequency that first drives to the transition condition estimated by the traditional N-factor method introduced by Smith and Gamberoni (1956).

5.1 Acoustic-wave wall-suction receptivity

The acoustic wave cannot by itself excite the TS wave so that the resonant condition is achieved interacting with the wall-suction disturbance by means of the adaptation mechanism. The resonant condition for this specific case is determined by,

$$\alpha_{\mathrm{TS}} = \alpha_\delta \qquad \text{and} \qquad \omega_{\mathrm{TS}} = \omega_\delta. \tag{46}$$

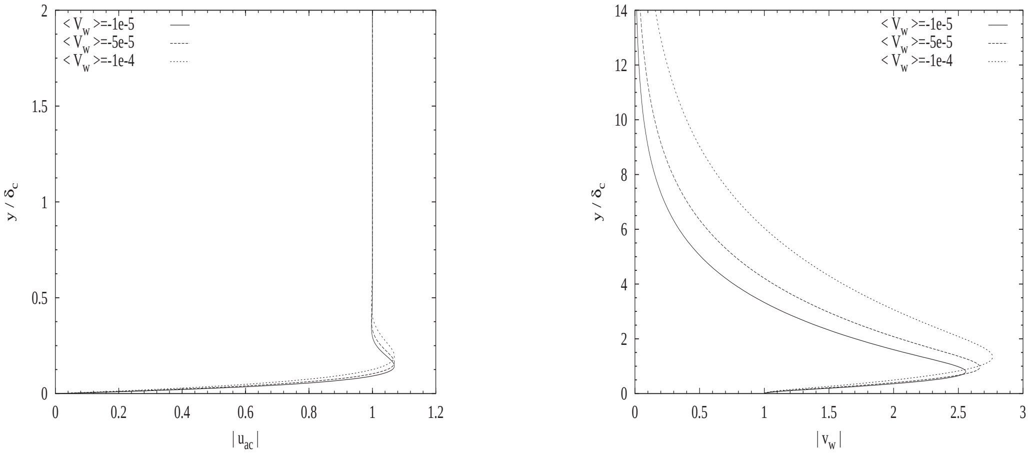

Figure 1: (Left) Modulus of the acoustic disturbance at neutral point. (Right) Modulus of the suction disturbance at neutral point. $\delta_c\alpha_\delta = 0.4030$, $\delta_c\alpha_\delta = 0.3258$ and $\delta_c\alpha_\delta = 0.2328$.

Figure 1 shows the solutions for the $\mathrm{O}(\epsilon)$ acoustic problem (left side) and for the $\mathrm{O}(\delta)$ wall-suction problem (right side). It is possible to appreciate the evolution of the acoustic wave into the boundary layer developed around the NACA-0012 airfoil for three different mean wall-suction velocities and the disturbance induced into the boundary layer due to the application of wall suction. These two solutions allow the calculation of the forcing term at $\mathrm{O}(\epsilon\delta)$. As can be observed from the figure the acoustic-wave profile represents a constant front except for a thin sublayer very near the wall where viscous effects induce the characterisitic Stoke's viscous layer. On the other hand, the wall-suction pertubation shows the wall-normal component where the maximum value lays approximately four times uper the acoustic-wave maximum.

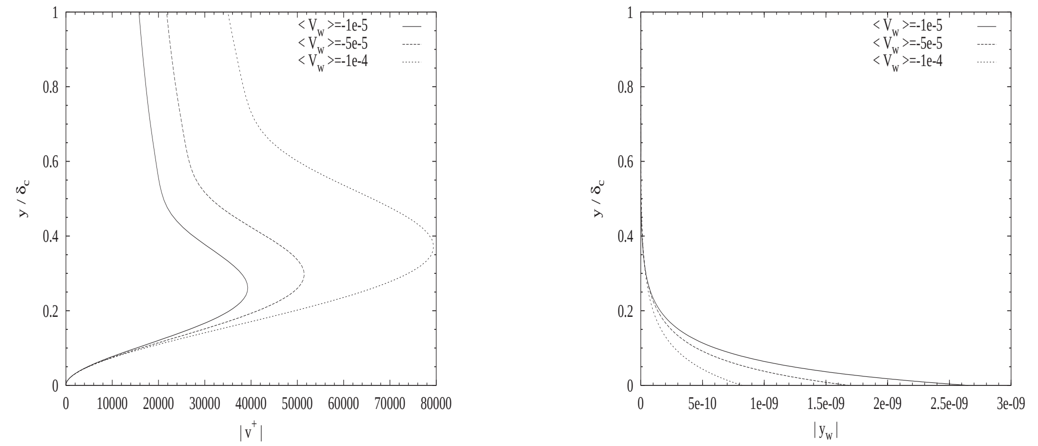

Figure 2: (Left) Adjoint eigenfunction at neutral point. (Right) Forcing term.

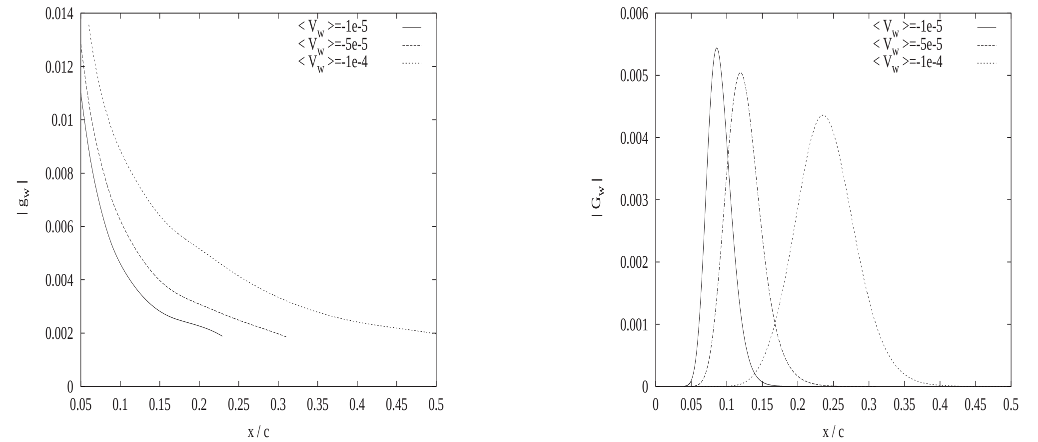

Figure 3: (Left) Receptivity coefficient $\mathrm{g_w}$, (Right) Receptivity function $\mathrm{G_w}$. Neutral points are: $x_n = 0.0899$, $x_n = 0.1244$ and $x_n = 2428$ for the respective $< V_w >$ values.

Figure 2 shows the modulus of left eigenfunction corresponding to the solution of the adjoint OS operator and the unsteady forcing created by the interaction of the solutions shown in Figure 1. It is possible to observe that the maximum of the left eigenfunction is also placed very near the wall while for the forcing term the maximum is reached at wall quickly vanishing as it leaves the wall.

Finally, the graphics in Figure 3 show the evolution of the receptivity coefficient $\mathrm{g_w}$ (left side) and the receptivity function $\mathrm{G_w}$ (right side) as a function of the streamwise direction x when it is normailzed by the chord length c.

5.2 Vorticity-wave wall-suction receptivity

The case where the possible external disturbance is a vorticity array convected by the free stream was studied for the NACA-0012 boundary-layer profile under the same mean wall-suction conditions and Reynolds number of the previous acoustic problem. The resonant condition in this case is,

$$\alpha_{\mathrm{TS}} = \alpha_\delta + \alpha_\epsilon \qquad \text{and} \qquad \omega_{\mathrm{TS}} = \omega_\epsilon. \tag{47}$$

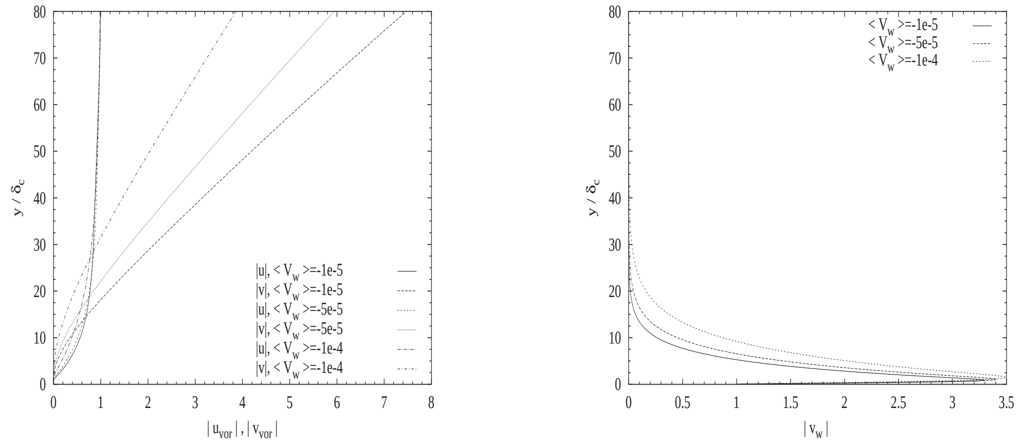

Figure 4: (Left) Modulus of the vorticity components u and v at neutral point for $\delta_c\alpha_\epsilon = 0.1188$, $\delta_c\alpha_\epsilon = 0.0933$ and $\delta_c\alpha_\epsilon = 0.0636$. (Right) Modulus of the wall-suction perturbation v at neutral point for $\delta_c\alpha_\delta = 0.2832$, $\delta_c\alpha_\delta = 0.2318$ and $\delta_c\alpha_\delta = 0.1688$.

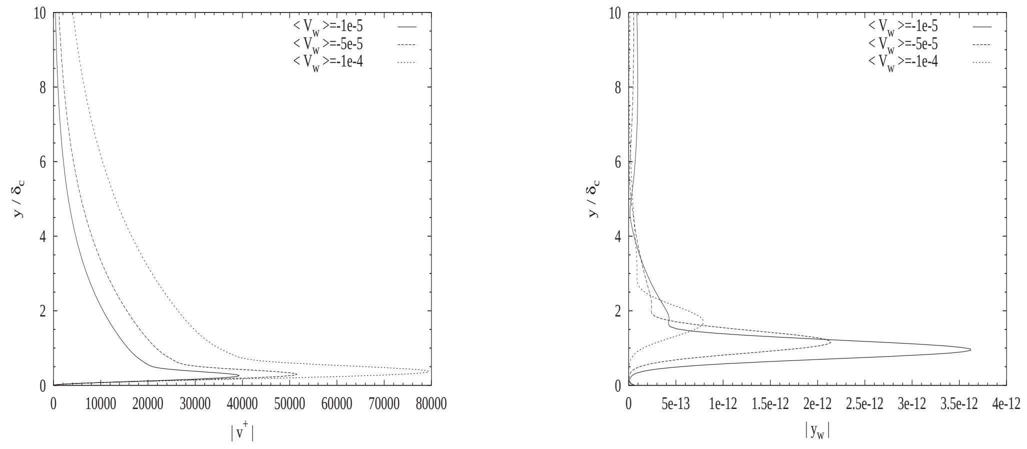

Figure 5: (Left) Modulus of the adjoint eigenvector v^+ at neutral point. (Right) Absolute value of the forcing vector y_w at neutral point.

In Figure 4, the modulus of components u_ϵ and v_ϵ of the vortical disturbance as well as the modulus of the wall suction disturbance are shown. It can be recognized from Figure 4 that the vorticity disturbance propagates far away from the boundary layer. A considerable numerical domain was needed in order to achieve the asymptotic condition imposed for the external boundary condition. The v component propagates faraway from the wall and the linear term of the asymptotic condition predominates. Moreover, the wall-suction disturbance goes out of the boundary layer with considerable values before reaching the homogeneous boundary condition as $y \to \infty$.

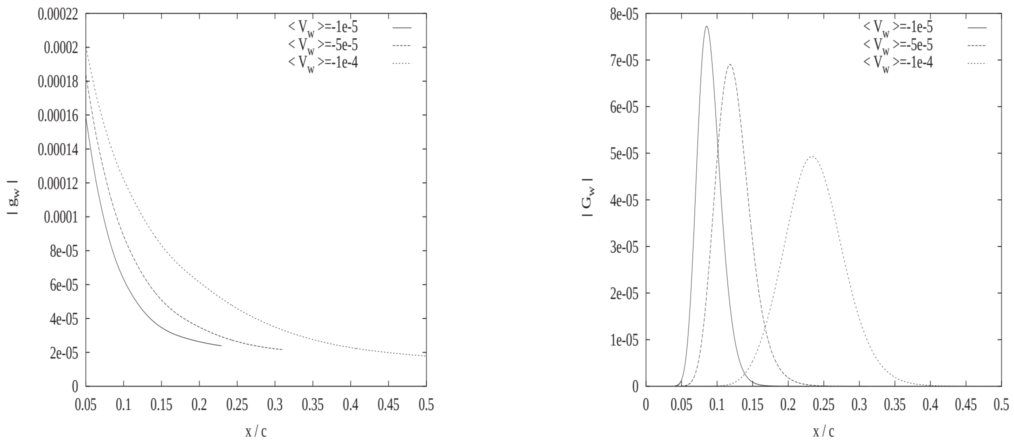

Figure 6: (Left) Receptivity coefficient g and (Right) receptivity function G, for the vorticity case.

The Figure 5 illustrates on the right, the forcing vector for the three different suction conditions. It can be noted that the maximum value of the forcing vector is no more at wall unlike the acoustic case. The modulus of the adjoint eigenfunction is shown on the left side of the figure. On the other hand, Figure 6 shows the receptivity parameters for the vorticity case.

The receptivity coefficient and the receptivity function give an estimation of the sensitivity of the boundary layer to the external perturbations when interacting with the wall-suction disturbance.

Looking at both the cases, the acoustic an the vortical case, greater values of $|\mathbf{G_w}|$ are obtained when an acoustic wave enters the boundary layer. The boundary layer developed around the airfoil is more receptive to acoustic disturbances than to a vortical field. Furthermore, for both the cases, as the mean wall suction increases, the maximum value of the receptivity-function curves moves downstream in the wing surface, but at the same time the base of the bell-shaped curve enlarges increasing in this way the receptive zone. Moreover, it can be noted that, as the mean wall suction increases the absolute values of the receptivity function decrease representing a more stable condition.

6 CONCLUSIONS

The natural receptivity analysis of the boundary-layer flow developed upon a particular geometry, namely a NACA-0012 airfoil was accomplished using a theoretical approach. The slow variation in the streamwise direction of the boundary layer permitted the application of the multiple-scale theory, which by means of the solvability condition allowed to solve the non-homogeneous differential system. The receptivity theory permitted to consider in the analysis different external disturbances like acoustic or vortical waves. It was demonstrated that their interaction with a pertubation caused by wall suction is capable of activating a wave modulation mechanism in order to excite a modal unstable wave whose amplitude spatially grows while convected down stream along the airfoil surface. After a numerical implementation of the model, the curves for the receptivity coefficient as well as for the receptivity function were obtained under two possible environmental conditions. The analysis showed that the boundary-layer flow over the NACA-0012 profile is more receptive to acoustic perturbations than to vortical ones. For both the cases, lower values of the receptivity function $|G_w|$ were found as mean wall-suction velocity was increased which indicates that the flow is less receptive when wall-suction is applied but unstable waves are not canceled. On the other hand, the values of the modulus of the receptivity coefficient, $|g_w|$, increased as the mean wall-suction increased. Finally, the amplitude of the excited Tollmien-Schlichting wave could directly be computed and consequently the transition position could be determined if a characteristic wall-suction velocity profile is adopted.

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