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AVOIDING FORMULATIONS WITH DELAYS TO SIMULATE CUTTING IN DRILLING OPERATIONS

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Abstract. The aim of this paper is to discuss some modelling strategies to account for the cutting in drilling operations. Two modelling techniques to deal with a uniformly distributed (symmetric) blade configuration are compared, one employing a set of delays and another based on an advection partial differential equation. A geometrical interpretation for each of them is provided through a schematic illustration, and by means of a simplified application case. The case is also used to highlight some of the advantages of the advection approach. Finally, the advection formulation is extended to tackle the problem considering arbitrarily distributed blades.

1 INTRODUCTION

The simulation of the cutting process in drill-string dynamics involves solving a free boundary problem: the soil elevation profile changes as the driller operates, producing a variation in the forces and torques generated by the interaction between the drilling structure and the soil. Richard et al. (2007) presents a bit-rock interaction model where the forces depend, among other variables, on an instantaneous depth-of-cut, which is a measure of the amount of soil that is being removed. One way to calculate this magnitude requires the evaluation of the position of the bit (both angular and axial) in previous time-steps, which leads to a system of delay-differential equations, as in Richard et al. (2007); Germay et al. (2009); Yan and Wiercigroch (2019). Another possibility is to consider the ideas presented in Wahi and Chatterjee (2008); Zhang and Detournay (2020), where instead of using delays, an additional advection equation to account for the soil dynamics is solved.

The objective of this work is to discuss some of the differences and advantages between the two approaches, by exploring the simulations obtained with a simplified application example. A geometrical interpretation of the advection equation and the boundary conditions employed is given. Finally, some hints on how to extend the formulation to treat the case of an arbitrary distribution of blades are provided.

1.1 The delay approach for a uniform and symmetric distribution of blades

The problem of solving drill-string dynamics with cutting requires that the equations of motion for the driller be solved together with those describing the evolution of the soil profile. At each time-step of the ODE solver (at time $\hat{t}$), one can obtain predictions for the axial position and the orientation of the driller, U and Φ. The delay approach employs these predictions to calculate an instantaneous depth-of-cut d. The cutting problem involves regenerative cutting, that is, the cutting tool passes many times over the already cut surface (leading to surface regeneration). In the delay approach, at every time-step of the solving process, the following procedure is used:

- First, at time $\hat{t}$, the equation $\Phi(t_i) - \Lambda_i = 0$ in the variable t_i is solved via interpolation at every time-step. In this equation, $i \in [1..N]$ where N is the number of blades in the bit, and Λ_i represents the angle distance between successive blades. If a bit with one blade is considered, $\Lambda_i = i2\pi$.
- Second, $d_i = U(\hat{t}) - U(t_i)$ is evaluated at each delayed time t_i, as in Fig. 2.
- Third, considering that cutting only occurs if the bit rotates in the positive direction, the sought depth-of-cut is found by calculating $d = \max\{\min\{d_i\}, 0\}$, if $\dfrac{\partial \Phi}{\partial t} > 0$, otherwise $d = 0$.

1.2 The advection approach for a uniform and symmetric distribution of blades

The free boundary problem of determining the evolution of the soil profile as the cutting operation takes place is tackled by solving the following advection equation along with the equations of motion of the bit:

$$\frac{\partial L_s}{\partial t} + \omega \frac{\partial L_s}{\partial \eta} = 0, \quad \text{with } \eta \in \Big[0, 2\pi\Big] \tag{1}$$

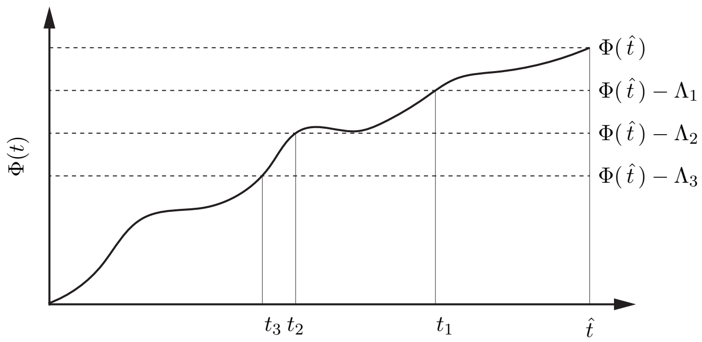

Figure 1: Illustrative graph. Angular position of the cutter $\Phi(t)$ and its position at previous passes $\Phi(t_i) = \Phi(\hat{t}) - \Lambda_i$, used to calculate the time delays.

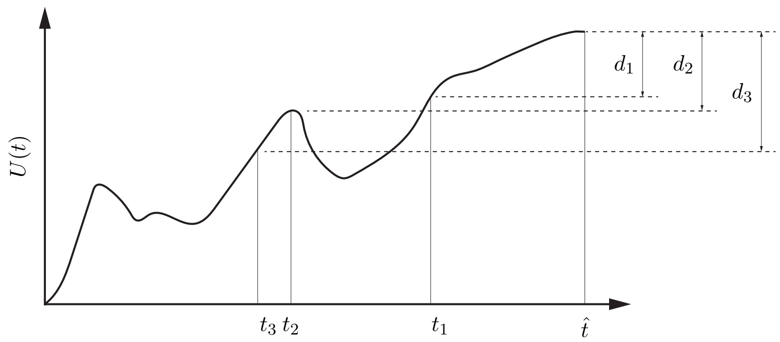

Figure 2: Axial position of the cutter $U(t)$. Example calculation of the depth-of-cut candidates d_i.

where $L_s(\eta, t)$ is the position of the soil with respect to some reference (zero). The problem is completed with some initial conditions (the initial soil profile) and the boundary condition below.

$$\text{If } \omega \geq 0, \quad L_s(\eta = 0, t) = \begin{cases} U(t), \ L_s(\eta = 2\pi, t) < U(t) \\ \\ L_s(\eta = 2\pi, t), \ \text{if } L_s(\eta = 2\pi, t) \geq U(t) \end{cases} \tag{2}$$

$$\text{and if } \omega < 0, \quad L_s(\eta = 2\pi, t) = L_s(\eta = 0, t), \tag{3}$$

considering that cutting occurs if the bit is rotating in the positive direction. In the previous equations, $U(t)$ is the position of the bit.

Finally, at all times the instantaneous depth-of-cut is given by

$$d = \max\Big\{L_s\Big(\eta = 2\pi, t\Big) - L_s\Big(\eta = 0, t\Big), 0\Big\} \tag{4}$$

1.3 Geometrical interpretation for the advection approach

A geometrical interpretation of the equation (1) and its boundary conditions (2) and (3) is presented in the sketch shown in Fig. 3.

While drilling, the blade rotates following a circular trajectory, removing soil as it advances. An advection equation represents a transport of some magnitude over its domain. In this particular application, it models how the soil elevation profile passing below the cutter evolves from the perspective of an observer that is fixed to the cutter itself. On the one hand, $L_s(\eta = 0, \hat{t})$ represents the elevation of the soil below the cutter after cutting has taken place. On the other hand, given the circular trajectory of the bit, $\eta = 2\pi$ also describes the soil below the cutter, but $L(\eta = 2\pi, \hat{t})$ represents the elevation before cutting has taken place. Therefore, the difference between the two, if positive, is the instantaneous depth-of-cut, as per (10).

The previous is illustrated through a graphical example given in Fig. 4 (a-c), where the behaviour of (1) for an off-bottom case, i.e. when the bit is rotating without contact with the soil, is shown. Meanwhile, Fig. 4 (d-f) depicts a case where cutting is taking place. In both situations, the initial condition is the one illustrated in Fig. 3, at time t_0.

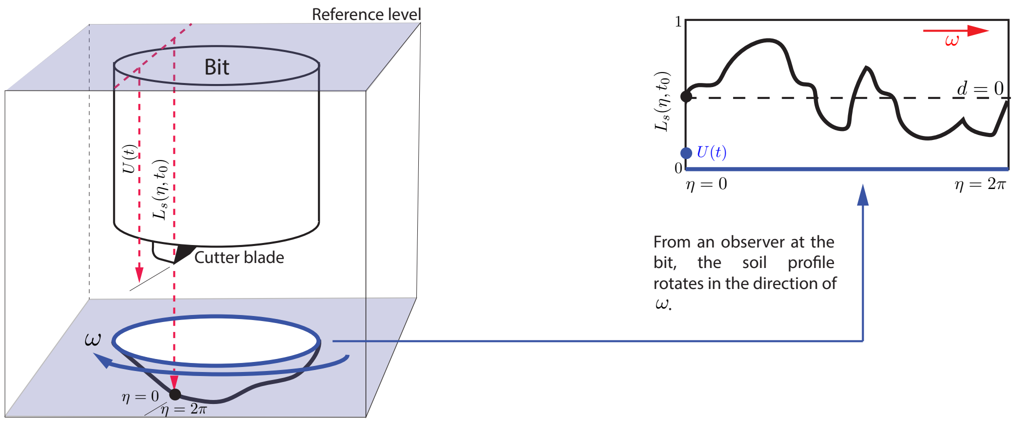

Figure 3: Sketch of a drill-bit with one blade, and initial condition for the illustrative examples.

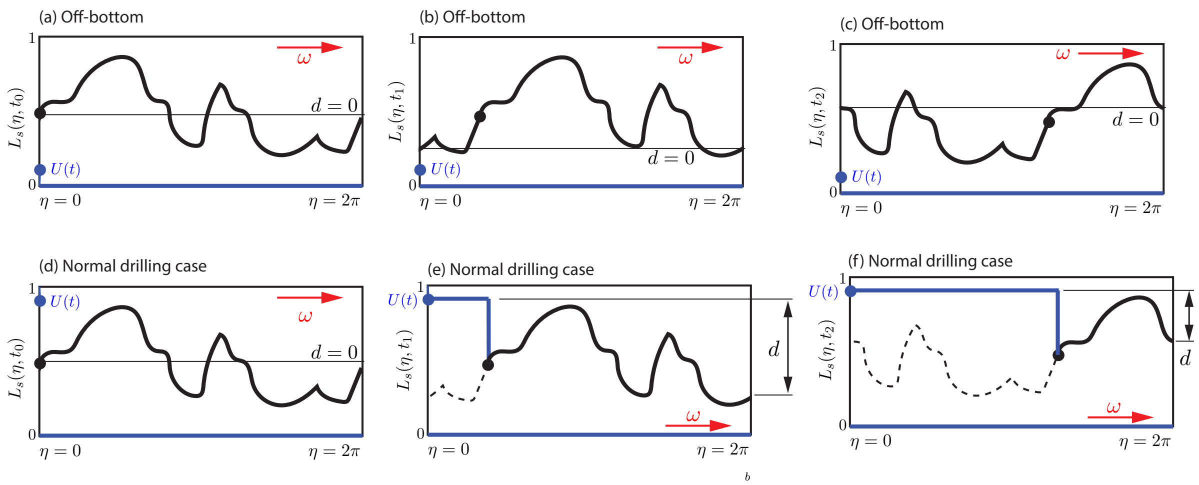

Figure 4: Schematic example of the behaviour of the advection equation to model the cutting process. (a-c) depict an off-bottom case, at times t_0, $t_1 > t_0$, and $t_2 > t_1$,, respectively. (d-e) show an example of a normal drilling condition. The red arrow depicts the direction in which the advection equation translate the soil profile with a speed ω.

Let (a) define the initial soil profile in an off-bottom case, $L_s(\eta, t = t_0)$. Let (b) and (c) be a snapshot of the soil profile at times $t_1 > t_0$ and $t_2 > t_1$, respectively. In the off-bottom case, the solution of the advection equation in conjunction with a periodic boundary condition is just a spatial translation of the initial profile (there is no change in the shape of the elevation profile). In this case, it is observed that the endpoints $\eta = 0$ and $\eta = 2\pi$ coincide, therefore the instantaneous depth-of-cut is exactly $d = 0$.

Now, Fig. 4 (d-f) depict a case where drilling is taking place. Let (d) show the initial state of the soil profile $L_s(\eta, t = t_0)$. Also, let the position of the bit $U(t) = U_0$ (constant), be represented by the blue dot. The fact that $U(t) > L_s(\eta = 0, t)$ means that cutting is taking place. Then, (b) and (c) show the evolution of the soil profile at times $t_1 > t_0$ and $t_2 > t_1$. As depicted in the sketch. Now there is a difference between the values of $L_s(\eta = 0, t)$, the soil elevation after cutting has taken place, and $L_s(\eta = 2\pi, t)$, the elevation before cutting. This difference is the so-called instantaneous depth-of-cut given by (10). In (e) and (f), the recently removed soil depth is depicted by a dashed line.

2 EXAMPLE APPLICATION

A simplified problem where the dynamics of the drill-string are known beforehand, thus decoupled, is treated. The focus is set on illustrating the calculation procedure to obtain the depth-of-cut, as well as pointing out the advantages or disadvantages of each approach.

Let the position of the bit given by

$$\Phi(t) = 15t + 5\sin(t) + \sin(2.21t) + 2\sin(\sqrt{2}t) + 1\sin(\sqrt{5}t) \tag{5}$$

and

$$U(t) = (3t+2\sin(10t)+(4\sin(7t)+2\sin(20.21t)+3\sin(\sqrt{20}t)+0.6\sin(\sqrt{50}t))\cdot 0.0001 \tag{6}$$

In the delay approach, the effect of each pass of the cutter is shown in Fig 5. Four cases, (a) to (d) are considered, representing each an additional pass of the blade. In a) the effect of the first pass is shown. The area of soil removed is depicted in grey. In b) the effect of the second pass of the blade is shown, the removed soil is depicted in darker grey. Then, c) and d) represent the third and forth pass of the blade. It is observed that both cover an area that has been previously removed, therefore they do not perform any cutting.

The solution following the delay approach depends on the amount of regenerative cuttings (N) that are considered, that is, how many previous passes of the blade are used in the calculation process. If too few were selected, then there is a risk of calculating an incorrect value of d, given that the effect of previous cuts would not be captured by the calculation algorithm. Fig. 5 illustrates the evolution of the elevation profile with an increasing number of passings of the blade. Thus, if $N < 4$, in the 4th iteration of the algorithm an error would be introduced in the solution as the correct depth-of-cut would require the information from the 1st and 2nd passes, which would not be available.

In Fig. 6, the depth-of-cut has been obtained using the delay approach with $N = 15$ (max). N_i represent the actual number of previous cutting steps that are required to correctly calculate the depth-of-cut. The graph shows that $max(N_i) = 9 < 15$, which assures that the solution has not introduced any error due to N being too small.

In Fig. 7 and Fig. 8 the calculated depth-of-cut using $N = 15$ and $N = 3$ is shown. The results are compared against the advection approach considering 200 linear elements in the FEM discretisation.

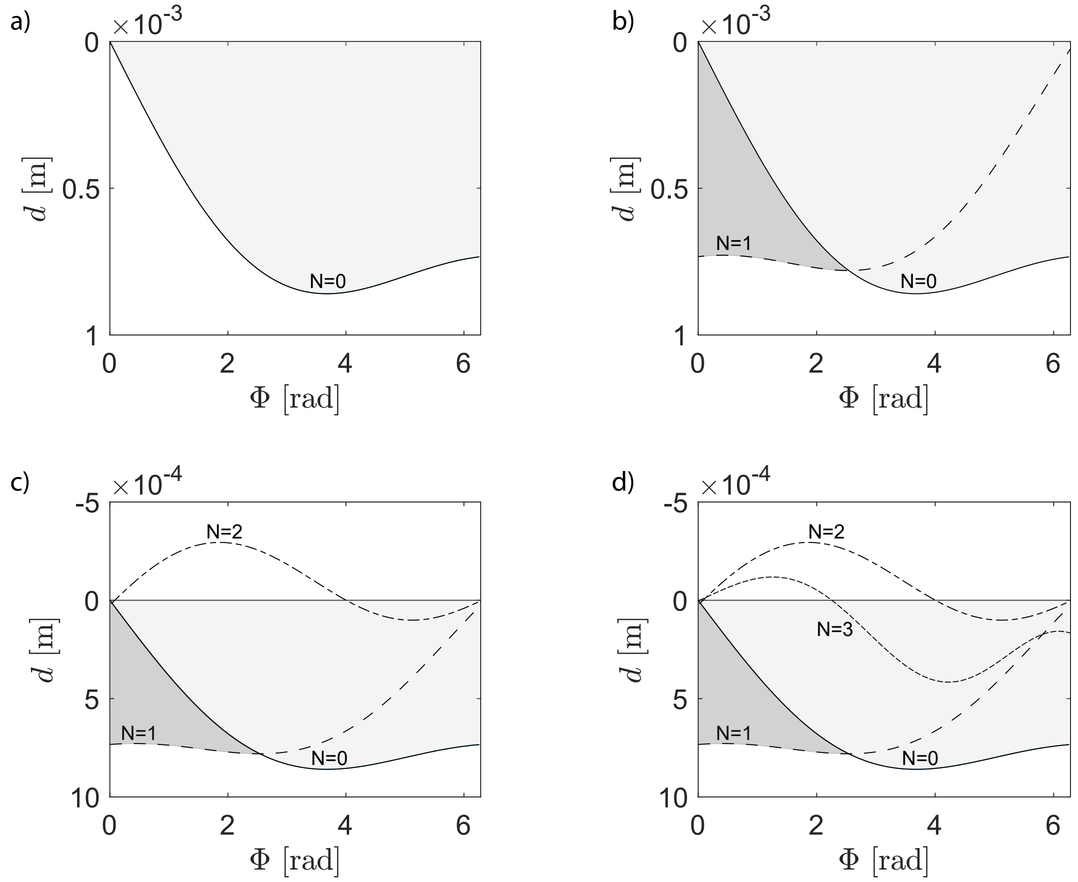

Figure 5: $U(\Phi)$ for: a) the 1st pass of the blade. The removed soil height is is shown in light grey; b) 2nd pass, the removed soil height is shown in dark grey; c) 3rd pass; and d) 4th pass. In c) and d) no soil is removed.

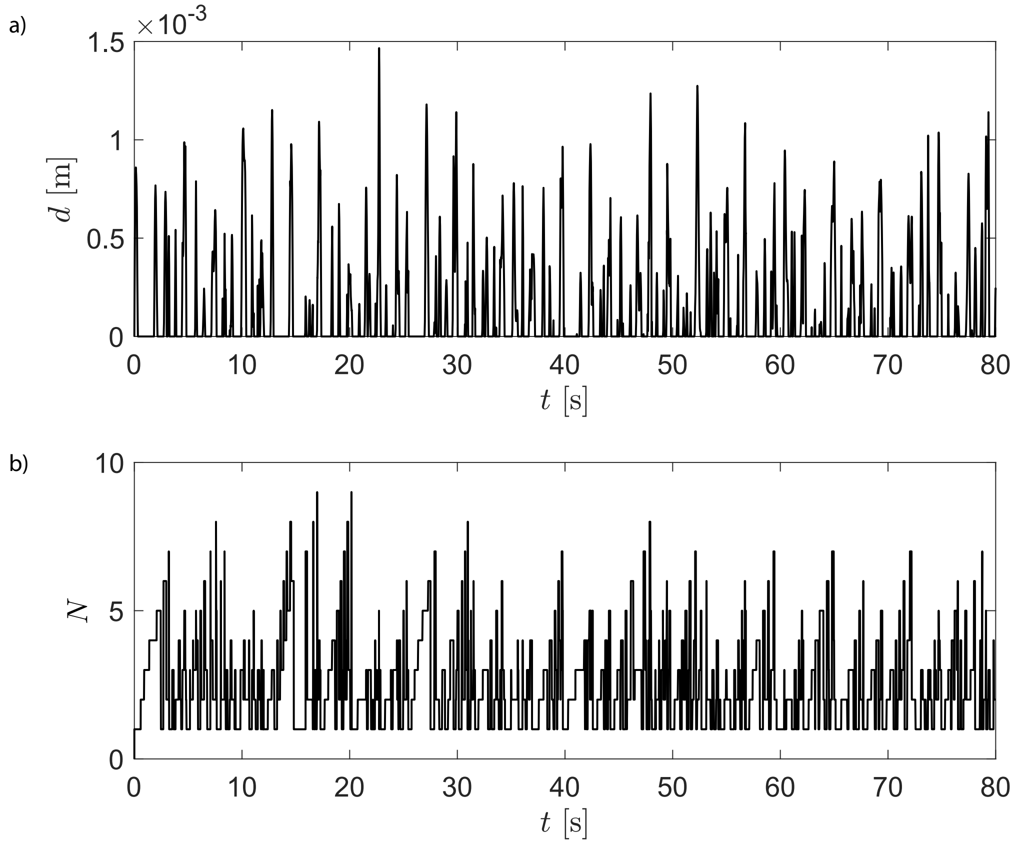

Figure 6: a) Depth-of-cut $d(t)$ as a function of time, considering $N = 15$. b) minimum number of previous passes required for the calculation of d.

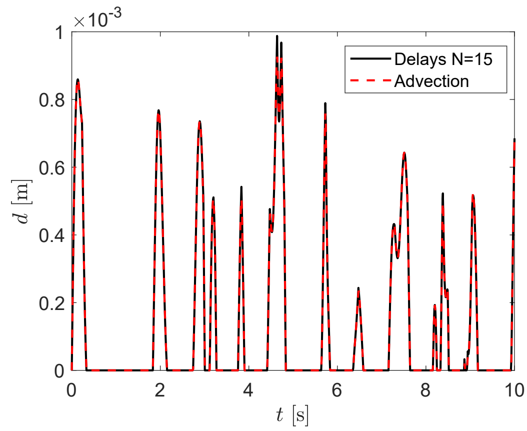

Figure 7: Depth-of-cut as a function of time $d(t)$, considering $N = 15$.

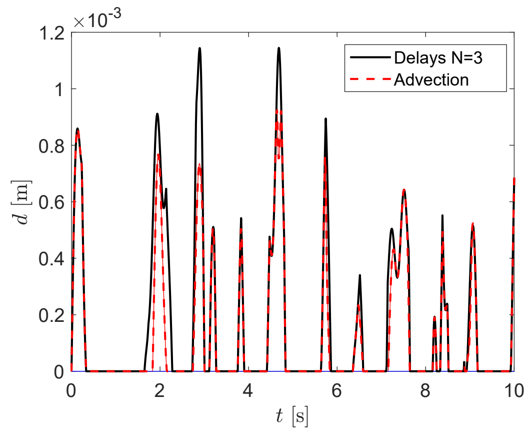

Figure 8: Depth-of-cut as a function of time $d(t)$, considering $N = 3$.

3 GENERALISATION TO MULTIPLE AND ARBITRARILY DISTRIBUTED NUMBER OF BLADES

The problem of cutting considering the delay approach has been extended to account for non-uniformly distributed blades in Yan and Wiercigroch (2019), but to the authors knowledge, it has not been tackled using the advection formulation. In what follows it is shown that this can be achieved in a simple way. The problem requires that an equation for each of the n_b blades is provided. Let Δ_i be the separation between two consecutive blades, then the equations are of the form

$$\frac{\partial L_{s,k}}{\partial t} + \omega \frac{2\pi}{\Delta_k} \frac{\partial L_{s,k}}{\partial \eta_k} = 0, \quad \text{with } \eta_k \in \Big[0, 2\pi\Big] \tag{7}$$

where $L_{s,k}(\eta_k, t)$ is the position of the soil with respect to some reference (zero). The problem is completed with some initial conditions (the initial soil profile) and the boundary conditions below.

If $\omega \geq 0$:

$$\begin{aligned}
\text{for } k = 1, \quad L_{s,1}(\eta_1 = 0, t) &= \begin{cases} U(t), \; L_{s,n_b}(\eta_{n_b} = 2\pi, t) < U(t) \\ \\ L_{s,n_b}(\eta_{n_b} = 2\pi, t), \; \text{if } L_{s,n_b}(\eta_{n_b} = 2\pi, t) \geq U(t) \end{cases} \\
\text{for } 2 \leq k \leq n_b, \quad L_{s,k}(\eta_k = 0, t) &= \begin{cases} U(t), \; L_{s,k-1}(\eta_{k-1} = 2\pi, t) < U(t) \\ \\ L_{s,k-1}(\eta = 2\pi, t), \; \text{if } L_{s,k-1}(\eta_{k-1} = 2\pi, t) \geq U(t) \end{cases}
\end{aligned} \tag{8}$$

if $\omega < 0$

$$\begin{aligned}
\text{for } 1 \leq k < n_b, \qquad & L_{s,k}(\eta_k = 2\pi, t) = L_{s,k+1}(\eta_{k-1} = 0, t) \\
\text{for } k = n_b, \qquad & L_{s,n_b}(\eta_{n_b} = 2\pi, t) = L_{s,1}(\eta_1 = 0, t)
\end{aligned} \tag{9}$$

In the previous equation, the hypothesis that cutting only occurs while the bit is rotating in the positive direction is enforced. Finally, at all times the instantaneous depth-of-cut at each blade is given by

$$d_k = \max\Big\{ L_{s,k}\Big(\eta_k = 2\pi, t\Big) - L_{s,k}\Big(\eta = 0, t\Big), 0 \Big\} \tag{10}$$

4 FINAL REMARKS

In this paper the delay and the advection approach to account for cutting dynamics in the simulation of drill-strings were compared. It is verified that the advection equation can successfully model the evolution of the soil elevation profile. The advection formulation can account for infinite regenerative effects ($N = \infty$), which is an advantage over the delay approach. The delay approach can be time costly if a large N needs to be used. In addition, the advection equation is easy to adapt for the case of multiple non-equiangular distribution of blades by considering one equation for each blade, and a lesser modification of the boundary conditions (so that the loop satisfies the periodic condition).

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