Mecánica Computacional Vol. XXII
M. B. Rosales, V. H. Cortínez y D. V. Bambill (Editores)
Bahía Blanca, Argentina, Noviembre 2003.

PROJECT METHODOLOGY FOR MULTIVARIABLE OPTIMAL CONTROLLERS AND STATE OBSERVERS APPLIED TO ELECTRICAL DRIVE SYSTEMS.

Fabiano Salvadori[*], Mauricio de Campos[*], José Eduardo Baggio[*] and Eliane S. E. Paim

[*] Departamento de Tecnologia, Universidade Regional do Noroeste do Estado do Rio Grande do Sul
Rua São Francisco, 501, Ijuí – RS, 98700-000, Tel: 55-55-3332-0250
e-mail: fabiano@unijui.tche.br, campos@unijui.tche.br, josebaggio@ieee.org, epaim@unicruz.edu.br

Key words: State Observer, Project Methodology and Optimal Controller

Abstract. *This works presents, the problem of estimate of states of multivariable systems, using complete order state observer optimal discretes. For study, it is considered the two-phase mathematical equivalent model of the three-phase asynchronous machine, their due to your characteristics (sixth-order model, non linear and hardly coupled), making possible to generalize for application in other systems. Due to the computing time necessary for discretization of the model, the procedure is realized off-line, resulting in a discrete model containing algebraic relations. The feedback matrix (gains) it is calculated off-line, using technique of optimal control. The gains are calculated for various frequencies of machine operation, resulting at various gain matrixes operating in a gain scheduling.*

1 INTRODUCTION

This work presents a states estimator applied to multivariable systems. For multivariable systems analysis, it is essential to reduce the complexity of the mathematical expressions and to use computers for the system project and analysis. For the analysis of multivariable systems, the focus of state variables (state spaces) is appropriate.

The theory of states observer designs here studied is applied to asynchronous machines aiming the implementation of high performance drive systems. This study can be use, not only in applications of control systems involving electric machines, but also in biological, biomedical and economical systems.

In high performance systems it is necessary the perfect knowledge of the used variables for feedback. The feedback signals can be divided in three groups: (a) easy measurement (stator voltages and currents); (b) intermediary (speed and position); and, (c) difficult measurement (electromagnetic torque and the magnetic flux).

For the (b) and (c) cases several, states estimator techniques were developed (Salvadori [2]) such as linear and non-linear, robust, adaptive, stochastics and deterministics estimators. These techniques allow, starting from easily measurable variables, to obtain another hard access variables. The flux acquisition is the most critical process in terms of time, and since the flux is the base for the strategies control with feedback it is wise to choose a reference for the observer's model so that the dear states are in the same reference of the actuation model (control), avoiding coordinates transformations.

This work proposes an off-line sampling method of the observer's continuous model. The gain matrixes are calculated, also in gain scheduling form, using optimal control techniques.

2 CONTINUOUS TIME MATHEMATICAL MODEL OF THE ASYNCHRONOUS MACHINE

It is possible to represent the asynchronous machine through a linear time variant (LTV) electric model, where the rotor frequency (w_R) behaves as a variant parameter in the time. The continuous time dynamic electric model, in generic reference (index g), of the asynchronous machine is given by (1), on state space representation,

$$\dot{\boldsymbol{\phi}}^g(t) = A^g(\omega_r)\boldsymbol{\phi}^g(t) + B^g \mathbf{v_s}^g(t) \tag{1}$$

$$\mathbf{i}_s^g(t) = C^g \boldsymbol{\phi}^g(t) \tag{2}$$

where, A^g is the system matrix (4x4 order); B^g is the input matrix (4x2 order); C^g is the output matrix (2x4 order); $\boldsymbol{\phi}^{\mathbf{g}}$ is the states matrix (4x1 order); $\mathbf{v_s}^g$ is the input matrix (2x1 order); $\mathbf{i}_s^g$ is the output matrix (2x1 order) and "·" represents the derive operator.

Considering the states matrix as being composed of the stator and rotor flux, *d,q* components, model flux/flux, then,

$$\phi^{g}(t) = \begin{bmatrix} \phi_{sd}^{g} & \phi_{sq}^{g} & \phi_{rd}^{g} & \phi_{rq}^{g} \end{bmatrix}^{T} \tag{3}$$

and, the input and output matrix, defined, respectively, as,

$$\mathbf{v}_{s}^{g}(t) = \begin{bmatrix} v_{sd}^{g} & v_{sq}^{g} \end{bmatrix}^{T} \tag{4}$$

$$\mathbf{i}_{s}^{g}(t) = \begin{bmatrix} i_{sd}^{g} & i_{sq}^{g} \end{bmatrix}^{T} \tag{5}$$

The representation of the machine equations in the form of states space (1) and (2) can be defined like,

$$A^{g}(\omega_{r}) = \begin{bmatrix} -\dfrac{r_s}{\sigma l_s} & \omega_g & \dfrac{r_s l_m}{\sigma l_s l_r} & 0 \\ -\omega_g & -\dfrac{r_s}{\sigma l_s} & 0 & \dfrac{r_s l_m}{\sigma l_s l_r} \\ \dfrac{r_r l_m}{\sigma l_s l_r} & 0 & -\dfrac{r_r}{\sigma l_r} & \omega_{gr} \\ 0 & \dfrac{r_r l_m}{\sigma l_s l_r} & -\omega_{gr} & -\dfrac{r_r}{\sigma l_r} \end{bmatrix} \tag{6}$$

$$B^{g} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{7}$$

$$C^{g} = \begin{bmatrix} \dfrac{1}{\sigma l_s} & 0 & -\dfrac{l_m}{\sigma l_s l_r} & 0 \\ 0 & \dfrac{1}{\sigma l_s} & 0 & -\dfrac{l_m}{\sigma l_s l_r} \end{bmatrix} \tag{8}$$

where, r_s and r_r are, respectively, the stator and rotor resistances; l_s, l_r and l_m are the stator, rotor and mutual inductances; σ is the coefficient of magnetic dispersion; ω_g and ω_r

are the rotor angular frequencies and the generic frequencies; and, ω_{gr} is the slipping (difference between the generic angular and rotor frequencies).

3 COMPLETE ORDER STATES OBSERVER

Fundamentally, closed loop states observer project in consists of a combination of the real time simulation with correction by an error signal. The dynamic behavior of the error vector is determined by the autovalues of the matrix (6). If the matrix is stable, the error vector will converge to zero with the desired speed. The problem to project a complete order observer is the determination of the gain matrix of the observer, such that the error dynamics are asymptotically stable with adequate answer speed. Considering the model equations (1), (2), (3) and (4} the observer's model in closed loop, continuous in the time, it can be presented in the form,

$$\dot{\hat{\boldsymbol{\phi}}}^g(t) = A^g \hat{\boldsymbol{\phi}}^g(t) + B^g \mathbf{v}_s^g(t) + K_e\left[\mathbf{i}_s^g(t) - \hat{\mathbf{i}}_s^g(t)\right] \tag{9}$$

$$\hat{\mathbf{i}}_s^g(t) = C^g \hat{\boldsymbol{\phi}}^g(t) \tag{10}$$

where, “^” represents estimated variables; and, K_e is the gain matrix. Substituting (10) in (9), results in the expression,

$$\dot{\hat{\boldsymbol{\phi}}}^g(t) = A^g \hat{\boldsymbol{\phi}}^g(t) + B^g \mathbf{v}_s^g(t) + K_e\left[\mathbf{i}_s^g(t) - C^g \hat{\boldsymbol{\phi}}^g(t)\right] \tag{11}$$

subtracting (11) of (1) and substituting (2) results,

$$[\dot{\boldsymbol{\phi}}^g(t) - \dot{\hat{\boldsymbol{\phi}}}(t)] = (A^g - K_e C^g)\boldsymbol{\phi}^g(t) - (A^g - K_e C^g)\hat{\boldsymbol{\phi}}^g(t) \tag{12}$$

this way it is possible to define the behavior of the observer's error

$$\Delta\boldsymbol{\phi} = \boldsymbol{\phi}^g(t) - \hat{\boldsymbol{\phi}}^g(t) \tag{13}$$

$$\dot{\Delta}\boldsymbol{\phi} = (A^g - K_e C^g)\Delta\boldsymbol{\phi} \tag{14}$$

Choosing the appropriately K_e matrix so that the autovalues have, in the continuous plan, big real negative part then, the problem with the value of the initial error does not exist ($\Delta\phi$ in $t(0)$) and that will tend quickly to zero.

4 SAMPLING OF THE MODEL OF THE COMPLETE ORDER STATES OBSERVER

Considering the continuous model of states defined by (1), it is obtained the discrete model in the form (15), where, I is the identity matrix whit the same order of the sampled system

$$\psi = \int_0^{t_a} (e^{A^{g(t)}})dt = It_a + \frac{A^g (t_a)^2}{2!} + \frac{(A^g)^2 (t_a)^3}{3!} + \cdots + \frac{(A^g)^i (t_a)^{i+1}}{(i+1)!} \tag{15}$$

Then, the discrete matrixes of the system are obtained in agreement with,

$$(a)\ F^g = I + A^g \psi \qquad (b)\ \psi B^g \qquad (c)\ H^g = C^g \tag{16}$$

And the discrete system of the machine, generic reference, with the operator "q" will be,

$$q\hat{\phi}^g[k + t_a] = F^g \hat{\phi}^g[k] + G^g \mathbf{v}_s^g[k] \tag{17}$$

Due to the reduced available time for signal sampling and control evaluation, it was chosen, to off-line discretize the model resulting in matrix whose elements are constants or dependents, only of the rotor frequency ω_r in agreement with the adopted referential.

The observer discrete model in closed loop,

$$\hat{\phi}^g[k + t_a] = F^g \hat{\phi}^g[k] + G^g \mathbf{v}_s^g[k] + K_e(\mathbf{i}_s^g[k] - H^g \hat{\phi}^g[k]) \tag{18}$$

The matrixes F^g and G^g are, square with order respectively 4x4 and 4x2. Defining the generic axis on the stationary referential (index g = s, consequently, $\omega_g = 0$, the elements F_{14}^g, F_{23}^g, F_{32}^g, F_{41}^g and F_{43}^g will be function of the rotor frequency (ω_r); the elements F_{12}^g and F_{21}^g will be zero; and the other elements will be constant. On the other hand the elements G_{11}^g, G_{22}^g, G_{31}^g, G_{41}^g will be constant different from zero and the others will be zero.

5 DETERMINATION OF THE GAIN MATRIX OF THE OBSERVER USING OPTIMAL CONTROL LAW

The determination of the gain matrix of the observer was accomplished using the laws of optimal control with gain scheduling. In problems of optimal control an performance index (**J**) is defined and the designer's objective is to design a controller / observer which will optimize this index. The index {**J**} is defined as,

$$J = \int_{t_0}^{t_1} \left\{ \left[\phi^g(t)\right]^T Q_o^g \phi^g(t) + (\mathbf{v}_s^g)^T R_o \mathbf{v}_s^g \right\} dt \qquad (19)$$

Being defined the optimal control Law, as Athams *et. Al* [1],

$$\mathbf{v}_s^g(t) = -(R_o^g)^{-1}(H^g)^T P_R \phi^g(t) = -K_e^g \phi^g(t) \qquad (20)$$

where, P_R is the semi-defined positive symmetrical matrix and solution of the *Riccati's* equation.

Considering the criterion of (19) where Q_0^g and R_0^g are diagonal matrixes, and, respectively, 4x4 order semi-defined positive and, 2x2 order defined positive, for whole $t_0 \leq t \leq t_1$ for which the criterion is minimum, the optimal linear deterministic observer's problem is defined. The selection of Q_0^g and R_0^g is based on the commitment and on the principle that variables strongly meditated tend to be small in closed loop. If the elements of the matrix Q_0^g are selected much larger than the elements of R_0^g, then, the components corresponding of ϕ^g they will be maintained small. In another way, if R_0^g is selected big, $\mathbf{v}_\mathbf{s}^g$ will be maintained small so that the smallest control effort is used.

The gain matrix of feedback can be projected from way to result in variants or invariants gains in the time [2]. In the first case, the dynamic equation is solved of *Riccati* (21); and, for the second case, the equation in permanent regime of *Riccati* (22}. It is necessary to solve the *Riccati´s* equation using a numeric integration method, this solution is simplified because $P_R(t_0, t_1)$ is symmetrical.

$$P_R F^g + (F^g)^T P_R - P_R (H^g)^T (R_o^g)^{-1} H^g P_R + Q_o^g = \dot{P}_R \qquad (21)$$

$$P_R F^g + (F^g)^T P_R - P_R (H^g)^T (R_o^g)^{-1} H^g P_R + Q_o^g = 0 \qquad (22)$$

Then it chooses to work with constant gains. The calculation of the matrix P_R was accomplished using MATLAB® (function DARE), for eight different rotoric frequencies, being varied of -400 for +400rad/s, in intervals of 40rad/s. With that it works with gain scheduling for the matrix K_e, varying the gains in agreement with the variation of the rotor frequency.

6 SIMULATION RESULTS

The numeric simulations were evaluated using programs specifically developed in C language. For the resolution of the mathematical model, the *Runge-Kutta* method was used (4 order with step of 0,5μs integration). The sampling period t_s was defined in 100,0μs. The time of simulation was of 1,0s permitting the machine obtains your maximum nominal speed and it reverted the rotation direction. With that the observer's behavior is verified in every group of operation frequencies. The components of the consideration matrix are project criteria. The used specifications were $Q_0^g(1,1) = Q_0^g(2,2) = 1.0e^{-3}$; $Q_0^g(3,3) = Q_0^g(4,4) = 1.0e^{-3}$ e $R_0^g(1,1) = R_0^g(2,2) = 1.0e^{-4}$.

On fig 1(a) is presented the speed, in (b) is presented the electromechanical torque. The stator flux (simulated and observed) is presented in (c) and (d) is presented the rotoric flux (simulated and observed). It can be seen that the estimated values follows with high precision the calculated values using the *Runge-Kutta* method. The difference of phase and amplitude present on figures simulation occurs due to the late inserted by the observer. It must be noticed that this late is not significant for the proposed objectives.

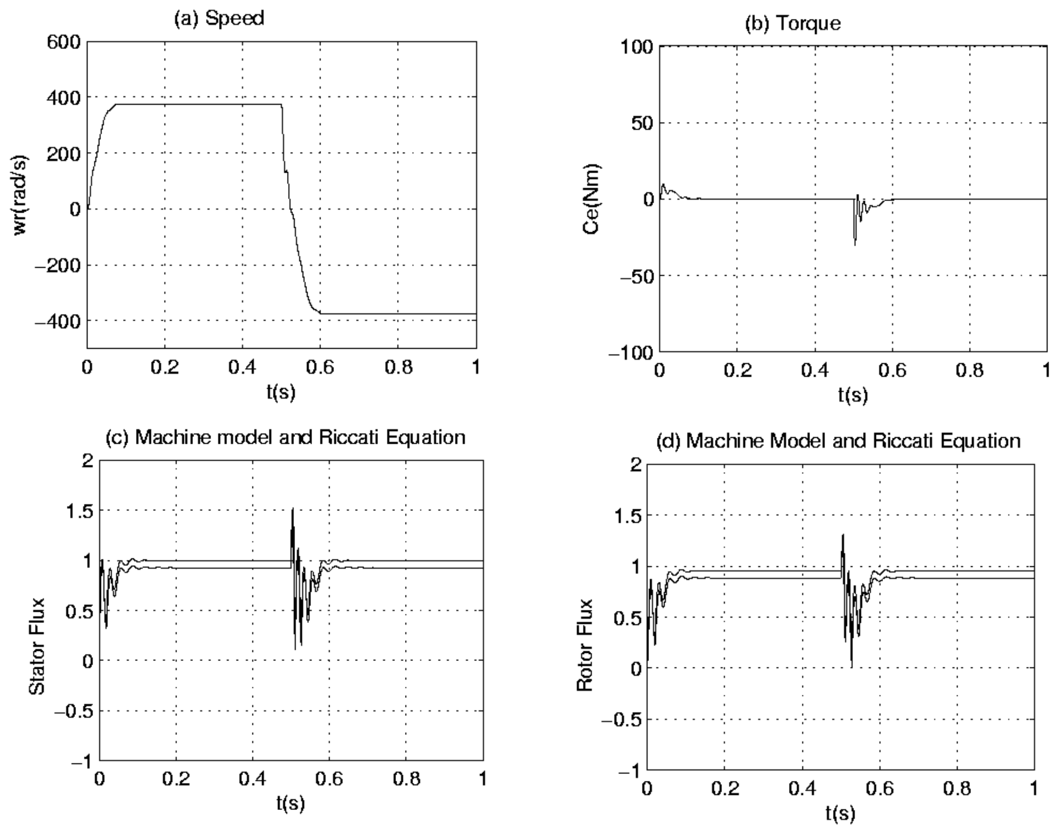

figure 1 – (a) Speed, (b) electromechanical torque, (c) stator flux and (d) rotor flux

7 CONCLUSION

The simulation results allows to conclude that: (a) The strategy of dicretize the observer model off-line is adequate to systems where the sample period is short; (b) The use of optimal control law strategy to calculate the observer gains has demonstrated good results; (c) The use of scheduled gains avoid that the variation of the closed loop system autovalues affect its performance. Therefore the obtained results present good perspectives to the use of the proposed discrete optimum observer in multivariable systems.

8 REFERENCES

[1] M. Athans e P. L. Falb, *Optimal Control - An Introduction to the Theory and Its Applications*, McGraw-Hill Book Company, New York, USA, 1966.

[2] Fabiano Salvadori, *Sistema de Acionamento Digital para Máquina Assíncrona Utilizando Estratégias de Controle Mono e Multivariáveis sem Sensor de Velocidade*, Tese Doutorado, DEE-CCT-UFPB, Campina Grande, PB, Brasil, 2000.