

# Ignition and combustion of diluted hydrogen mixtures in a flow past an array of catalytic wires

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**Abstract.** *In this paper we study the heat transfer and heterogeneous combustion problem of the flow of a reacting mixture (in this case we address the case of hydrogen/oxygen mixtures) flowing through an array of catalytic wires. The fluid velocity, vorticity, temperature and species concentration of the reactants have been obtained for different flow Reynolds numbers and for a Prandtl number of 0.72. The flow Reynolds number based on the wire separation is assumed to be small but not larger than 40. We have obtained the ignition temperature of the surface reactions as a function of the Peclet number for the case of a wire radius of  $a = 0.02$  cm. and a wire separation of  $l = 0.1$  cm. The required wire temperature increases with the Peclet number, beginning with a critical Peclet number of 0.056 corresponding to the auto-ignition condition (ignition at room temperature).*

# 1 Introduction

The study of the flow dynamics of a gas through an array of wires represents an important contribution to the development in the design of thermal devices due to the constant need to maximize their efficiency and therefore to reduce the operation costs. The flow through an array of catalytic wires has a number of important industrial applications, including the modeling of fibrous fibers, flow through porous media, design of catalytic converters and heat exchangers. The understanding of these processes helps to reduce the contact area between the fluid and the wire array, helps to minimize the pressure drop and therefore increase the efficiency of the system and coupled with the catalytic combustion (hydrogen-air), helps to reduce considerably the emission of pollutants to the atmosphere, specially the nitrogen oxides.

The problem of flow across of a simple wire at uniform temperature and suspended in an infinite medium has been studied experimentally and analytically in the last decades. Cole and Roshko<sup>1</sup> employed the Oseen approximation coupled to the energy equation to obtain the heat transfer in a circular cylinder. The temperature field was obtained using the first terms of the Bessel function. Illingworth<sup>2</sup> was able to obtain the next terms of the expansion using a similar technique. Hieber and Gebhart<sup>3</sup> solved the problem using asymptotic techniques and matching theory by modifying the Prandtl number and assuming that the Reynolds number tends to zero. Dennis et al.<sup>4</sup> evaluated different Prandtl numbers solving the complete form of the Navier-Stokes and energy equations. The precision of their calculations matches with the results obtained experimentally by Collins and Williams<sup>5</sup>. Recently, Kurdymov and Fernández<sup>6</sup> found the heat transfer in a tube immersed in an infinite medium at low Reynolds numbers, of order unity.

All of the previous studies were done assuming that the wire is in an infinite medium. The problem of an array of wires has not been studied as often as with the single wire. Gordon<sup>7</sup>, solves numerically the fluid field of the stream function and vorticity transport equations, using a Cartesian coordinate system and a finite difference scheme. Wung and Cheng<sup>8-9</sup> obtained the periodic solution of the fluid flow and fluid temperature using finite analytic methods. Wang<sup>10</sup> found a series of solutions for an array of wires using the same method employed by Wung and Cheng, obtaining the evolution of the Nusselt number and the viscous stress over the surface of the wires. Along with the development in the dynamics of the fluid past an array of wires, the catalytic combustion process has been studied in different configurations, such as plates, tubes, and different geometries.

The catalytic ignition of fuel/air mixtures has received considerable attention in the literature in the past few years, due to its role in the start-up in the automobile catalytic converters and in the catalytic and catalytically assisted combustors<sup>11-15</sup>. Experimental and theoretical studies of catalytic ignition have been published increasing our knowledge about several aspects related with this type of critical nonlinear process. Vesper and Schmidt<sup>14</sup> published experimental results of the catalytic ignition of different hydrocarbons on platinum using a stagnation-point flow configuration. They showed that for

methane, the critical ignition temperature decreases as the mixture becomes richer. From the theoretical point of view, the ignition process has been studied by either numerical simulations using elementary chemistry<sup>13,16–19</sup> or by large activation energy asymptotic analyses using an one-step overall reaction mechanism<sup>20–24</sup>. The basic principles of heterogeneous catalysis has been described elsewhere<sup>16,25</sup>. Williams et al.<sup>16</sup> presented a model for the catalytic combustion of hydrogen at high temperatures. They presented the rate parameters of a detailed surface chemistry. Warnatz and co-workers<sup>17–18</sup> studied in a series of papers, the catalytic combustion and ignition of hydrogen using detailed kinetic mechanisms for both surface and gas-phase reactions. They compared their results with previously reported data<sup>26</sup>. Deutschmann et al.<sup>13</sup> studied the catalytic ignition of different fuels on different catalyst materials. They indicated that the ignition process is an abrupt transition from kinetically controlled system to one controlled by mass transport and depends mainly on the adsorption - desorption reaction steps. In their numerical simulations they showed that one or the other reactant almost covered the surface prior to ignition. There is a need to obtain reduced kinetic schemes for the catalytic combustion, which help to build a bridge between the full numerical works and the theories developed using an overall one-step reaction for the surface kinetics. In a previous work by Treviño et al.<sup>27</sup> an asymptotic analysis is presented for the catalytic ignition of hydrogen mixtures using a simplified model for the heterogeneous chemistry. The critical conditions for ignition has been deduced and obtained in a closed form the parametric influence on this critical process.

The understanding of the catalytic combustion coupled to the dynamics of a gas past an array of wires is a field that has received very little attention. Vera and Liñán<sup>28</sup> described the diffusion controlled combustion of a reacting mixture flowing past a two-dimensional array of catalytic wires. Assuming first that the gas phase homogeneous reactions are frozen, the analysis provides the dependence of the downstream values of the temperature and the reactant mass fractions on the Peclet number, the Lewis number of the reactants, the temperature of the wires, and the ratio of the wire radius to the wire spacing. The analysis includes the effect of the homogeneous reactions, showing that the rate of fuel consumption in the gas phase grows with the ratio of the adiabatic laminar flame velocity to the upstream flow velocity, the product of the Zeldovich number and the Peclet number, and the nondimensional temperature of the wires. The objective of this work is to extend the results obtained by Vera and Liñán<sup>28</sup> by including the heterogeneous kinetics for the hydrogen and oxygen mixtures and by increasing the Reynolds numbers up to 40.

## 2 Formulation

The physical model under study is shown in Fig. 1. A diluted premixed combustible mixture of air-hydrogen, with mass concentrations denoted by  $Y_{F0}$  and  $Y_{O_20}$ , for the

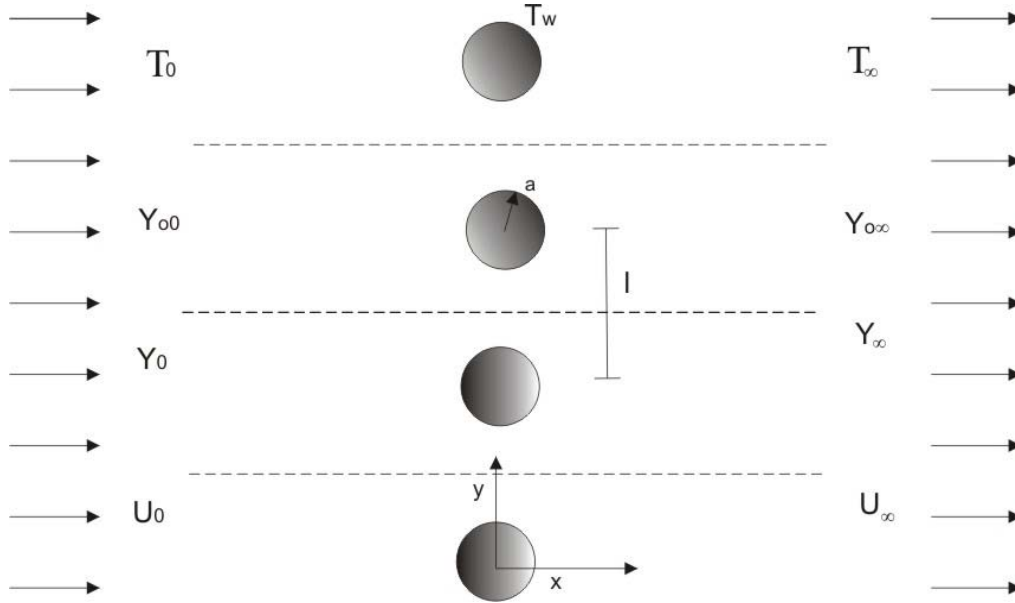


Figure 1: Schematics of the flow problem

hydrogen and oxygen, respectively, flows with a uniform velocity  $U_0$  and temperature  $T_0$ , perpendicular to an infinite array of catalytic (platinum) wires of radius  $a$ , with the space between the center of neighbor wires denoted by  $l$ . The physical properties of the highly diluted mixture such as density  $\rho_g$ , kinematic viscosity  $\nu_g$ , specific heat at constant pressure  $C_{pg}$ , thermal conductivity  $\lambda_g$  as well as the thermal conductivity of the wire  $\lambda_w$ , for simplicity are assumed to be constants. The Reynolds number,  $Re = U_0 l / \nu_g$  based on the wire separation will be assumed to be of order unity or at least lower to 40, in order to achieve a laminar steady flow. The governing equations are made nondimensional using the scales of  $l$  and  $U_0$  for the spatial coordinates and the velocity components, respectively. The pressure difference to the uniform upstream ambient pressure is scaled with  $\rho_g U_0^2$ . The nondimensional temperature is defined by  $\varphi = (T - T_0) / (T_e - T_0)$ , where  $T_e$  corresponds to the adiabatic temperature achieved by consuming completely the fuel, that is  $T_e = T_0 + Y_{F0} Q / C_{pg} W_F$ , where  $Q$  and  $W_F$  are the heat release per unit of mol of fuel consumed and the molecular weight of the fuel, respectively. The mass fractions of the main species have been normalized with the corresponding values at the upstream conditions,  $Y = Y_F / Y_{F0}$  and  $Y_O = Y_{O_2} / Y_{O_20}$ . The resulting governing equations take the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right], \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\text{Re}} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right]. \quad (3)$$

$$u \frac{\partial \varphi}{\partial x} + v \frac{\partial \varphi}{\partial y} = \frac{1}{\text{Pe}} \left[ \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} \right]. \quad (4)$$

$$u \frac{\partial Y}{\partial x} + v \frac{\partial Y}{\partial y} = \frac{1}{\text{Pe} L_F} \left[ \frac{\partial^2 Y}{\partial x^2} + \frac{\partial^2 Y}{\partial y^2} \right]. \quad (5)$$

$$u \frac{\partial Y_o}{\partial x} + v \frac{\partial Y_o}{\partial y} = \frac{1}{\text{Pe} L_O} \left[ \frac{\partial^2 Y_o}{\partial x^2} + \frac{\partial^2 Y_o}{\partial y^2} \right]. \quad (6)$$

In the above equations, the resulting nondimensional parameters are the Reynolds number, the Peclet number,  $\text{Pe} = \text{Re} \nu_g / \alpha_g$ , where  $\alpha_g$  denotes the thermal diffusivity of the combustible gas. The Lewis numbers of both reactants are given by  $L_F = \alpha_g / D_F$  and  $L_O = \alpha_g / D_O$ , where  $D_F$  and  $D_O$  are the binary diffusion coefficients of the fuel and the oxidizer, respectively. Also in these equations the Cartesian coordinates are  $x$  normal to the array and  $y$  parallel to the array, with the origin placed in the symmetry plane and the line joining the centers of the wires (see Fig. 1). The nondimensional velocity components are  $u$  and  $v$  along the  $x$  and  $y$  coordinates, respectively. The nondimensional pressure difference is denoted by  $p$ .

The nondimensional boundary conditions are given as follows

$$x \rightarrow -\infty : (u, v) = (1, 0), \quad \varphi = Y - 1 = Y_O - 1 = p = 0 \quad (7)$$

$$x \rightarrow +\infty : \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial \varphi}{\partial x} = \frac{\partial Y}{\partial x} = \frac{\partial Y_O}{\partial x} = 0 \quad (8)$$

$$y \rightarrow \pm \frac{1}{2} : v = \frac{\partial u}{\partial y} = \frac{\partial \varphi}{\partial y} = \frac{\partial p}{\partial y} = \frac{\partial Y}{\partial y} = \frac{\partial Y_O}{\partial y} = 0 \quad (9)$$

At the wire surface,  $r = (x^2 + y^2)^{1/2} = a/l = \epsilon$ , the following boundary condition apply

$$r = \epsilon : (u, v) = (0, 0), \quad \varphi - \varphi_w = \frac{\partial Y}{\partial r} - \varpi \frac{L_F}{L_O \phi} = \frac{\partial Y_O}{\partial r} - \varpi = 0, \quad (10)$$

where  $\varpi$  is the nondimensional reaction rate at the surface of the wire defined by

$$\varpi = \frac{sW_O L_O l}{\rho_g \alpha_g Y_{O_2 0}} \omega, \quad (11)$$

$s$  is the molar stoichiometric coefficient of the overall surface reaction  $F + sO_2 \rightarrow \text{products} + Q$ ,  $\phi$  corresponds to the equivalence ratio of the mixture at the upstream conditions,  $\phi = (sW_O / W_F) (Y_{F0} / Y_{O_2 0})$ ,  $W_O$  is the molecular weight of the oxidizer and  $\omega$  is the

dimensional reaction rate. The maximum possible value of  $\varpi$ ,  $\varpi_0$ , is such that the fuel concentration vanish at the outlet conditions ( $x \rightarrow \infty$ ), that is

$$\varpi_0 = \frac{Pe \phi L_O}{2\pi\epsilon}. \quad (12)$$

The difference between the fuel concentrations at the inlet and outlet conditions is related to the overall reaction rate as  $1 - \overline{Y_\infty} = \overline{\delta}$ , where  $\overline{Y_\infty} = \lim_{x \rightarrow \infty} (1/2) \int_0^{1/2} Y_\infty(x, y) dy$  and  $\overline{\delta} = \overline{\varpi}/\varpi_0 = 1/(\pi\varpi_0) \int_0^\pi \varpi(\epsilon, \beta) d\beta$ , are the mean values of the fuel concentration at the outlet conditions and the reduced nondimensional reaction rate at the surface of the wire, respectively.  $\beta$  is the angle formed by the radial line from any point in the surface of the wire with the origin and the horizontal line ( $x$  axis).  $\delta$  is therefore the normalized Damköhler number, with  $0 \leq \delta \leq 1$ , thus measuring the overall conversion efficiency. From the overall stoichiometry, the final normalized concentrations of both reactants are related by  $(1 - \overline{Y_\infty}) \phi = 1 - \overline{Y_{O_\infty}}$ . The overall energy conservation gives:  $\varphi_w = (Pe/2\pi\epsilon\overline{Nu})\overline{\varphi_\infty}$ , where  $\overline{Nu}$  is the mean value of the Nusselt number,  $\overline{Nu} = Q_w/2\pi\epsilon\lambda_g(T_w - T_0)$ , where  $Q_w$  is the overall heat loss per unit length of the wire and finally,  $q_E = \overline{\varphi_\infty} - \overline{\delta}$ , where  $q_E$  is the nondimensional externally added heat flux, defined by  $q_E = Q_E/Pe\rho_g\alpha_g C_{pg}(T_e - T_0)$ , with  $Q_E$  being the added external heat flux per unit length of the wire. For not very lean mixtures, the catalytic reaction rate can be shown to be written as<sup>27</sup>

$$\omega = \frac{2\Gamma k_{2a} K^2}{(1 + K)^2}, \quad (13)$$

where  $K = \sqrt{k_{1d}/(k_{1a} - 2k_{2a})}$ , with

$$k_{1d} = \Gamma A_{1d} \exp\left(-\frac{E_{1d}}{RT_w}\right) \quad (14)$$

and

$$k_{ia} = \frac{S_i p Y_{iw} W}{\Gamma W_i^{3/2} \sqrt{2\pi RT_w}}, \quad \text{for } i = 1(H_2), 2(O_2). \quad (15)$$

Here  $\Gamma \simeq 1.6603 \cdot 10^{-9}$  mol/cm<sup>2</sup> is the surface molar concentration of active sites and  $R$  is the universal gas constant.  $E_{1d}$  and  $A_{1d}$  are the activation energy and the preexponential factor of the desorption reaction for the adsorbed atomic hydrogen, respectively. These parameters can be found elsewhere<sup>27</sup>. The rates of the adsorption reactions, on the other hand, are expressed in terms of sticking probabilities or accommodation coefficients,  $S_i$ , which represent the fraction of the collisions with the surface that leads to successful adsorption. The rate of collisions for species  $i$  is  $Z_w = p_i/\sqrt{2\pi m_i kT}$ , from classical kinetic theory, where  $p_i$  and  $m_i$  are the partial pressure and the mass of species  $i$ , and  $k$  is the Boltzmann constant,  $k = 1.38 \times 10^{23}$  J/K. The rates of the desorption reactions are  $\omega_{1a} = k_{1a}\theta_v^2$ ,  $\omega_{2a} = k_{2a}\theta_v^2$ , and  $\omega_{6a} = k_{6a}\theta_v$ , where  $\theta_v$  is the fraction of empty or vacant

sites. This formulation is only valid for not very lean mixtures, with the adsorption rate of hydrogen being very large compared with that of oxygen, that is  $k_{1a} \gg 2k_{2a}$ , everywhere in the wire surface. The parametric values of the hydrogen mixtures are:  $L_F = 0.3$ ,  $L_O = 1$ ,  $W_F = 2$ ,  $W_O = 32$ ,  $Q = 2.3781 \times 10^5$  J/mol,  $s = 1/2$ .

### 3 Numerical solution of the governing equations

It is preferable to introduce the stream function-vorticity formulation for the heat transfer and catalytic combustion problem, by defining the nondimensional vorticity  $\Omega = \partial v/\partial x - \partial u/\partial y$  and the stream function  $\psi$  such as  $u = \partial\psi/\partial y$  and  $v = -\partial\psi/\partial x$ . The nondimensional governing equations transform to

$$-\Omega = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} \quad (16)$$

$$\frac{\partial\psi}{\partial y} \frac{\partial\Omega}{\partial x} - \frac{\partial\psi}{\partial x} \frac{\partial\Omega}{\partial y} = \frac{1}{\text{Re}} \left[ \frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2} \right] \quad (17)$$

$$\frac{\partial\psi}{\partial y} \frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x} \frac{\partial\theta}{\partial y} = \frac{1}{\text{Pe}} \left[ \frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} \right]. \quad (18)$$

$$\frac{\partial\psi}{\partial y} \frac{\partial Y}{\partial x} - \frac{\partial\psi}{\partial x} \frac{\partial Y}{\partial y} = \frac{1}{\text{Pe} L_F} \left[ \frac{\partial^2 Y}{\partial x^2} + \frac{\partial^2 Y}{\partial y^2} \right]. \quad (19)$$

$$\frac{\partial\psi}{\partial y} \frac{\partial Y_o}{\partial x} - \frac{\partial\psi}{\partial x} \frac{\partial Y_o}{\partial y} = \frac{1}{\text{Pe} L_O} \left[ \frac{\partial^2 Y_o}{\partial x^2} + \frac{\partial^2 Y_o}{\partial y^2} \right], \quad (20)$$

where  $\theta = \varphi/\varphi_w$ , to be solved with the corresponding boundary conditions. The Cartesian coordinate system  $(x, y)$  is not well suited for the studied problem. It is then convenient to transform the Cartesian physical coordinate system to a boundary-fitted coordinate, where the grid lines coincide with the body contour in the flow region. This new grid can provide an accurate representation of the boundary conditions in the region of greatest sensitivity. Thompson et al.<sup>29,30</sup> introduced a method for numerical generation of curvilinear coordinate systems with coordinate lines conformal with all boundaries of arbitrarily shaped bodies in a two-dimensional flow region. In this grid generation technique, the Cartesian coordinates  $(x, y)$  of the boundary-fitted grid points in the complex physical space are computed as solutions of an elliptic system of partial differential equations with Dirichlet boundary conditions. They proposed Poisson equations for the coordinate generating system, such as

$$\begin{aligned} \frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2} &= P(\xi, \eta) \\ \frac{\partial^2\eta}{\partial x^2} + \frac{\partial^2\eta}{\partial y^2} &= Q(\xi, \eta), \end{aligned} \quad (21)$$

where  $P(\xi, \eta)$  and  $Q(\xi, \eta)$  are to be selected. Since it is desired to execute all numerical computations in the uniform rectangular transformed plane, the roles of the dependent and independent variable must be interchanged in equation (21). This yields an elliptic system of non linear equations in the transformed plane

$$\begin{aligned} \alpha \frac{\partial^2 x}{\partial \xi^2} - 2\beta \frac{\partial^2 x}{\partial \xi \partial \eta} + \gamma \frac{\partial^2 x}{\partial \eta^2} &= -J^2 \left( P(\xi, \eta) \frac{\partial x}{\partial \xi} + Q(\xi, \eta) \frac{\partial x}{\partial \eta} \right) \\ \alpha \frac{\partial^2 y}{\partial \xi^2} - 2\beta \frac{\partial^2 y}{\partial \xi \partial \eta} + \gamma \frac{\partial^2 y}{\partial \eta^2} &= -J^2 \left( P(\xi, \eta) \frac{\partial y}{\partial \xi} + Q(\xi, \eta) \frac{\partial y}{\partial \eta} \right) \end{aligned}$$

where  $\alpha, \beta$ , and  $\gamma$  are the deformation coefficients

$$\begin{aligned} \alpha &= \left( \frac{\partial x}{\partial \eta} \right)^2 + \left( \frac{\partial y}{\partial \eta} \right)^2 \\ \beta &= \frac{\partial x}{\partial \eta} \frac{\partial x}{\partial \xi} + \frac{\partial y}{\partial \eta} \frac{\partial y}{\partial \xi} \\ \gamma &= \left( \frac{\partial x}{\partial \xi} \right)^2 + \left( \frac{\partial y}{\partial \xi} \right)^2 \end{aligned}$$

and  $J$  denotes the Jacobian of the transformation

$$J = x_\xi y_\eta - x_\eta y_\xi \quad (22)$$

Utilizing exponential functions to achieve the attraction of coordinate lines to the contour of the complex geometry

$$P(\xi, \eta) = \sum -a_i \text{sign}(\xi - \xi_i) \exp(-c_i |\xi - \xi_i|) \quad (23)$$

$$Q(\xi, \eta) = \sum -b_i \text{sign}(\eta - \eta_i) \exp(-d_i |\eta - \eta_i|) \quad (24)$$

employing the appropriate constants  $a_i, b_i, c_i$ , and  $d_i$ , a nearly orthogonal boundary-fitted coordinate system is obtained on the cylinder, while in the rest of the boundaries, the system of coordinates is orthogonal. Fig. 2 shows a typical near orthogonal grid generated for this problem.

### 3.1 Governing Equations in Boundary-Fitted Coordinate Systems

Using the transformation in equation (22), the transport, energy and concentration equations are, respectively

$$-\tau \frac{\partial \psi}{\partial \xi} - \sigma \frac{\partial \psi}{\partial \eta} = \alpha \frac{\partial^2 \psi}{\partial \xi^2} + \gamma \frac{\partial^2 \psi}{\partial \eta^2} - \left( 2\beta \frac{\partial^2 \psi}{\partial \eta \partial \xi} - J^2 \Omega \right) \quad (25)$$



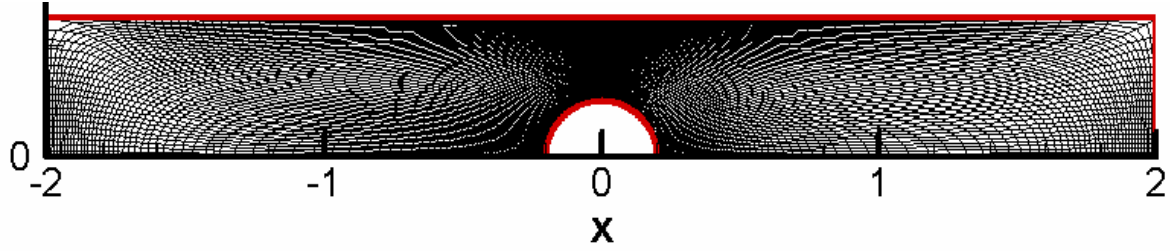


Figure 2: Nearly orthogonal grid structure for a value of  $\varepsilon = 0.2$ .

$$\left[ J^2 \operatorname{Re} \frac{\partial \psi}{\partial \eta} - \tau \right] \frac{\partial \Omega}{\partial \xi} + \left[ -J^2 \operatorname{Re} \frac{\partial \psi}{\partial \xi} - \sigma \right] \frac{\partial \Omega}{\partial \eta} = \alpha \frac{\partial^2 \Omega}{\partial \xi^2} + \gamma \frac{\partial^2 \Omega}{\partial \eta^2} - 2\beta \frac{\partial^2 \Omega}{\partial \eta \partial \xi} \quad (26)$$

$$\left[ JPe \frac{\partial \psi}{\partial \eta} - \tau \right] \frac{\partial \theta}{\partial \xi} + \left[ -J^2 Pe \frac{\partial \psi}{\partial \xi} - \sigma \right] \frac{\partial \theta}{\partial \eta} = \alpha \frac{\partial^2 \theta}{\partial \xi^2} + \gamma \frac{\partial^2 \theta}{\partial \eta^2} - 2\beta \frac{\partial^2 \theta}{\partial \eta \partial \xi} \quad (27)$$

$$\left[ JPeL_H \frac{\partial \psi}{\partial \eta} - \tau \right] \frac{\partial Y}{\partial \xi} + \left[ -J^2 PeL_H \frac{\partial \psi}{\partial \xi} - \sigma \right] \frac{\partial Y}{\partial \eta} = \alpha \frac{\partial^2 Y}{\partial \xi^2} + \gamma \frac{\partial^2 Y}{\partial \eta^2} - 2\beta \frac{\partial^2 Y}{\partial \eta \partial \xi} \quad (28)$$

$$\left[ JPeL_o \frac{\partial \psi}{\partial \eta} - \tau \right] \frac{\partial Y_o}{\partial \xi} + \left[ -J^2 PeL_o \frac{\partial \psi}{\partial \xi} - \sigma \right] \frac{\partial Y_o}{\partial \eta} = \alpha \frac{\partial^2 Y_o}{\partial \xi^2} + \gamma \frac{\partial^2 Y_o}{\partial \eta^2} - 2\beta \frac{\partial^2 Y_o}{\partial \eta \partial \xi} \quad (29)$$

where the coefficients are given by

$$\begin{aligned} \sigma &= \frac{1}{J} \left( \frac{\partial y}{\partial \xi} D_x - \frac{\partial x}{\partial \xi} D_y \right) \\ \tau &= \frac{1}{J} \left( \frac{\partial y}{\partial \eta} D_x - \frac{\partial x}{\partial \eta} D_y \right) \\ D_x &= \alpha \frac{\partial^2 x}{\partial \xi^2} - 2\beta \frac{\partial^2 x}{\partial \xi \partial \eta} + \gamma \frac{\partial^2 x}{\partial \eta^2} \\ D_y &= \alpha \frac{\partial^2 y}{\partial \xi^2} - 2\beta \frac{\partial^2 y}{\partial \xi \partial \eta} + \gamma \frac{\partial^2 y}{\partial \eta^2} \end{aligned}$$

Considering that the equations of the transformation and the physical system are elliptic, only one method of solution can be employed. The system can be solved in an implicit form by discretizing using centered finite differences. The diffusion terms of the equations are used to iterate in order to obtain the convergence of the solution of the system.

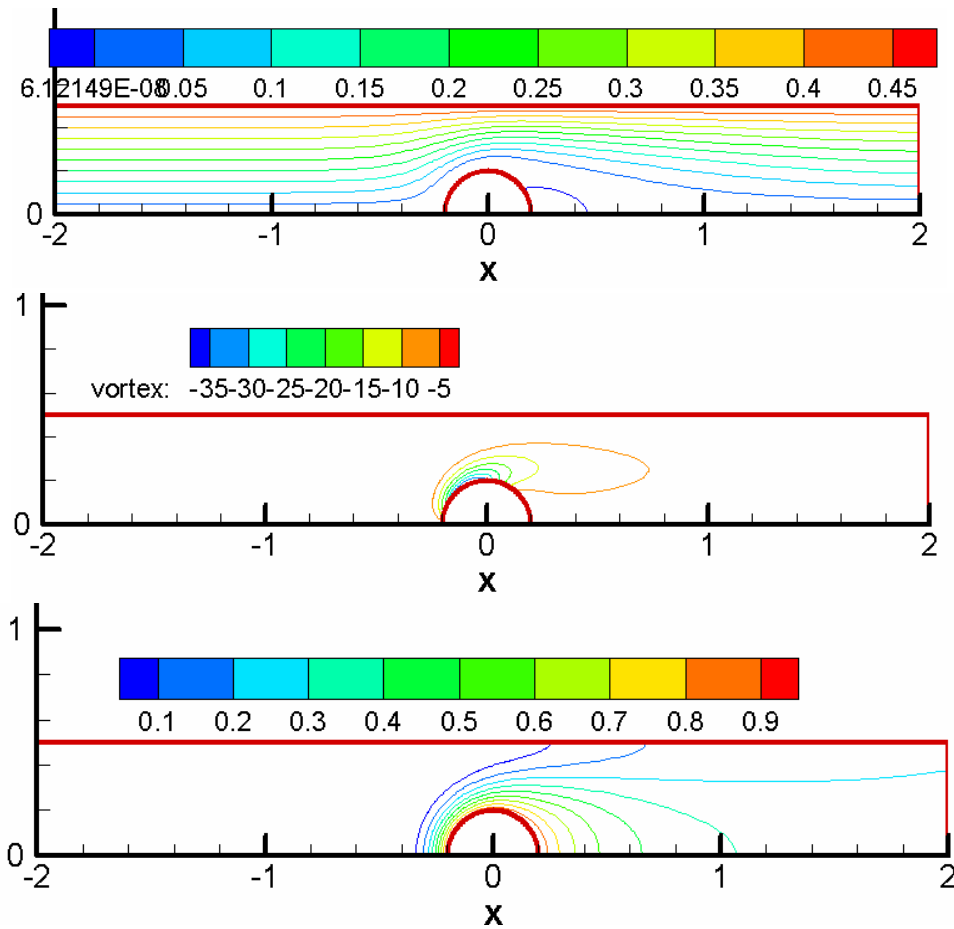


Figure 3: Numerical results of the stream function, vorticity and temperature fields for  $Re=40$  and  $Pr=0.72$ .

## 4 Results and Conclusions

All the numerical calculations shown in this paper have been obtained for a wire radius of  $a = 0.02$  cm. Fig. 3 shows the results obtained for the nondimensional stream function  $\Psi$ , the vorticity  $\Omega$  and the temperature  $\theta$  obtained for a Reynolds number of  $Re = 40$ , a Prandtl number of  $Pr = 0.72$  and a nondimensional wire radius of  $\varepsilon = 0.2$ . A recirculation bubble can be seen at the back of the wire. Using these profiles, the mass fractions for the hydrogen and oxygen can be obtained after solving the corresponding mass conservation equations for both species. For a given wire temperature we can compute the overall nondimensional surface reaction rate  $\bar{\delta}$ , the nondimensional heat loss per unit length of the wire,  $q_w$  and the external heat per unit length of the wire,  $q_E$  that must be added to maintain the assumed wire temperature. Fig. 4 shows the nondimensional external

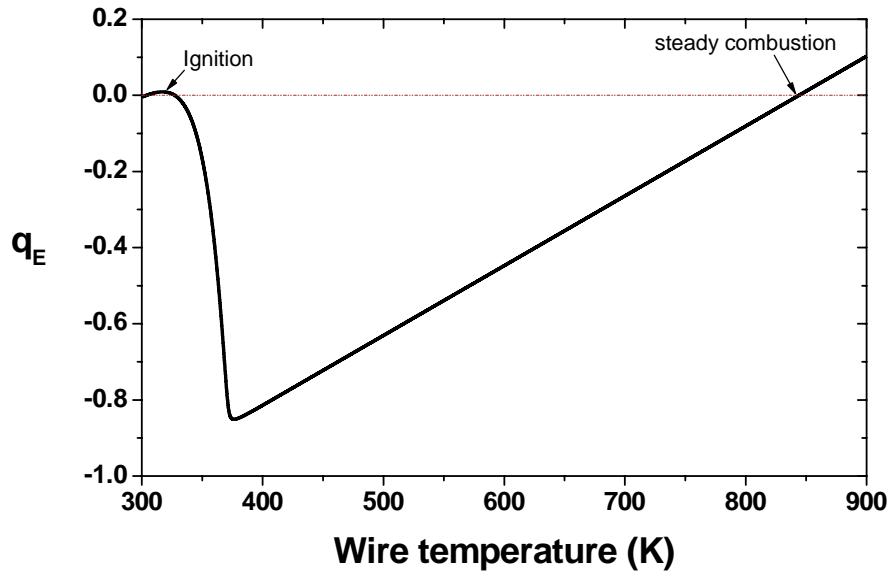


Figure 4: Nondimensional external heat flux per unit length of the wire as a function of the wire temperature.

heat flux per unit length of the wire  $q_E$  as a function of the wire temperature, showing the ignition condition ( $dq_E/dT_w$ ) and the steady combustion condition with  $q_E = 0$ . Finally, in Fig. 5 it is shown the ignition temperature as a function of the Peclet number. The lower line corresponds to the wire temperature without considering the surface reaction heat release, while the upper is considering it. There is a minimum in the Peclet number of 0.056 below which the catalytic ignition takes place at room temperature. As the Peclet number increases, the ignition temperature also increases, indicating the need of external energy to heat up the wire to reach the required minimum wire temperature. The open rectangles are obtained using the asymptotic limit of low Peclet numbers. The solid quadrangles are obtained numerically using the techniques explained in this paper.

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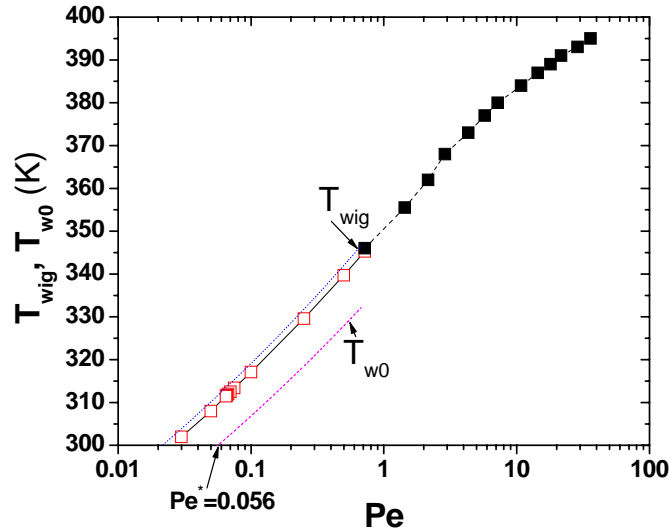


Figure 5: Ignition wire temperature as a function of the Peclet number for a highly diluted hydrogen/air mixture.

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