KNOWLEDGE DIFFUSION PATHS IN A RESEARCH CHAIN

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Abstract. Knowledge impacts all human actions and is responsible for survive and development of human race. Knowledge is both cause and solution for human’s changes in the environment and the technological development. It is transmitted by a nested complex net, with nested relations closely related to human knowledge generation. Analysis and solutions of how knowledge is generated and permeates into a research chain needs innovation, mainly in the context of comprehension of knowledge generation, knowledge diffusion and knowledge transferring processes. In this work we deal with the behavior of knowledge diffusion processes in a research chain, in the sense of the influence of preferential paths of knowledge diffusion among the chain net. A knowledge chain is adopted as the environment in which a research group deals and is affected by those phenomena. We present a model, developed from the continuum formulation, which established a partial difference system of equations that guide the problem, to the discrete formulation. This approach allows a numerical study, in finite elements, of knowledge generation and diffusion, together with the consideration of those paths inside the research chain. It is identified with the knowledge preferential connections and transfer like process as result of researches leaders’ efforts, result of human resources formation and knowledge transferring to the others researches of the chain. Accumulation and feedback of knowledge as also as the paths of knowledge transfer are considered. Some numeric experiments are given, related to scientific knowledge generation data, as also with the changes in diffusion coefficients, generation behavior, and knowledge inductance. The model considers a transient two dimension domain as the researches chain of knowledge. Results behavior denotes the influence of the connection and the preferential diffusion paths. We consider that this approach could be, also, a step to develop a computational model concerning to the evaluation of innovation impact in human society.
1 INTRODUCTION

Knowledge permeates and guides every action of human kind. Across all the evolution of the self called humanity, conditioning our technologic and social development, knowledge allows the solution of the problems the human society deals with. In this paper we present a computational model intended to help understanding the process of knowledge transfer in a research chain. That net of relations and individual researches or research groups belongs to a large amount of adjacent segments of a same society or possibly to the interaction process between different societies (Bevilacqua, 1980, 2001). A relatively recent sequence of papers (Cavalli-Sforza, et al., 1981, 1982, 1993) exposed ideas concerning to the quantification of knowledge transferring process, related to cultural transmission networks, inside and between social groups.

We consider that as ideas the behavior of this problem have been explored for about the last thirty years. Our target is the study of the influence of paths of preferential diffusion propagation at the knowledge chain, as a component of the net of relations between researches of the chain, evaluating the influence of the transmission, diffusion and generation, together with the parameters that introduces those aspects in the problem and play main roles in those processes.

A general complete approach (Cavalli-Sforza, 1993; Feldmann et al., 1994, 1995) considers both genotype and phenotype factors to study the evolution of social groups. That is, inheritance and social interaction are both shaping the history of the group. Inductance, diffusion and generation of knowledge are key aspects of social interaction in the related chain net.

We are interested here in a relatively short period of time, through interaction among groups or sub-groups, the horizontal aspect of the knowledge diffusion problem, that constitutes the predominant factor. In this way, some examples will be showed and analyzed, considering a two dimension domain that represents the research chain. In this domain, the diffusion, generation and transmission of knowledge occur between the researches of this net. The problem is considered full transient. The mathematical model incorporates a diffusion coefficient, a generation, and a knowledge inductance, or impedance, coefficient as the parameters that guide the diffusion of knowledge in the research chain.

Results behavior denotes the influence of the connection and of the preferential diffusion paths. In the section 2 the mathematical model is considered, together with an analysis of the physical meaning of the knowledge conditioning parameters. Section 3 contains the variational and finite element model, with considerations about the solution method. At section 4 some numerical examples are developed and comments are related to the computational solutions of the problem.

The approach adopted in this work is a step in the development of the study of the knowledge propagation in social groups. Other steps will be needed until a computational model concerning to the evaluation of innovation impact in human society could be operational. In this way retention terms must be considered, individual cognitive process must be modeled, and correlation with experimental data is recommended to validate the computational solutions, as most near as possible to the real behavior of the problem. This goal could allow the design of a model to target some grants and revenues in research support.

2 MATHEMATICAL MODEL: A TRANSIENT DIFFUSION MODEL

The equation that guides the transient diffusive problem of knowledge transfer, with generation, reads:
In eq. (1) we have: $u(x,y,t)$ as the function of knowledge level, or knowledge density in time, of the researches or research groups of the chain net, $c_p$ as the cognitive inductance, or what is the same, the cognitive impedance, $K_0$ being the diffusion coefficient, i.e., cognitive permeability, of the chain net, and $r = \alpha C_0$ to be the coefficient of knowledge generation of the researches or research groups. In this approach, div (...) and $\nabla$ (...) are respectively the divergence and gradient operators, restricted to the bi-dimensional chain domain, as considered. Into this domain the researches or groups are distributed in an uniform density to this step of the model.

The analysis of the qualitative behavior of the chain, considers the relations and the influence of the cognitive impedance, diffusion and generation across the net. In the sense of development of a sensitivity analysis of the problem to the variation of those key parameters, we consider a non-uniform evaluation to $c_p$, $K_0$ and $r$. With this approach, those functions are defined as:

$$
c_p \frac{\partial u}{\partial t} = \text{div} \left( K_0 \nabla u \right) + \alpha C_0 u
$$

Those parameters have the following meaning:

- $K_0$: Cognitive Permeability. It controls the knowledge flow in the production chain. It depends of the net researches competence to transfer knowledge from one segment to another and the willingness of the individuals in the group to teach and learn. Increasing values of $K_0$ leads to increasing values of the knowledge flow in the chain.

- $C_0$: Creativity. This parameter reflects the creativity inherent to the groups of individuals in the chain. It translates the amount of knowledge produced. The higher the value of $C_0$, the more creative will be the individuals in the group.

- $c_p$: Cognitive impedance. This parameter expresses the velocity of knowledge absorption in the chain. The lower the value of $C_p$ the faster will be the ability of individuals or groups to learn and master new knowledge. High values of $C_p$ mean slow learning capacity. It is a kind of intellectual inertia or impedance, meaning not creativity. It is possible to have a creative person with relatively low ability to learn.

- $\alpha$: Investment or any other mechanism intended to enhance productivity. This parameter is related to the investments and incentives to maximize the output of each segment in the production chain.

Knowledge is, in this approach, analogous to a form of energy that flows along the chain. The difference in knowledge production between two segments generates knowledge flow in these segments. In a perfectly, hypothetic, homogeneous society where the knowledge production would be evenly distributed, no knowledge flow could occurs.

The knowledge level, $u$, in this model, is a continuous function that answers to the impact of the knowledge density, in the bi-dimensional domain. This function is also the measurement of the variation of knowledge in time at each cell of the knowledge network. In this sense a point or cell of this space may contains one or more than one researches, with it’s own level of knowledge. Knowledge density at a section cell of the network net represents the characteristic knowledge level as result of the knowledge production, proper to each segment in the chain, affected by cognitive processes, ranging from scientific papers to hardware.
With this consideration, the flow is always in the opposite direction of increasing \( u(x,y,t) \). In other words, the flow runs in the direction of decreasing potentials, from plus to minus. As a consequence, there is no privileged flow direction regarding the nature of the output of the different groups in the chain. Therefore, knowledge flow intensity is proportional to \( \nabla u(x,y,t) \), to \( K_0(x,y) \), by \( c_p(x,y) \), and by the intensity of knowledge generation, \( r(x,y) \). Meanwhile \( K_0 \) is the proportionality factor of knowledge diffusion from one point to another of the knowledge network.

### 3 VARIATIONAL AND FINITE ELEMENT FORMULATIONS

Let the knowledge transfer network domain be represented by \( \Omega \subset R^2 \), the bi-dimensional real space. This domain refers to a planar chain of interconnected research cells, as established before, where each one is connected to their neighborhood, meaning that there is interchange of knowledge only if there are connections between researches or groups of this social chain. So, even if restricted by a boundary, this medium consists of an infinity number of points, or cells, whole coupled by the diffusion and induction cognitive processes. Let us denote:

\[
\begin{align*}
\mathcal{U} & = \{ w(x,y,t) \in H^1(\Omega) \mid w = \mathcal{W} \text{ in } \Gamma = \partial \Omega \} \\
\hat{u} & = \{ w(x,y,t) \in H^1_0(\Omega) \mid w = 0 \text{ in } \Gamma = \partial \Omega \}
\end{align*}
\]

respectively the spaces of trial, \( u \), and weight functions, \( \hat{u} \), for a variational formulation of (1), where \( \mathcal{W} \) is a prescribed value in the boundary, \( \Gamma \), of the domain.

An implicit formulation was adopted to deal with the solution of the diffusive problem with generation of knowledge. A Galerkin formulation was then developed to deal with the weak form of eq. (1). To develop a finite element formulation of the problem given by eq. (1), we also assume that \( U^h \) is the finite subspace counterparts of \( U \). A mesh of finite elements was then adopted, to solve the problem in a finite number of points of the discrete network domain. At this point, the domain \( \Omega \) was then discretized into a discrete number of finite elements. Defining a partition \( P^h(\Omega) \) given by a limited number, \( nelm \), of triangular elements on \( \Omega \), that are subdomains of the whole domain, such that the new discrete domain is given by:

\[
\overline{\Omega} = \bigcup_{\epsilon = 1}^{nelm} \Omega^\epsilon
\]

Considering that, in eq. (4), each of the subdomains \( \Omega^\epsilon \) has a smooth piecewise boundary, such that \( \Gamma = \partial \Omega^\epsilon \), and \( h \) be the characteristic mesh size (Donea and Huerta, 2003).

The solution is given in each \( n^{th} \) time step, with respect to the \( (n-1)^{th} \) of knowledge density, \( u \), in the nodes of the mesh. Under these definitions the finite element semi-discrete formulation of the continuous problem (1), reads:

- For each \( t \in [0,T_{\text{max}}[ \), given \( (\alpha,C_0,c_p,K_0) \), find \( u^h \in U^h \), such that for \( \forall \hat{u}^h \in U^h_0 \):

\[
\begin{align*}
\int_{\Omega^\epsilon} c_p \left( u^{h,n} \hat{u}^h - u^{h,n-1} \hat{u}^h \right) d\Omega^\epsilon + \int_{\Omega^\epsilon} K_0 \nabla u^{h,n} \cdot \nabla \hat{u}^h d\Omega^\epsilon = \\
\int_{\Omega^\epsilon} \alpha C_0 u^{h,n} d\Omega^\epsilon + \int_{\Gamma} K_0 \hat{u}^h (n \cdot \nabla u^{h,n}) d\Gamma
\end{align*}
\]

for:

\[
\begin{align*}
u^h &= \overline{u} \quad \text{on } \Gamma \\
u(x,t,0) &= u_0 \quad \text{in } \overline{\Omega} \text{ at } t = 0
\end{align*}
\]
4 RESULTS AND COMMENTS

To evaluate the efficiency of the proposed computational model three numerical experiments were performed.

All of the experiments performed on those three examples are given at an arbitrary adopted 5 x 5 unities square domain, with an initial uniform condition of \( u_0 = 0 \) in all this domain; with exception of the regions that represents the main researches knowledge density concentrations, where some arbitrary values distinct of \( u_0 \) were given. Those regions are five circular areas in the x-y plane, of radius 0.2. The main one is in the geometric center of the domain, and the others four, accordingly with the examples characteristics are centered at
The domain is discretized in 10,784 P1 triangular finite elements, where the P1 element is concerned to a linear polynomial interpolation function. The maximum time of simulation may be variable for the experiments, but was fixed in this work for the three experiments, at \( T_{\text{max}} = 1 \), and is representative of the time evolution of the knowledge chain, with time discretization given in one hundred time steps.

Figure 2 – Two knowledge main nuclei linked by a preferential diffusion path: finite element mesh (left), and initial condition (right)

The first experiment is associated with the check of the validity of the solution as the knowledge chain net evolves in time, as resultant of the diffusion of the knowledge in the bi-dimensional chain domain. In fig. 1 is showed the mesh for a one central nucleus of knowledge density from which the knowledge is affected by diffusivity as time runs on, as also the isolines of the evolution of the knowledge chain. The knowledge density of this center is 8 and zero into the outside of this nucleus. Cognitive impedance was taken as 1.25 in all the whole domain, and knowledge generation as 1.95 only in central nucleus and zero from its periphery to the border. A null boundary condition was adopted. We consider that the computational model presents a well evolution behavior, with a symmetric pattern of isolines, for the only presence of a central knowledge density in the research network.

The second experiment consists of a main region and a secondary one, formally nuclei of knowledge, where initial knowledge level is 8.0 and 3.0, respectively, which are distinct from that of the whole domain that is zero. In fig. 2 are viewed the finite element mesh, in left side, and the isolines of the initial condition. Boundary condition is a Dirichlet one and taken as zero in the four boundaries. The central (main) and left side (secondary) regions of initially concentrated knowledge density are connected by a preferential path where knowledge diffusion is taken as 57% higher than the diffusion coefficient outside of this path which value is one. Cognitive inductance is also taken with unitary value and knowledge generation was not considered in this example.

The evolution of the knowledge network of this second experiment is seen in fig.3 at four time steps, from top left in clockwise direction. We see that the preferential path, with a greater diffusivity, is favorable to the growth of the secondary group of knowledge.
concentration that grows from 3 to 5.4 as time goes on. The connection between those knowledge groups of cells is evident with a strong density of isolines evolving across the preferential path. The impulse to the growth of the knowledge density in the whole knowledge chain is also showed, with isolines dispersing from the two nuclei to the border.

The last example deals with a knowledge chain domain with five nuclei of initial knowledge level, with distinct values from that of the rest of the network. The central knowledge tower is at 8.0 level when the chain evolution begins, and the four laterals concentrations initially at 3.0 level, while the region outside of those five research concentrations has an initial zero density of knowledge, as the border condition at the frontier of the domain. In this network, the right and the left positioned regions of knowledge density are connected by a path of preferential diffusion with the central main region of knowledge

Figure 3 – Isolines of evolution of knowledge level of a two knowledge nucleus connected by a preferential path. Beginning in left to, and following the clockwise direction: 0.02, 0.1, 0.5 and 0.99 time steps
density. The groups of cells in the top and in the bottom of the domain have not connection with preferential paths or diffusion with the domain outside themselves. Cognitive impedance is fixed at one for the entire domain, and knowledge generation is at 1.00, except for the non-preferential paths non-connected regions, in which it is zero.

The preferential path connection is determinant for the development behavior of the knowledge network. The knowledge diffuses mainly along this path, and as a locally isolated density for the groups non-connected by a preferential path. The absence of interaction between those (in top and in bottom) knowledge densities, with the outside space, indicates that they do not perform with a growth in their knowledge behavior as time grows up.

Figure 4 – Five knowledge main nucleus linked by one preferential diffusion path, with two non-linked nucleus: finite element mesh (top left), and evolution of the network: at 0.01 time step (top right), at 0.45 (bottom right) and 0.85 (bottom left) time steps

The absence of interaction of a social group, as in research, is an indication of the stagnation of the behavior of the group. In other sense, groups that has strong interactions and knowledge exchange with a more developed one, with relation to knowledge density, have a
high growth performance in it’s knowledge density level, as is the situation between the central and the laterals groups in the last example. In this panorama the laterals preferential paths connected social groups evolve from a 3.00 knowledge density, at 0 time step, to more than 44.0, at time step 1.00, when they permutated knowledge continuously with a strong research group, such as the central main group, which develops also to this last step of knowledge density.

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